# Dividing with indices

Students will explore simplifying expressions by dividing numerical terms with indices. They will investigate why a numerical term with a zero index is equal to one.

## Visible learning

The learning intention and success criteria are to be shared after the lesson Launch.

### Learning intention

* To understand how to apply the index laws to simplify numerical expressions.

### Success criteria

* I can write numbers in index form.
* I can simplify numerical expressions using the division-index law.
* I can simplify numerical expressions using the zero-index law.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* represents and operates with fractions, decimals and percentages to solve problems
**MA4-FRC-C-01**
* operates with primes and roots, positive-integer and zero indices involving numerical bases and establishes the relevant index laws **MA4-IND-C-01**

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Table 1: lesson summary

|  |  |  |  |
| --- | --- | --- | --- |
| ****Section**** | ****Summary of activity**** | ****Teaching strategies**** | ****Teaching points**** |
| ****Launch**** | Students are asked to find an expression that equates to one when given 2 fives, or 3 fives, 4 fives, 5 fives or 6 fives. | Visibly random groups of 3Vertical non-permanent surfacesGallery walk | Students are reminded of fractions that are equivalent to one. |
| ****Explore**** | Students are given the expression $3^{8}÷3^{3}$ and are asked to write that expression in as many ways as possible before discussing each approach as a class. Students then complete the banner task questions from [Appendix A](#_Appendix_A) to develop a pattern to show the index laws involving division and the zero-index law. | Visibly random groups of 3Vertical non-permanent surfacesThink-Pair-ShareNotice and wonder | Students devise the divisional index law and the zero-index law by expanding and eliminating fractions equivalent to one. |
| ****Summarise**** | Students look at different ways to simplify an expression from slide 5 from the PowerPoint Dividing with indices before making notes on shortcuts and consolidating rules through the card sort activity from [Appendix B](#_Appendix_B). | Pose-Pause-Pounce-BounceNotes to future forgetful selves | Students formalise the rules used to simplify expressions and draw out the elimination of fractions equivalent to one. |
| ****Apply**** | Students return to the Launch section to apply what they now know about division and zero-index laws. Students create questions using the index laws, including ones that involve multiple terms. | Visibly random groups of 3Vertical non-permanent surfacesGallery walkTwo stars and a wish | Students use the index laws for dividing with indices and the zero-index and revisit the multiplying and power of a power index laws. |

### Activity structure

Please use the associated PowerPoint Dividing with indices to display images in this lesson.

### Launch

1. Assign students to visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) and position groups at vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)) around the classroom.
2. Ask groups to write an expression that equals one using: 2 fives, 3 fives, 4 fives, 5 fives and 6 fives.
3. After some time, allow students to do a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) to observe other students' work.
4. Unpack any answers drawing out the unit fraction in a class discussion.

$$\frac{5}{5}=1,\left(\frac{5}{5}\right)^{5}=1, \frac{5}{5}×\frac{5}{5}=1,\left(\frac{5}{5}\right)×\left(\frac{5}{5}\right)^{5}=1,\frac{5}{5}×\frac{5}{5}×\frac{5}{5}=1$$

1. Reveal the learning intention and success criteria to students. Inform students that they will develop a new method through this lesson.

### Explore

1. Write the expression $3^{8}÷3^{3}$ on the board and ask students how many ways they can write this expression.
2. Using a finger vote, ‘one’ for one way, ‘2’ for 2 ways and so on, ask students to indicate how many different ways they have found to write the expression.
3. Starting with a student with one way, have them write it on the board. Continue choosing different students until there are no more strategies left from the class to show.
4. Display slide 3 from the PowerPoint Dividing with indices and have students consider, in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), which expression would help them solve the equation without a calculator.

Students should recognise that writing the expression as a fraction and grouping unit fractions together would be helpful.

1. Back in their visibly random groups of 3 at their non-permanent surfaces, have students complete the banner task questions from Appendix A ‘Banner task questions’. ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)).
2. Bring the class back together and ask students what they noticed and wondered ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)).

Students should have noticed that when dividing indices with the same base, the powers can be subtracted. They may wonder what will happen if the 2 indices are the same.

