# Arc length

Students explore arc lengths as a proportion of circumference to establish the arc length formula.

Students will need at least one digital device per pair to interact with Desmos during this lesson. A non-digital alternative is provided, which requires Appendix A to be printed.

## Visible learning

Learning intentions and success criteria should be shared with students later in the learning episode.

### Learning intentions

* To understand the connection between arc length and circumference.
* To be able to solve problems involving arc length.

### Success criteria

* I can explain what an arc is.
* I can explain where the formula for arc length comes from.
* I can calculate the arc length of a circle.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* represents and operates with fractions, decimals and percentages to solve problems  **MA4-FRC-C-01**
* applies knowledge of the perimeter of plane shapes and the circumference of circles to solve problems **MA4-LEN-C-01**

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Table 1: lesson summary

|  |  |  |  |
| --- | --- | --- | --- |
| Section | Summary of activity | Teaching strategies | Teaching points |
| Launch | Students watch a video about the Nardò Ring ([bit.ly/NardoRing](https://bit.ly/NardoRing)) up to 1:07 and discuss what they notice and wonder. | Think-Pair-Share  Notice and wonder | Identify what students notice and wonder about the circle terminology featured in the video. |
| Explore | In pairs, students complete a Desmos activity ([bit.ly/ArcLengthDesmos](https://bit.ly/ArcLengthDesmos)) where they make a range of predictions related to the proportion of the circumference an arc will make. For a non-technology option, see [Appendix A.](#_Nardo_Ring_approximations) | Pose-Pause-Pounce-Bounce | Develop an understanding that an arc is a proportion of the circumference and use this to make a formula. |
| Summarise | Students are formally introduced to the terminology and the formula using the PowerPoint. Arc length (AL PPT). Students complete independent practise using Variation Theory in [Appendix B](#_Variation_theory) before completing their notes. | Worked examples (Your turn)  Variation Theory | Build fluency with the arc length formula and provide answers in terms of and rounded to a specified number of decimal places. |
| Apply | In randomised groups of 3, students review a series of solutions to arc length questions using [Appendix C](#_Two_truths,_one). Students then complete [Appendix D](#_Open_Middle) which includes 2 Open Middle Problems using arc length. | Visibly random groups of 3  Vertical non-permanent surfaces  Gallery walk | Interrogate common errors related to calculating arc length before using the formula the solve non-routine problems. |

## Activity structure

Please use the associated PowerPoint *Arc Length* (AL PPT) to display images in this lesson.

### Launch

1. Show students the video ‘The Nardò Ring: A Curved Geoglyph Amidst Italian Beauty’ (2:01) ([bit.ly/NardoRing](https://bit.ly/NardoRing)) up to 1:07.
2. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), ask students to consider what they notice and wonder ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy))about the Nardò Ring.

Students might notice that the circumference is 12.6 km, the diameter is 8 km, and that a driver can drive at 240 km/h and feel as though they are driving on an infinite straight line.

Students might wonder what the circle radius is or how long it would take for different vehicles to drive around the circle.

### Explore

1. With one device between pairs of students, assign the Desmos activity ‘Arc Length’ ([bit.ly/ArcLengthDesmos](https://bit.ly/ArcLengthDesmos)) for students to complete.

Before completing this activity, you will need to set up a Desmos Classroom ([bit.ly/createdesmosclassroom](https://bit.ly/createdesmosclassroom)) and use the pacing feature to restrict the students to screen 2.

If technology is unavailable, Appendix A could be printed double-sided and placed in a plastic sleeve. Students could then use an erasable marker to complete the activity, similar to the Desmos activity. After making their guesses, students could flip the sheet to check their accuracy.

It might also be beneficial to remind students that the angle at the centre of a circle is 360.

1. Use the pacing feature to pace students to screens 2–7 and ask the students to complete the activities.
2. Once most students have completed screens 2–7, extend the pacing feature to screen 8 and ask students to respond to the question.
3. Use the Pose-Pause-Pounce-Bounce questioning strategy (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) to initiate a class discussion about what information would help make their predictions more precise and why.

