# Point of intersection

Students explore intersecting lines and their point of intersection through the context of distance-time graphs.

Students will need at least one digital device per pair to interact with Desmos during this lesson.

## Visible learning

### Learning intention

* I can find the point of intersection of 2 intersecting lines.

### Success criteria

* I can identify the point of intersection of 2 graphs.
* I can verify a point of intersection.
* I can explain what the point of intersection tells us about 2 graphs.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* creates and displays number patterns and finds graphical solutions to problems involving linear relationships **MA4-LIN-C-01**

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Table 1: lesson summary

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| Section | Summary of activity | Teaching strategies | Teaching points |
| Launch | Play the video ‘120m Women's Stawell Gift Final 2024 - 3rd Place’ (1:36) ([bit.ly/StawellGift2024](http://www.bit.ly/StawellGift2024)) to discuss the Stawell Gift race. Replay the video, focusing on the moment Chloe (red) passes Jennifer (pink), and ask students to time it. | Notice and wonderPose-Pause-Pounce-Bounce | The purpose of the Launch is to introduce the concept of a handicap race, setting the stage for analysing the intersection of distance-time graphs through real-world context. |
| Explore | Present slide 3 of the PowerPoint Point of intersection to discuss the graphs, identifying the runners and interpreting how the graphs represent the race. Slide 4 is then used to facilitate class discussion of the significance of the point of intersection. Students verify the point of intersection using both equations. | Think-Pair-SharePose-Pause-Pounce-Bounce | Deepen students' understanding of graph interpretation by having them analyse the intersection point between 2 runners, making connections to prior knowledge of equations and speeds. |
| Summarise | Display graphs comparing 2 electric scooter companies (slide 6) and ask students to find the point of intersection. Students use mini whiteboards to share answers. Randomly selected students explain the meaning of the point of intersection in the given context. Repeat for slides 7–10. |  | Reinforce students’ learning by applying their knowledge to practical scenarios, allowing them to practise identifying and interpreting points of intersection and understanding their significance in different real-life contexts. |
| Apply | Set up the Desmos activity ([bit.ly/desmospoi](https://bit.ly/desmospoi)) and model how to complete screen 2. Students work in pairs to predict intersection points on screens 3–6. Facilitate a discussion on strategies and confidence using screens 7–8. Extension challenges are on screens 9–10. | Pose-Pause-Pounce-Bounce | To give students experience using technology to model and predict intersection points, encourage them to apply their knowledge to solve problems. |

## Activity structure

Please use the associated PowerPoint *Point of intersection* (POI PPT) to display images in this lesson.

### Launch

1. Show students the video ‘120m Women's Stawell Gift Final 2024 - 3rd Place’ (1:36) ([bit.ly/StawellGift2024](https://bit.ly/StawellGift2024)).

Once the video has played, return to the start of the video and display the start list for students.

1. Ask students to consider what they notice and wonder ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy))about the race.

Students may notice the race is 120 m long, that the runners start in different positions, the time for the first-place runner and that the lanes are marked by rope and not lines on the grass.

Students may wonder what the distances mean next to each runner’s name in the start list, or how their starting position is determined.

1. Explain to students that the Stawell Gift uses a handicap system so that the runners cross the finish line at roughly the same time. The runner’s start position in the race is based on their performance in previous races. The fastest runner receives the largest handicap.

The information provided in the explanation comes from the Stawell Gift page in Wikipedia (2024) ([bit.ly/StawellGiftInfo](https://bit.ly/StawellGiftInfo)).

1. Ask students if they have participated in or seen the gift race at school athletics carnivals.
2. Show the video again starting at the 1:15 minute mark, and ask students to write down how long it takes for Chloe Mannix-Power (runner in the red top in lane 1) to pass Jennifer King (runner in the pink top in lane 6).

Students might use different strategies such as counting the seconds from the start or trying to use the time on the video.

1. Initiate a sharing of ideas and reasoning using the Pose-Pause-Pounce-Bounce questioning strategy (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) to ask students if there is a more accurate way to find the time when Chloe passes Jennifer.

Students may suggest finding the speeds for both runners which they learned about previously in Lesson 8 – distance-time graphs of Unit 13 – ratios and rates.

### Explore

1. Display slide 3 of the PowerPoint (POI PPT) which shows the graphs of both runners.
2. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), ask students to discuss what they notice and wonder about the 2 lines.

During the discussion, students should contribute what the axes represent – that the coordinate points on the $y$-axis represent the runner’s starting position and thus identify the red line as Chloe and the pink line as Jennifer. The coordinate points with $y$-values of 120 show the finish of the race and the corresponding $x$-values are their race times.

1. Display slide 4 of the PowerPoint (POI PPT) which shows the graphs of both runners and their point of intersection.

Display the Desmos graph Stawell Gift ([bit.ly/StawellGiftGraph](https://bit.ly/StawellGiftGraph)) if you would like to zoom in on the point of intersection.

1. Initiate a sharing of ideas and reasoning using the Pose-Pause-Pounce-Bounce questioning strategy to ask students what they think this coordinate point represents in the context of the race.

Explain to students that the point of intersection has been written as a fraction to maintain accuracy. Calculators could be used to find the decimal values or approximations of the point of intersection.

