Mathematics Stage 5 (Year 10) – unit of learning

Elective – the unit circle

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# Rationale

The NSW Department of Education publishes a range of curriculum support materials, including samples of lesson sequences, scope and sequences, assessment tasks, examinations, student and teacher resource booklets, and curriculum planning and curriculum evaluation templates. The samples are not exhaustive and do not represent the only way to complete or engage in each of these processes. Curriculum design and implementation is a dynamic and contextually-specific process. While the mandatory components of syllabus implementation must be met by all schools, it is important that the approach taken by teachers is reflective of their needs, and faculty or school processes.

The NSW Education Standards Authority (NESA) defines [programming](https://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/understanding-the-curriculum/programming) as ‘the process of selecting and sequencing learning experiences which enable students to engage with syllabus outcomes and develop subject specific skills and knowledge’ (NESA 2024b). A program is developed collaboratively within a faculty. It differs from a unit in important ways, as outlined by NESA on their [Advice on units](https://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/understanding-the-curriculum/programming/advice-on-units) page. A unit is a contextually-specific plan for the intended teaching and learning for a particular class for a particular period. The organisation of the content in a unit is flexible and it may vary according to the school, the teacher, the class and the learning space. They should be working documents that reflect the thoughtful planning and reflection that takes place during the teaching and learning cycle. There are mandatory components of programming and unit development, and this template provides one option for the delivery of these requirements. The NESA and department guidelines that have influenced this template are elaborated upon at the end of the document.

This resource has been developed to assist teachers in NSW Department of Education schools to create learning that is contextualised to their classroom. It can be used as a basis for the teacher’s own program, assessment, or scope and sequence, or be used as an example of how the new curriculum could be implemented. The resource has suggested timeframes that may need to be adjusted by the teacher to meet the needs of their students.

# Overview

**Description**: this program of learning addresses content from the focus areas of Surds and Trigonometry. The lessons and sequences in this program of learning are designed to allow students to explore surds, exact values, the unit circle and graphing trigonometric functions.

**Duration**: this program of learning is designed to be completed over a period of approximately 5 weeks but can be adapted to suit the school context.

**Explicit teaching**: suggested learning intentions and success criteria are available for some lessons provided. Learning intentions and success criteria are most effective when they are contextualised to meet the needs of students in the class. The examples provided in this document are generalised to demonstrate how learning intentions and success criteria could be created.

**Accessing the resources**: this program of learning includes a range of student-facing and teacher resources. All resources can be accessed from the [Stage 5 – Elective – the unit circle](https://education.nsw.gov.au/teaching-and-learning/curriculum/mathematics/mathematics-curriculum-resources-k-12/mathematics-7-10-resources/year-10-elective-the-circle-unit) catalogue page.

# Outcomes

## Core

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**

## Path

A student:

* **describes and performs operations with surds and fractional indices MA5-IND-P-02**
* establishes and applies the properties of trigonometric functions and finds solutions to trigonometric equations **MA5-TRG-P-02**

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**Prior to planning for teaching and learning, please consider the following**:

**Engagement**

* How will I provide authentic, relevant learning opportunities for students to personally connect with lesson content?
* How will I support every student to grow in independence, confidence and self-regulation?
* How will I facilitate every student to have high expectations for themselves?
* How will I identify and provide the support each student needs to sustain their learning efforts?

**Representation**

* What are some different ways I can present content to enable every student to access and understand it?
* How will I identify and address language and/or cultural considerations that may limit access to content for students?
* How will I make lesson content and learning materials more accessible?
* How will I plan learning experiences that are relevant and challenging for the full range of students in the classroom?

**Expression**

* How will I provide multiple ways for students to respond and express what they know?
* What tools and resources can students use to demonstrate their understanding?
* How will I know every student has understood the concepts and language presented in each lesson?
* How will I monitor if every student has achieved the learning outcomes and learning growth?

# Lesson sequence and details

## Learning episode 1 – simplifying surds

### Teaching and learning activity

Students learn how to simplify surds visually and algebraically.

### Syllabus content

* Establish and apply the following results for and
* Expand and simplify expressions involving surds

Table 1 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **Simplifying surds**  **Duration:** 1 lesson  **Learning intention**   * To know how to simplify surds.   **Success criteria**   * I can explain what a surd is. * I can identify surds that can be simplified. * I can express surds in their simplest form. * I can compare values that contain surds. | * *Simplifying surds* PowerPoint * Appendix A, printed on A3 paper in a plastic pocket, (one copy per group of 3) * Appendix B and C, printed (one copy per student) |  |

## Learning episode 2 – rationalising the denominator

### Teaching and learning activity

Students explore how to estimate the magnitudes of values that involve fractional surds to develop why we rationalise the denominator.

### Syllabus content

* Expand and simplify expressions involving surds
* Rationalise the denominators of surds of the form

Table 2 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **Rationalising the denominator**  **Duration:** 1–2 lessons  **Learning intention**   * To understand how to rationalise the denominators of fractions.   **Success criteria**   * I can estimate the magnitude of a surd. * I can explain what to multiply a fraction by to get an integer denominator. * I can rationalise fractions that have surds in their denominators. | * Appendix A, printed on A3 paper in a plastic pocket (one copy per group of 3) * Appendix B and C, printed |  |

## Learning episode 3 – exact ratios

### Teaching and learning activity

Students learn where exact values originate from, why they always work and how to easily recall them.