Students should recognize that knowing the angle size aids in predictions, as they can calculate the fraction of the circle and use it to determine the proportion of the arc length.

1. Extend the pacing to screens 9–14 and ask students to reattempt their predictions.
2. Once most students have completed screens 9–14, extend the pacing feature to screen 15 and ask students to respond to the question.
3. Use the Pose-Pause-Pounce-Bounce questioning strategy to initiate a class discussion about whether there was an improvement in predictions and what caused that improvement.

Some students might identify that the angle size assisted them to be more precise. Other students might point out that they had also seen the distances before and could use that to predict how far it went.

1. Extend the pacing feature to include screen 16. Encourage students to guess and check the angle size.

The correct answer is 60, which students could determine as 60 is of 360, as 2.1 km is of 12.6 km.

1. Extend the pacing to include screens 17–23 and ask students to complete the activity.
2. Use the Pose-Pause-Pounce-Bounce questioning strategy to initiate a class discussion on how students can use the angle, expressed as a fraction of the circle, to calculate the arc length.

Students should be able to identify that you will need to find an angle out of 360°.

1. Reveal the learning intentions and success criteria for this lesson.

### Summarise

1. Explain to the students that the mathematical term for part of the circle’s circumference is ‘arc’, so the cars in the Desmos activity were driving along the arc length.
2. Ask students to add ‘arc’ to their glossary from Lesson 1 – circumference and diameter.

An arc is part of a circle’s circumference.

It is also noted that in Mathematics Extension 1 this definition will extend from a circle’s circumference to include all curves.

1. Use slides 3–10 from the PowerPoint (AL PPT) for explicit teaching using the Worked examples (Your turn) method ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)).
2. Distribute Appendix B ‘Variation Theory’ to each student, which uses Variation Theory ([variationtheory.com/introduction/](https://variationtheory.com/introduction/)) to highlight changes in the radius and angle size when calculating arc length.

This activity has been adapted from the activity created by Sam Webster for the Variation Theory website ([bit.ly/VTarcLength](https://bit.ly/VTarcLength)).

1. Students are to create notes to their future forgetful selves ([bit.ly/notestofutureself](https://bit.ly/notestofutureself)) on defining an arc and how to calculate its length.

### Apply

During this section, students will work in visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) on vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)).

#### Two truths and a lie

1. Distribute one A3 copy of Appendix C ‘Two truths, one lie’ ([bit.ly/twotruthsoneliestrategy](https://bit.ly/twotruthsoneliestrategy)) to each group. Students are to display Appendix C on the vertical non-permanent surface using adhesive putty.
2. Students are required to read through each question and determine which solutions are correct and which solution has an error. Each group should annotate the work sample to explain what the error is and how it could be corrected.

The final line of each solution is intentionally left incomplete, requiring students to either calculate the missing value or review the steps to identify any errors.

1. Students are to do a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) of each other’s solutions. Ask the groups to discuss whether they agree with each group’s annotations.

#### Open Middle

Distribute or display the Open Middle problems contained in Appendix D for students to complete in their groups at vertical non-permanent surfaces.

## Assessment and differentiation

### Suggested opportunities for differentiation

##### ****Launch****

* A notice and wonder strategy is used where there is no correct answer so that all students can participate in the discussion.

##### Explore

* Pair students of mixed abilities so that stronger students can assist others, promoting peer learning.
* High achieving students could be challenged to incorporate more complexity into their written explanations.

##### Summarise

* Questions could be adapted to meet the needs of students.
* Students could work in pairs or visibly random groups of 3 to collaborate on the Variation Theory task.

##### Apply

* Using random groups to promote collaboration among students of different ability levels, allows more capable students to support those who need additional help.
* The final answers could be provided in Appendix C, allowing students to concentrate on explaining the reasoning behind the error.
* Solutions could be adapted in Appendix C to address other misconceptions, such as not using in final calculations, different representations of leaving answer in terms of or rounding errors.
* Students could be challenged to find as many unique solutions to Appendix D as they possibly can.
* Students could be challenged to maximise or minimise arc length, radius or angle size.
* Students could be challenged to maximise or minimise the difference between either arc length, radius or angle size.
* Students could be challenged to explain why particular solutions, such as 5, cannot be achieved in Appendix D.
* Students could be challenged to use factors to make an exhaustive list of solutions to problem 1 in Appendix D.
* Students could be challenged to create a spreadsheet to verify all possible solutions for a particular angle on problem 2 of Appendix D.
* Students could be given prompts to consider factors of 360 that could result in simplified fractions.