1. Draw students' attention to the last 2 questions of the banner task. In a Think-Pair-Share, ask students what they notice and wonder about these 2 examples.

Students should notice that when the powers are the same value, they subtract to equal zero but when they eliminated the unit fractions, the answer was one.

1. In pairs, ask students to reason why this will work, no matter which number is used as the base or the power. Students should consider fractions and decimals as the base.

### Summarise

1. Display slide 5 of the PowerPoint Dividing with indices, telling students that it shows how 4 different teachers solved the question $3^{6}÷3^{4}$.
2. In pairs, have students discuss the benefits and limitations of each method.
3. Bring the class back together and use a Pose-Pause-Pounce-Bounce questioning strategy (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) for students to discuss the different methods. Prompting questions could include:
* Which method was the quickest? Which was the best to show understanding? Which was the best to explain to a friend?
* Do you think any of the methods are not acceptable? Why or why not?
1. Students are to create notes to their future forgetful selves ([bit.ly/notestofutureself](https://bit.ly/notestofutureself)) on how to divide numerical terms with indices and the zero index.
2. Distribute Appendix B ‘Card sort’ to pairs of students. Students match the cards to show the division question and answer.

Cards are not in order, so students could be issued with scissors to cut the cards themselves.

The card sort contains questions that will result in a negative index if students are just using the division rule. Students have not been exposed to negative indices yet. Teachers will need to challenge students to return to basics and look for fractions that are equivalent to one to be able to find the matching answer.

### Apply

1. Draw students' attention back to the problem from the Launch section. In a Think-Pair-Share, have students discuss how they could equate the different numbers of fives to equal one.

$$\frac{5}{5}×5^{0}=1,\frac{5}{5}×\frac{5}{5}×5^{0}=1$$

1. Assign students into new visibly random groups of 3 and position groups at vertical non-permanent surfaces around the classroom, with one marker between each group.
2. Pose the following question to students: ‘If $2^{5}$ is the answer, what could the question be?’
3. Write some suggestions on the board.
4. Suggest that these questions are very straightforward and ask what a creative or challenging question could look like.

A discussion may be useful about what a creative or challenging question could look like. The teacher could suggest to students that questions could include more than 2 terms or may use more than one operation.

Groups may benefit from having a calculator to check their questions.

1. Distribute Appendix C ‘Answers for creative questions’ to each group. Tell students that for each answer, they need to create 3 questions: one using division, one using multiplication and one which they believe is creative or challenging.
2. Students are to do a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) of the questions developed and give peer feedback using the Two stars and a wish strategy ([bit.ly/DLSpeerfeedback](https://bit.ly/DLSpeerfeedback)).

## Assessment and differentiation

### Suggested opportunities for differentiation

**Explore**

* Prior to the lesson, the teacher could set up Polypad ([polypad.amplify.com/p](https://polypad.amplify.com/p)) with a visual representation of an expression with a smaller base to visualise the division.
* The task in Appendix A could be modified to include multiple applications of division.
* The task in Appendix A could be modified such that $3$ is written as $3^{1}$.

**Summarise**

* **The teacher could provide a list of word prompts, such as base, index, division and subtraction or a sentence starter such as ‘To divide expressions with the same base …’ to help students write their notes to future forgetful selves.**
* The card sort activity from Appendix B could be simplified by reducing the number of pairs.
* The card sort activity from Appendix B could be modified to increase the challenge by adapting the questions to have cards that have no matches and challenging students to identify matches.
* Students could be challenged to consider what other expressions could be created to match the answer cards.
* Students could be challenged to explore negative indices when these naturally arise during the card sort activity.

**Apply**

* Students should be challenged to connect with prior knowledge such as measurement, including areas, volumes and converting units.
* Students can be extended by being challenged to create an expression, resulting in the creative question’s answer due to a misconception and explain the error.

### Suggested opportunities for assessment

**Launch**

* The teacher can take photographs of students’ work from the vertical non-permanent surfaces during the gallery walk.

**Explore**

* The teacher can take photographs of students’ work in the banner task questions to reflect on understanding of the index laws.

**Summarise**

* The teacher could make a copy of students’ notes for their future forgetful selves.
* Check that students have correctly matched the expressions on the cards. Show particular focus to the cards that have a base of 3, as these could reflect the misconception of dividing the numbers in the index.