### Syllabus content

* Derive and apply the exact sine, cosine and tangent ratios for angles 30°, 45° and 60°

Table 3 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **Exact ratios**  **Duration:** 1–2 lessons  **Learning intentions**   * To understand the origin of the exact values of trigonometric ratios for specific angles. * To be able to solve problems using exact values for trigonometric ratios.   **Success criteria**   * I can determine the sine, cosine, and tangent of given angles within a right-angled triangle using the exact trigonometric ratios. * I can draw and label the special right-angled triangles to find the exact values of trigonometric ratios. * I can substitute exact values into trigonometric equations. * I can simplify trigonometric expressions involving the exact values. | * *Exact ratios* PowerPoint * Appendix A, printed (one copy per pair) * Appendix B, C and D, printed (one copy per student) |  |

## Learning episode 4 – special trigonometric relationships

### Teaching and learning activity

Students investigate complementary angles and why .

### Syllabus content

* Derive and apply the exact sine, cosine and tangent ratios for angles 30°, 45° and 60°
* Verify and use the relationships between the sine and cosine ratios of complementary angles in right-angled triangles: and
* Verify that the tangent ratio can be expressed as a ratio of the sine and cosine ratios

Table 4 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **Special trigonometric relationships**  **Duration:** 1 lesson  **Learning intentions**   * To use complementary angles to find results. * To verify that   **Success criteria**   * I can identify pairs of complementary angles. * I can explain why and * I can prove why . | * *Special trigonometric relationships* PowerPoint * Appendix A and B, printed (one per student) |  |

## Learning episode 5 – a circle and 4 quadrants

### Teaching and learning activity

Students are introduced to the unit circle, the 4 quadrants and measuring angles in each quadrant.

### Syllabus content

* Redefine the sine and cosine ratios in terms of the unit circle

Table 5 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **A circle and 4 quadrants**  **Duration:** 1–2 lessons  **Learning intention**   * To understand the unit circle and how it applies to trigonometry.   **Success criteria**   * I can accurately draw and label the unit circle. * I can label the 4 quadrants in a unit circle. * I can identify different angles around the unit circle. | * *A circle and 4 quadrants* PowerPoint * Appendix A and B, printed (per student) |  |

## Learning episode 6 – exact values in 4 quadrants

### Teaching and learning activity

Students link exact values and the Cartesian plane to produce unit circles.

### Syllabus content

* Redefine the sine and cosine ratios in terms of the unit circle

Table 6 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **Exact values in 4 quadrants**  **Duration:** 1–2 lessons  **Learning intention**   * To investigate angles and trigonometric values in quadrants 2–4 of the unit circle.   **Success criteria**   * I can accurately draw and label the unit circle. * I can find trigonometric ratios using the unit circle. * I can plot points on the Cartesian plane corresponding to the exact trigonometric values. | * *Exact values in 4 quadrants* PowerPoint * Appendix A, printed (one copy per pair) * Appendix B and C, printed A3 (one copy per group of 3) * Appendix D and E, printed (per student) * A3 plastic sleeves * Paper plate with crimpled rim around the edge (per student) |  |

## Learning episode 7 – the unit circle

### Teaching and learning activity

Students will determine the sign of any angle in the unit circle and practise matching equivalent ratios for positive angles up to 360°.

### Syllabus content

* Use the unit circle or graphs of trigonometric functions to establish and apply the relationships ,   
  and for obtuse angles when

Table 7 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **The unit circle**  **Duration:** 1–2 lesson  **Learning intention**   * To identify equivalent trigonometric ratios in the unit circle.   **Success criteria**   * I can identify in which quadrants trigonometric ratios are negative or positive. * I can explain why different trigonometric ratios in the unit circle are positive or negative. * I can match trigonometric ratios that are equivalent. | * *The unit circle* PowerPoint * Appendix A and B, printed (one copy per student) * Appendix C (one copy per group of 3) * Adhesive putty for each group |  |

## Learning episode 8 – graphing using the unit circle

### Teaching and learning activity

Students use spaghetti to construct the graphs of sine and cosine from the unit circle.

### Syllabus content

* Use graphing applications to examine the sine, cosine and tangent ratios for (at least)  and graph the results
* Use graphing applications to examine graphs of the sine, cosine and tangent functions for angles of any magnitude, including negative angles

Table 7 – lesson sequence and details

|  |  |  |
| --- | --- | --- |
| Teaching and learning activities | Required resources | Registration, adjustments and evaluation notes |
| **Graphing using the unit circle**  **Duration:** 1–2 lessons  **Learning intentions**   * To be able to graph trigonometric functions. * To understand the relationship between the unit circle and trigonometric graphs.   **Success criteria**   * I can draw a graph of the trigonometric functions over one or more periods. * I can explain the connection between the unit circle and the graphs of trigonometric functions. | * *Graphing using the unit circle* PowerPoint * Appendix A, printed (one copy per student) * 2 sheets of graph paper (per student) * 40 lengths of spaghetti (per student), scissors and glue * Appendix B and C, printed (one copy per student) |  |

# References

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NESA holds the only official and up-to-date versions of the NSW Curriculum and syllabus documents. Please visit the NSW Education Standards Authority (NESA) website <https://educationstandards.nsw.edu.au/> and the NSW Curriculum website <https://curriculum.nsw.edu.au/>.

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NESA (2024b) ‘[Programming](https://educationstandards.nsw.edu.au/wps/portal/nesa/k-10/understanding-the-curriculum/programming/advice-on-units)’, Understanding the curriculum, NESA website, accessed 28 January 2025.

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