### Suggested opportunities for assessment

##### ****Launch****

* A Think-Pair-Share provides students with the opportunity to reflect on their understanding.
* Monitor responses in class discussions to check for student understanding of circumference and diameter.

##### Explore

* Responses are collected by Desmos and can be reviewed for student understanding of proportionality.
* The teacher could facilitate class discussions and observe students’ reasoning and justification in response to the provided prompts. The teacher should particularly focus on how students refer to proportionality regarding what fraction of the circle is represented in each activity.

##### Summarise

* When discussing with students, teachers can encourage their students to think about their decisions around what has changed in each question, and what they expect each variation to do because of the solution.
* Appendix B could be collected to observe student understanding of the arc length formula and how it is applied.
* Review students’ notes to future forgetful selves for understanding of arc length.

##### Apply

* When placed in groups of 3, students provide and receive peer feedback on their understanding.
* Students working at vertical non-permanent surfaces means the teacher can assess student progress and provide support where appropriate.

## Appendix A

### Nardò Ring approximations

The total circumference of the Nardò Ring is 12.6 km. Place this sheet in a plastic sleeve. Use a marker to trace the required distance from the start point in a clockwise direction.

|  |  |
| --- | --- |
| A circle with a line marked 'start' for students to estimate the proportion of circumference driven by the car.  **Drive 1**: 6.3 km | A circle with a line marked 'start' for students to estimate the proportion of circumference driven by the car.  **Drive 2**: 3.15 km |
| A circle with a line marked 'start' for students to estimate the proportion of circumference driven by the car.  **Drive 3**: 9.45 km | A circle with a line marked 'start' for students to estimate the proportion of circumference driven by the car.  **Drive 4**: 4.2 km |
| A circle with a line marked 'start' for students to estimate the proportion of circumference driven by the car.  **Drive 5**: 8.8 km | A circle with a line marked the start for students to estimate the proportion of circumference driven by the car.  **Drive 6**: 1.92 km |
| A circle with the arc length of 6.3 km drawn.  **Drive** 1: 6.3 km | A blue circle with 2 equal radii.  **Drive 2**: 3.15 km |
| A blue circle with 2 equal radii.  **Drive 3**: 9.45 km | A blue circle with 2 equal radii.  **Drive 4**: 4.2 km |
| A blue circle with 2 equal radii.  **Drive 5**: 8.8 km | A blue circle with 2 equal radii.  **Drive 6**: 1.92 km |

## Appendix B

### Variation Theory

Calculate the length of each arc. Give your solution in terms of , before rounding your solution to 2 decimal places.

|  |  |
| --- | --- |
| A sector with a radius of 14 cm and an angle of 36°. |  |
| A sector with a radius of 10 cm and an angle of 36°. |  |
| A sector with a radius of 36 cm and an angle of 10°. |  |
| A sector with a radius of 72 cm and an angle of 10°. |  |
| A sector with a radius of 36 cm and an angle of 20°. |  |
| A sector with a radius of 30 cm and an angle of 26°. |  |
| A sector with a radius of 3 cm and an angle of 260°. |  |
| A sector with radius 3 cm and an angle of 110°. |  |