Students should interpret that Chloe caught up to Jennifer after approximately 10 seconds and 91 metres.

1. In a Think-Pair-Share, ask students to discuss the graphs and equations. Question prompts could include:
* Which runner is quicker? How do you know?
* Who would win in a 50 m race?
* If you graphed all 6 runners, would they intersect at the point ($10\frac{5}{67},91\frac{13}{67}$)?
1. Ask pairs to discuss and determine how they could algebraically verify that the point ($10\frac{5}{67},91\frac{13}{67}$) satisfies both equations.

Students should substitute the $x$ value $10\frac{5}{67}$ into each equation, resulting in a $y$ value of $91\frac{13}{67}$ for both equations.

### Summarise

1. Use slide 6 of the PowerPoint (POI PPT) to display the graphs and equations of 2 electric scooter companies. Working in pairs:
2. have students record the point of intersection on mini whiteboards.
3. ask students to hold up their whiteboards. Address any misconceptions or errors.
4. ask students how they can verify that their answer is correct. Model substituting $x=5$ into both equations, resulting in a $y$ value of 3.5.
5. randomly select students using a questioning strategy such as Pose-Pause-Pounce-Bounce to explain what the point of intersection represents in the context of the graph.

In this example, the point of intersection represents the time when both company’s price models are equal.

1. Continue through slides 7–10 of the PowerPoint (POI PPT) repeating steps a–d for each slide.

**Slide 6:** (5,3.5) represents that renting an electric scooter for 5 minutes is when the 2 payment options are equal. If renting a scooter for more than 5 minutes, option A is more cost effective.

**Slide 7:** (5,50) represents that purchasing 5 tickets in a year is when the 2 payment options are equal. If paying for more than 5 tickets, option B is more cost effective.

**Slide 8:** (7.5, 87.5) represents that paying for 7.5 classes in a year is when the 2 payment options are equal. As you can’t pay for half a class, if paying for more than 7 classes, option B is more cost effective.

**Slide 9:** (4,60) represents that hiring a court 4 times in a year is when the 2 payment options are equal. If hiring the court more than 4 times, option A is more cost effective.

**Slide 10:** (16,960) represents that playing 16 games of golf in a year is when the 2 payment options are equal. If playing more games, option A is more cost effective.

### Apply

1. With one device between pairs of students, direct students to the Desmos Classroom activity ‘Point of intersection’ ([bit.ly/desmospoi](https://bit.ly/desmospoi)).

Before completing this activity, you will need to set up a Desmos Classroom ([bit.ly/createdesmosclassroom](https://bit.ly/createdesmosclassroom)) and use the pacing feature to restrict the students to screen 2.

1. Display and model completing screen 2 before allowing students to complete screen 2 on their own device.
2. Pace students to screens 3–6 for them to continue plotting where they think the lines will intersect.
3. Pace students to screens 7–8 to explain how the results are displayed and compare their final answer to the actual point of intersection.
4. Use a questioning strategy such as Pose-Pause-Pounce-Bounce to facilitate a class discussion of how students predicted the point of intersection and how they knew they were correct.
5. If completing the extension activity, pace students to screens 9–10. Model how to use the Desmos calculator to complete screen 9, then allow students to create their own challenge to swap with a partner.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* Provide sentence starters or a structured template for students who need support in communicating their observations and questions about the race.
* Students may benefit from physically acting out the race to understand the premise.
* Using the data from the video, challenge students to calculate estimated speeds for the runners based on their starting positions and race times.

**Explore**

* Use concrete manipulatives, like moving counters along a racetrack diagram, to demonstrate how runners’ positions change over time and lead to intersections.
* Challenge students to match given equations to the correct runners' graphs and justify their reasoning based on starting positions and gradients.

**Summarise**

* Use role-playing scenarios where students act as customers choosing between different pricing plans, allowing students to more conversationally explain their reasoning.
* Provide sentence starters for students to explain their reasoning.
* Allow students to use different formats to represent their understanding of intersection points, such as drawing a diagram, writing a mathematical explanation, or creating a story problem that uses the graphs.
* Use probing questions to challenge students to think deeper about their answers. For instance, ask, ‘What does it mean if the intersection point changes?’ or ‘How would you explain the significance of this point to someone unfamiliar with graphs?’

**Apply**

* Use the Desmos teacher dashboard to monitor student responses. Snapshot certain misconceptions to address with small groups or the whole class.

### Suggested opportunities for assessment

**Launch**

* Observe students’ initial notice and wonder responses to gauge their understanding of the race setup and the concept of a handicap system.
* Observe student responses to assess their prior understanding of distance-time graphs.

**Explore**

* Observe students’ discussions about the graphs and equations, taking notes on how well they understand and interpret the axes and lines.
* Observe students’ ability to substitute and solve equations to verify intersection points. Additional explicit instruction may be required.

**Summarise**

* Use student answers on mini whiteboards to quickly assess their understanding of intersection points in various contexts. Look for patterns in misconceptions and address them immediately.
* Observe student contributions within the Pose-Pause-Pounce-Bounce questioning to assess their understanding of the point of intersection within each context.

**Apply**

* Desmos’ teacher dashboard can be used to assess students’ answers and reasoning in real time or following the activity.

## References

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