# Unitary method

This activity introduces how to use the unitary method to find unknown amounts.

## Visible learning

Learning intentions and success criteria should be shared with students later in the learning episode.

### Learning intention

* To be able to use the unitary method to answer questions involving percentages.

### Success criteria

* I can find a percentage of a number.
* I can find 1% of a quantity.
* I can calculate 100% of a quantity using the unitary method.
* I can calculate any percentage of a quantity using the unitary method.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* represents and operates with fractions, decimals and percentages to solve problems
**MA4-FRC-C-01**

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Table 1: lesson summary

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| --- | --- | --- | --- |
| Section | Summary of activity | Teaching strategy | Teaching points |
| Warm-up | Students work in visibly random groups of 3 to find the highest common factor of 12, 25, and 63, before creating their own lists. | Visibly random groups of 3Vertical non-permanent surfaces | A review to remind students that all numbers have a factor of 1. |
| Launch | Students watch the video [bit.ly/PotatoParadoxProblemEmbedded](https://bit.ly/PotatoParadoxProblemEmbedded) to preview the Potato Paradox before attempting to solve it. | Visibly random groups of 3Vertical non-permanent surfacesGallery walk | Students consider a challenging problem where the unitary method makes the solution easier to find. |
| Explore | In pairs have students attempt the questions from [Appendix A](#_Appendix_A_–) The students engage in a discussion to draw out that all percentages can be reduced to 1. | Goal-free problemsThink-Pair-Share | The purpose of this task is to encourage students to use multiplicative thinking when appropriate but develop a need to divide to make 1. |
| Summarise | Students study worked examples from slides 3 to 6 of the PowerPoint *Unitary method* before completing the questions in [Appendix B](#_Faded_examples_(1)). They then complete four quadrant notes on the unitary method ([Appendix C](#_Frayer_Diagram)). | Worked examples (Your turn)Faded examplesThink-Pair-ShareFour Quadrant Notes | Explicitly teach the unitary method and consolidate this understanding using the four quadrant notes. |
| Apply | Students apply Polya’s problem-solving process([Appendix D](#_Faded_examples_(2))) to a series of cards with word problems ([Appendix E)](#_Problem-solving_cards) before applying their new skills to the potato paradox from the launch. | Polya’s problem-solving processVisibly random groups of 3Vertical non-permanent surfaces  | Extend the unitary method to apply to word problems and solve the paradox from earlier in the lesson. |

### Activity structure

Please use the associated PowerPoint *Unitary method* to display images in this lesson.

### Warm up

1. By working in visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) on vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)), ask students to find the highest common factor of the numbers 12, 25, and 63.
2. Ask students if they can find another set of 3 numbers where 1 is the highest common factor.
3. Use a Pose-Pause-Pounce-Bounce (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) questioning strategy to ask students what they notice about the types of numbers they chose?

This is an opportunity for students to revise that 1 is a factor of all numbers.

### Launch

1. Introduce the Potato Paradox by showing the video ‘The Potato Paradox’ (1:00) ([bit.ly/PotatoParadoxProblemEmbedded](https://bit.ly/PotatoParadoxProblemEmbedded)).
2. Continuing with students in the same groups of 3 as the warm-up activity, give students time to attempt the Potato Paradox question on their vertical non-permanent surfaces.
3. Have students complete a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) to view other group’s work.
4. Reveal to students, the learning intentions and success criteria.

### Explore

1. Distribute Appendix A ‘Percentage questions’ to pairs of students and allow students time to attempt the questions.
2. Once students have attempted most questions, conduct a class discussion using the following questions:
* Which questions did your group find easier?
* Which questions did your group find more challenging?
* What strategies did your group use?

Students are expected to identify that questions from Appendix A that involved factors of 100 were easier to answer than other questions, particularly those involving prime numbers.

1. Using the strategy goal-free problem ([bit.ly/goalfreeproblems](https://bit.ly/goalfreeproblems)), on vertical non-permanent surfaces, ask students to respond to the statement, ‘12% of a quantity is 30. Calculate what you can.’

Students would be expected to realise that if they find 1% of the quantity, they could calculate any other percentage total.

Students should identify that finding 1% has been common in their calculations.

### Summarise

1. Use slides 3 to 6 from the PowerPoint *Unitary method* to explicitly teach the unitary method using the worked examples (Your turn) method ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)).
2. Distribute Appendix B ‘Faded examples’ and have students complete the 5 questions.
3. Initiate a Think-Pair-Share asking students if any of the questions from Appendix B ‘Faded examples’ could be completed differently and why we still use the unitary method for all questions.