## Appendix C

### Two truths, one lie

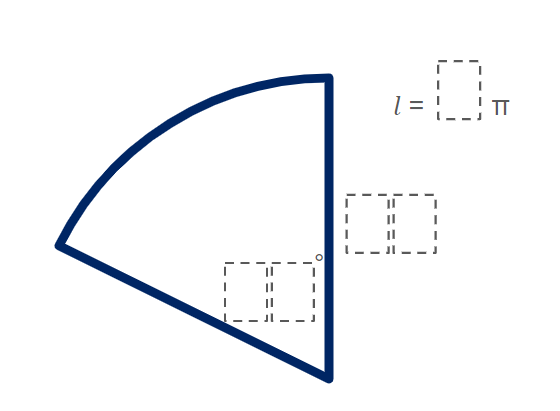
|  |  |  |  |
| --- | --- | --- | --- |
| Calculate the arc length. Leave your solution in terms of  A sector with a radius of 15 cm and an angle of 60°. | **Solution 1** | **Solution 2** | **Solution 3** |
| Calculate the arc length. Round your solution to one decimal place.  A quadrant with a radius of 20 cm. | **Solution 1** | **Solution 2** | **Solution 3** |
| Calculate the arc length. Round your solution to one decimal place.  A semicircle with a diameter of 22 cm. | **Solution 1** | **Solution 2** | **Solution 3** |
| Calculate the arc length. Leave your solution in terms of  A sector with a radius of 6 cm and the non included angle of 150°. | **Solution 1** | **Solution 2** | **Solution 3** |

## Appendix D

### Open Middle

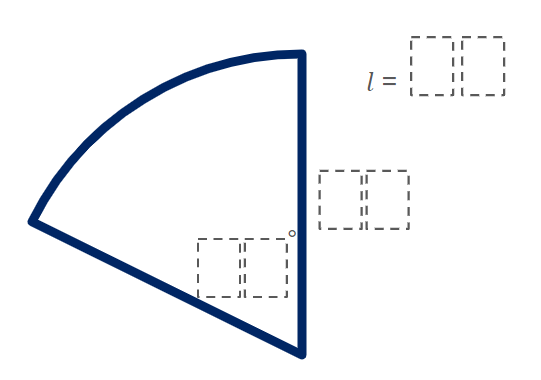
#### Problem 1

Using the digits 0–9 at most one time each, place a digit in each box to create part of a circle with the corresponding arc length in terms of .



#### Problem 2

Using the digits 0–9 at most one time each, place a digit in each box to create part of a circle with the corresponding arc length rounded to the nearest whole number.



## Sample solutions

### Appendix B – Variation Theory

|  |  |
| --- | --- |
| *A sector with a radius of 14 cm and an angle of 36°.* |  |
| *A sector with a radius of 10 cm and an angle of 36°.* |  |
| *A sector with a radius of 36 cm and an angle of 10°.* |  |
| *A sector with a radius of 72 cm and an angle of 10°.* |  |

|  |  |
| --- | --- |
| *A sector with a radius of 36 cm and an angle of 20°.* |  |
| *A sector with a radius of 30 cm and an angle of 26°.* |  |
| *A sector with a radius of 3 cm and an angle of 260°.* |  |
| *A sector with a radius of 3 cm and an angle of 110°.* |  |

### Appendix C – Two truths, one lie

|  |  |  |  |
| --- | --- | --- | --- |
| Calculate the arc length. Leave your solution in terms of  A sector with a radius of 15 cm and an angle of 60°. | **Solution 1** | **Solution 2**  **This solution is incorrect as the student has used diameter instead of radius in the formula.** | **Solution 3** |
| Calculate the arc length. Round your solution to one decimal place.  A quadrant with a radius of 20 cm. | **Solution 1**  **This solution is incorrect as the student has halved the radius.** | **Solution 2** | **Solution 3** |
| Calculate the arc length. Round your solution to one decimal place.  A semicircle with a diameter of 22 cm. | **Solution 1**  **This solution is incorrect as the diagram has provided the diameter of the semicircle, and the student has substituted this as the radius.** | **Solution 2** | **Solution 3** |
| Calculate the arc length. Leave your solution in terms of  A sector with a radius of 6 cm and the non included angle of 150°. | **Solution 1** | **Solution 2** | **Solution 3**  **This solution is incorrect has substituted the external angle in the arc length.** |

### Appendix D – Open Middle

In all the possible solutions below, the values for and can be swapped.

#### Problem 1 – possible solutions

|  |  |  |
| --- | --- | --- |
|  |  |  |

#### Problem 2 – possible solutions

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
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|  |  |  |
|  |  |  |

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