# Linear equations

Students explore linear relationships through a problem of stacking cups. They use their understanding to develop an equation that models the height of the stack of cups, given the number of cups stacked.

## Visible learning

### Learning intention

* To recognise that a linear relationship can be represented by an equation.

### Success criteria

* I can represent a linear pattern using an equation.
* I can plot linear relationships on the Cartesian plane.
* I can justify a prediction using my understanding of linear relationships.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* creates and displays number patterns and finds graphical solutions to problems involving linear relationships **MA4-LIN-C-01**

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Table 1: lesson summary

|  |  |  |  |
| --- | --- | --- | --- |
| Section | Summary of activity | Teaching strategies | Teaching points |
| Launch | Display slide 3 from the PowerPoint *Linear equations* (LE PPT) or bring in disposable cups. In groups of 3, students observe and discuss the cup’s features, then predict the height of a 10-cup stack. | Think-Pair-ShareNotice and wonderGallery walk | Engage students’ curiosity and introduce the concept of pattern recognition using a concrete manipulative. |
| Explore | Use slide 6 from the PowerPoint (LE PPT) to model constructing Table 2. Discuss which aspects of the stack are constant and what changes. Introduce the equation ‘Height = 5 + number of cups × 1.5’ and have students decide on symbols to use to represent ‘height’ and ‘number of cups’. Students solve for various stack heights, including estimating cups needed to reach the moon. | Pose-Pause-Pounce-Bounce | Guide students in identifying patterns and constructing a table to understand linear relationships and introduce equation building. |
| Summarise | Use slides 12–19 from the PowerPoint (LE PPT) for the explicit teaching of constructing linear equations from tables and graphs. Students write notes to their future forgetful selves. | Worked examples (Your turn)Notes to future forgetful selves | Consolidate understanding of constructing linear equations by explicitly teaching constants and the rate of increase/decrease through worked examples. |
| Apply | In pairs, students complete [Appendix B](#_Appendix_B) by constructing and verifying linear equations from tables and graphs. |  | Allow students to independently practice constructing and verifying linear equations from tables and graphs. |

## Activity structure

Please use the associated PowerPoint *Linear equations* (LE PPT) to display images in this lesson.

### Launch

1. Display Figure 1, which is available on slide 3 of the PowerPoint (LE PPT) or preferably, bring in some disposable cups for students to see and manipulate.

Figure 1: disposable cup



1. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), students discuss what they notice and wonder ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) about the cup.

Students might notice there are 2 parts of the cup: the base and the lip.

Students might wonder:

* Why these cups have a lip?
* How could you stack these cups?
* How much taller is the base than the lip?
* Why are we starting a Maths lesson with a disposable cup?
1. Assign students to visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) at vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)). Provide each group with a set of 3 disposable cups and a ruler.
2. Pose the question to students ‘How tall do you think a stack of 10 cups would be?’

If unable to provide disposable cups to students, slide 4 can be used to display measurements of a sample cup. Appendix A ‘Cup template’ may be printed to provide a template for students to draw on or cut out and manipulate the cup shape.

Students should be able to solve this problem by using a table of values to generate a pattern.

1. Conduct a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) to allow students to observe how their peers approached the task. Encourage students to consider how each group represented their thinking. For example, if they used a table, symbols or drawings.

### Explore

If using your own disposable cups, the heights in the following table will need to be adjusted.

1. With students remaining in their groups of 3, use slide 6 of the PowerPoint (LE PPT) to model completing Table 2. Allow randomly selected students to suggest values.

Table 2: number of cups and height of stack (cm)

|  |  |
| --- | --- |
| Number of cups | Height of stack (cm) |
| 0 | 0 |
| 1 | 6.5 |
| 2 | 8 |
| 3 | 9.5 |

1. Use a questioning strategy such as Pose-Pause-Pounce-Bounce (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) to facilitate a class discussion, using prompts such as:
* How much does the height of stack increase with each cup?
* Can you describe how the height changes in words?
* What part of the pattern is ‘constant’ and what part ‘increases or decreases’?

It may be beneficial to co-construct definitions of ‘constant’ and ‘rate of increase/decrease’ with students based on their prior learning in mathematics, other subjects and real-world contexts.

In this example, the base height of the cup is constant as it is being counted once, no matter how many cups are added to the stack.

The total lip height is the rate of increase as it changes based on how many cups are added to the stack.

1. Use slide 7 to display the heights: 14 cm, 15 cm, and 1 m.
2. Ask groups to calculate, on their vertical non-permanent surfaces, the number of cups required to reach each height.

Exactly 6 cups are required to reach 14 cm.

7 cups are required to reach 15 cm as 6 cups is less than 15 cm.

64 cups are required to reach 1 m. The hope is that students will recognise the need for an algebraic approach to determine this result.

1. Use a questioning strategy such as Pose-Pause-Pounce-Bounce to discuss the need to construct an equation, as continuing the pattern in the table or reading off plotted points is unreasonable for large or decimal values.
2. Use slide 8 to show the equation $Height = 5+number of cups × 1.5$.
3. Referring to the previous class discussion, ask students to define which part of the equation is ‘constant’ and which part has a rate of increase/decrease. The following prompts could be used to facilitate conversation of the equation:
* Decide as a class on appropriate symbols to use instead of ‘height’ and ‘number of cups’.
* Discuss if order matters. Is $1.5×number of cups +5=5+number of cups×1.5$?
1. Challenge groups to use the equation to find the height of 3 cups, 10 cups and 100 cups.
2. Challenge groups to use the equation to find the number of cups required to reach the heights on slide 7 (14 cm, 15 cm, 1 m).
3. Use slide 9 of the PowerPoint (LE PPT) to display an image of the moon and ask students ‘How many cups would it take to reach the moon?’. Have students discuss in their groups and record their estimate in the top right corner of their working surface to return to later.
4. Use slide 10 of the PowerPoint (LE PPT) to display that the distance of the moon from Earth is 384 400 kilometres.
5. Students use the equation to calculate the number of cups required to reach the moon (25 626 666 664 cups).
6. Facilitate a class discussion, comparing students’ initial estimates to their calculated answers. Possible question prompts might include:
* What reasoning did you use to make your initial prediction?
* How did your reasoning change after completing the activity?
* Could we use similar reasoning to make predictions about any linear pattern?