Students may have identified that the fourth question could have been completed by dividing through by 4 first and then multiplying by 10. Finding the unit amount is beneficial as it allows mathematicians to find any percentage of the quantity.

1. Students work in their visibly random group of 3, and complete 4 quadrant notes to their future forgetful selves ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)) from Appendix D ‘Four quadrant notes’ on a vertical non-permanent surface.

### Apply

#### Word problems

1. Back in their random groups of 3, distribute Appendix D ‘Problem-solving mat’ in a plastic sleeve with an erasable marker.
2. Use slides 8 to 12 from the PowerPoint to introduce Polya’s problem-solving process ([bit.ly/PoylasPSP](https://bit.ly/PoylasPSP)).
3. Distribute a problem-solving card from Appendix E ‘Problem-solving cards’ to each group and give them time to complete. Students should be encouraged to check their answers by finding the original percentage of the total quantity.

The cards will need to be cut up before the lesson.

1. Once a card has been answered, groups are to exchange their problem-solving card with another group.

#### Potato paradox

1. Ask students if they can recall the details of ‘The Potato Paradox.’

If students are unable to recall details, show the video ‘The Potato Paradox’ (1:00) again ([bit.ly/PotatoParadoxProblemEmbedded](https://bit.ly/PotatoParadoxProblemEmbedded)).

1. In their groups of 3, ask students to draw a Problem-solving mat on their vertical non-permanent surface and apply their understanding of the unitary method to solve the Potato Paradox.
2. After sufficient time has passed for all groups to make a satisfactory attempt at the problem, ask a successful group to share their solution with the class.

If no groups are successful, watch the remainder of the video ([bit.ly/PotatoParadoxProblem](https://bit.ly/PotatoParadoxProblemEmbedded)) and pose the new question, ‘A 50 kg cactus is 98% water. After some rain, the cactus is now 99% water. How much will it weigh?’.

Answer: The cactus will now weigh 100kg.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Warm-up**

* If the teacher notices a misconception, they can temporarily combine 2 groups, telling the groups that at least one of them is incorrect. The teacher then walks away, leaving students to explain and justify their solution.
* The teacher could ask guiding questions, such as ‘What do these numbers have in common?’
* The teacher could encourage the use of Venn diagrams to visualise the lists of factors.
* Students could be challenged to develop sets of numbers with larger numbers.

**Launch**

* Encourage students to try and visually represent the situation.

**Explore**

* Bar models could be used to assist students to visualise some of the questions.
* Additional questions could be provided to explore percentages that have a fraction in the percentage or other decimal numbers.
* In the goal-free problem, the teacher could prompt student thinking by asking them to consider what are the factors and multiples of 12.
* In the goal-free problem, the teacher could challenge students to find unique and challenging percentage values, including values that have fractions and decimals.

**Summarise**

* Students could work in pairs to support progression in the task.
* Questions could be adjusted to a suitable level of challenge for the faded examples task.

**Apply**

* Questions could be adjusted to a suitable level of challenge on the problem-solving cards.

### Suggested opportunities for assessment

**Warm-up**

* The teacher can listen to the discussions and observe the work on the vertical non-permanent surfaces
* The teacher could take photographs of the work as work samples.

**Launch**

* The teacher should listen to the discussions for use of language and problem-solving strategies.

**Explore**

* The teacher can listen to the discussions and observe the work on the vertical non-permanent surfaces
* The teacher could take photographs of the work as work samples.

**Summarise**

* Student work could be collected from either ‘Faded examples’ or ‘Four quadrant notes’ to check student understanding and use of appropriate language.

**Apply**

* Students give and apply feedback on their peer’s work providing opportunities for students to reflect on their learning.
* Teachers could take a photograph of completed Problem-solving mats as evidence of a students working mathematically skills.

## Appendix A

### Percentage questions

1. 20% = 50, find 100%.
2. 10% = 50, find 100%.
3. 5% = 50, find 100%.
4. 25% = 50, find 100%.
5. 200% = 50, find 100%.
6. 40% = 50, find 100%.
7. 75% = 50, find 100%.
8. 30% = 50, find 100%.
9. 0.5% = 50, find 100%.
10. 17% = 50, find 100%.
11. 37% = 50, find 100%.
12. 127% = 50, find 100%.