**Apply**

* When placed in visibly random groups of 3, students provide and receive peer feedback on their understanding.
* Monitor responses when sharing solutions to check for student understanding of dividing indices and the zero index.

## Appendix A

### Banner task questions

1. $\frac{5^{6}}{5^{2}}$
2. $\frac{3^{4}}{3^{2}}$
3. $\frac{4^{4}}{4}$
4. $3^{5}÷3^{4}$
5. $3^{7}÷3^{5}$
6. $3^{5}÷3^{1}$
7. $7^{6}÷7^{4}$
8. $7^{9}÷7^{3}$
9. $4^{7}÷4^{4}$
10. $3^{5}÷3^{5}$
11. $7^{3}÷7^{3}$
12. $(\frac{2}{3})^{4}÷(\frac{2}{3})^{4}$

## Appendix B

### Card sort

Cut the cards below and arrange them in matching pairs.

|  |  |  |  |
| --- | --- | --- | --- |
| $$5^{9}÷5^{2}$$ | $$5^{5}÷5^{3}$$ | $$3^{8}÷3^{2}$$ | $$3^{5}÷3^{2}$$ |
| $$3^{8}÷3^{6}$$ | $$5^{3}÷5^{8}$$ | $$5^{5}÷5^{4}$$ | $$3^{8}÷3^{4}$$ |
| $$3^{3}÷3^{8}$$ | $$5^{5}÷5^{5}$$ | $$5^{2}$$ | $$3^{4}$$ |
| $$1$$ | $$3^{2}$$ | $$3^{3}$$ | $$5^{7}$$ |
| $$\frac{1}{5^{5}}$$ | $$\frac{1}{3^{5}}$$ | $$5$$ | $$3^{6}$$ |

## Appendix C

### Answers for creative questions

|  |  |  |  |
| --- | --- | --- | --- |
| Answer | Dividing with indices | Multiplying with indices | Creative or challenging question |
| 1. $2^{8}$
 |  |  |  |
| 1. $3^{4}$
 |  |  |  |
| 1. $4^{4}$
 |  |  |  |
| 1. $5^{3}$
 |  |  |  |
| 1. $1$
 |  |  |  |
| 1. $3$
 |  |  |  |
| 1. $4$
 |  |  |  |
| 1. ****6****
 |  |  |  |
| 1. $\frac{2^{3}}{3^{2}}$
 |  |  |  |

## Sample solutions

### Appendix A – banner task questions

1. $\frac{5^{6}}{5^{2}}=5^{4}$
2. $\frac{3^{4}}{3^{2}}=3^{2}$
3. $\frac{4^{4}}{4}=4^{3}$
4. $3^{5}÷3^{4}=3$
5. $3^{7}÷3^{5}=3^{2}$
6. $3^{5}÷3^{1}=3^{4}$
7. $7^{6}÷7^{4}=7^{2}$
8. $7^{9}÷7^{3}=7^{6}$
9. $4^{7}÷4^{4}=4^{3}$
10. $3^{5}÷3^{5}=3^{0}=1$
11. $7^{3}÷7^{3}=7^{0}=1$
12. $(\frac{2}{3})^{4}÷(\frac{2}{3})^{4}=\frac{2}{3}^{0}=1$

### Appendix B – card sort

|  |  |  |  |
| --- | --- | --- | --- |
| $$3^{8}÷3^{6}$$ | $$3^{2}$$ | $$5^{5}÷5^{4}$$ | $$5$$ |
| $$3^{3}÷3^{8}$$ | $$\frac{1}{3^{5}}$$ | $$5^{3}÷5^{8}$$ | $$\frac{1}{5^{5}}$$ |
| $$3^{5}÷3^{2}$$ | $$3^{3}$$ | $$5^{5}÷5^{3}$$ | $$5^{2}$$ |
| $$3^{8}÷3^{4}$$ | $$3^{4}$$ | $$5^{5}÷5^{5}$$ | $$1$$ |
| $$3^{8}÷3^{2}$$ | $$3^{6}$$ | $$5^{9}÷5^{2}$$ | $$5^{7}$$ |

## References

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