### Summarise

1. Use slides 12–19 of the PowerPoint (LE PPT) for explicit teaching of constructing an equation to represent a linear relationship using the Worked examples (Your turn) method ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)).
2. Students write notes to their future forgetful selves on constructing an equation to represent a linear relationship for a table or graph.

Students could suggest a series of steps such as: determine the constant value, determine the rate of increase/decrease, write the equation in the format $y=rate of increase/decrease x+constant$.

### Apply

1. Distribute Appendix B ‘Linear equations’ to pairs of students.
2. Pairs work together to construct a linear equation to represent each of the tables and graphs.
3. Once students have constructed their equations, challenge students to verify that each equation is correct by substituting values into the equation.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Explore**

* Students could be provided with scaffolds such as partially filled tables or an equation.
* Provide sentence starters for observations, for example, ‘I noticed that each cup adds…’.
* The task could be presented with fewer scaffolds, allowing students to more openly explore the problem.
* Provide students with opportunities to compare different cups and different ways of stacking, and to construct equations to compare each variation.

**Summarise**

* The worked examples could be adjusted to have non-zero values to further challenge students.
* Non-examples could be used to highlight patterns that are not linear.
* Offer a structured outline for constructing equations, including labelled prompts for constants and rates of increase/decrease.
* Use colour-coded notes or examples on what stays constant versus what changes.
* Challenge students to predict what would change in the equation if the cup’s dimensions were altered or if it had a different shape.

**Apply**

* Provide a sentence starter for verifying each equation, for example, ‘If I substitute $…$, I should get…’.
* Provide students with access to the Desmos graphing calculator to explore the connection between linear equations and their graphs.
* Challenge students to explain what they notice and wonder about pairs of graphs or tables to highlight the impact of changing the values in the equation.

### Suggested opportunities for assessment

**Launch**

* Record initial predictions for the height of a 10-cup stack, noting reasoning to assess students' approaches to estimation.

**Explore**

* Observe that students can correctly complete Table 2 (number of cups versus height of stack (cm)). If students have a misconception at this stage, it should be addressed before continuing with the lesson.
* Students’ answers to questions about constants and rate of increase/decrease can reveal their grasp of these concepts and identify areas needing clarification.

**Summarise**

* An exit ticket could be used to have students define important terms from the lesson. For example, constant, rate of increase/decrease and linear.
* Review students' notes to see if they accurately captured the equation construction process and steps.

**Apply**

* Collect Appendix B for formative or summative assessment, focusing on the accuracy of constructed equations.

## Appendix A

### Cup template



## Appendix B

### Linear equations

Construct a linear equation to represent each pattern.

|  |  |  |
| --- | --- | --- |
| 1. | $$x$$ | $$y$$ |
|  | 0 | 7 |
|  | 1 | 9 |
|  | 2 | 11 |

|  |  |  |
| --- | --- | --- |
| 2. | $$x$$ | $$y$$ |
|  | 0 | 2.5 |
|  | 1 | 3 |
|  | 2 | 3.5 |

|  |  |  |
| --- | --- | --- |
| 3. | $$x$$ | $$y$$ |
|  | 0 | 3 |
|  | 1 | -2 |
|  | 2 | -7 |

|  |  |  |
| --- | --- | --- |
| 4. | $$x$$ | $$y$$ |
|  | 1 | 6 |
|  | 2 | 8 |
|  | 3 | 10 |

|  |  |  |
| --- | --- | --- |
| 5. | $$x$$ | $$y$$ |
|  | 1 | $$c$$ |
|  | 2 | $$c+m$$ |
|  | 3 | $$c+2m$$ |

|  |  |
| --- | --- |
| 6. | Graph of y=x+2. |
| 7. | Graph of y=2x+1. |
| 8. | Graph of y=-2x+1. |
| 9. | Graph of y=-2x-1. |
| 10. | Graph of y=-0.5x+1.5. |

## Sample solutions

### Appendix B – linear equations

|  |  |
| --- | --- |
|  | constant = 7rate of increase/decrease = 2$$y=2x+7$$ |
|  | constant = 2.5rate of increase/decrease = 0.5$$y=0.5x+2.5$$ |
|  | constant = 3rate of increase/decrease = -5$$y=-5x+3$$ |
|  | constant = 4rate of increase/decrease = 2$$y=2x+4$$ |
|  | constant = $c$rate of increase/decrease = $m$$$y=mx+c$$ |
|  | constant = 2rate of increase/decrease = 1$$y=x+2$$ |
|  | constant = 1rate of increase/decrease = 2$$y=2x+1$$ |
|  | constant = 1rate of increase/decrease = 2$$y=-2x+1$$ |
|  | constant = 1rate of increase/decrease = 2$$y=-2x-1$$ |
|  | constant = 1rate of increase/decrease = 2$$y=-0.5x+1.5$$ |

## References

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