## Appendix B

### Faded examples

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 17% of a quantity is 82.45. Find 100% of that quantity.  | 5.6% of a quantity is 84. Calculate 100% of that quantity.  | 48% of a quantity is 12. Calculate 100% of that quantity. | 40% of a quantity is 28. Calculate 100% of that quantity.  | 31% of a quantity is 257. Calculate 100% of that quantity.  |
|  |  |  |  |  |

Additional questions

1. 135% of a quantity is 85. Find 100% of that quantity.
2. 23% of a quantity is 64. Find 85% of that quantity.

## Appendix C

### Four quadrant notes

|  |  |
| --- | --- |
| **Example 1**37% of a quantity is 233.1. Calculate 100% of that quantity.  | **Example 2**45% of a quantity is 441. Find 86% of that quantity. |
| **Things to remember** | **Example 3** |

## Appendix D

### Problem-solving mat



## Appendix E

### Problem-solving cards

|  |
| --- |
| A shop sells red, blue, and black pens. The shop sells 148 blue pens in a week, which is 37% of their pen sales. How many pens did they sell in total? |
| There are 8 girls in a Year 8 Maths class, which represents 32% of all children in the class. How many children are in the class? |
| A local soccer player scored 12 goals in the season which represented 30% of her total attempts. How many attempts did she have in the season? |
| A child spent 75% of his savings on video games. If he spent $288 on video games, how much money did he have saved? |
| A small tub of yoghurt has 180 mg of calcium, which is 23% of the recommended dietary intake. How much calcium should an average adult consume each day? |
| A bottle of orange juice has 81 mg of vitamin C which is 180% of the recommended dietary intake for an adult. How much vitamin C should an adult consume each day? |
| % of an amount of money is $75 000. What is 55% of the amount of money? |

## Sample solutions

### Appendix A – banner task

1. 20% = 50, find 100%. Answer = 250
2. 10% = 50, find 100%. Answer = 500
3. 5% = 50, find 100%. Answer = 1000
4. 25% = 50, find 100%. Answer = 200
5. 200% = 50, find 100%. Answer = 25
6. 40% = 50, find 100%. Answer = 125
7. 75% = 50, find 100%. Answer = 66.67 (2dp)
8. 30% = 50, find 100%. Answer = 166.67 (2dp)
9. 0.5% = 50, find 100%. Answer = 10 000
10. 17% = 50, find 100%. Answer = 294.1176471 (7dp)
11. 37% = 50, find 100%. Answer = 135.1351351 (7dp)
12. 127% = 50, find 100%. Answer = 39.37007874 (7dp)

### Appendix B – faded examples

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 17% of a quantity is 82.45. Find 100% of that quantity.  | 5.6% of a quantity is 84. Calculate 100% of that quantity.  | 48% of a quantity is 12. Calculate 100% of that quantity. | 40% of a quantity is 28. Calculate 100% of that quantity.  | 31% of a quantity is 257. Calculate 100% of that quantity.  |
|  |  |  |  |  |

1. 135% of a quantity is 85. Find 100% of that quantity.

100% = 62.96296296…

1. 23% of a quantity is 64. Find 85% of that quantity.

85% = 236.5217391…

### Appendix C – four quadrant notes

|  |  |
| --- | --- |
| **Example 1**37% of a quantity is 233.1. Calculate 100% of that quantity.  | **Example 2**45% of a quantity is 441. Find 86% of that quantity. |
| **Things to remember**A technique used where you find the value of 1 unit (1%) and multiply this to find a particular amount. | **Example 3**22% of a quantity is 154. Calculate 100% of that quantity.  |

### Appendix E – problem-solving cards

A shop sells red, blue, and black pens. The shop sells 148 blue pens in a week, which is 37% of their pen sales. How many pens did they sell in total?

There are 8 girls in a Year 8 Maths class, which represents 32% of all children in the class. How many children are in the class?

A local soccer player scored 12 goals in the season which represented 30% of her total attempts. How many shots did she make in the season?

A child spent 75% of his savings on video games. If he spent $288 on video games, how much money had he saved?

A small tub of yoghurt has 180 mg of calcium, which is 23% of the recommended dietary intake. How much calcium should an average adult consume each day?

A bottle of orange juice has 81 mg of vitamin C which is 180% of the recommended dietary intake for an adult. How much vitamin C should an adult consume each day?

% of an amount of money is $75 000. What is 55% of the amount of money?

## References

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