# Rationalising the denominator

Students explore how to estimate the magnitudes of values that involve fractional surds to understand why we rationalise the denominator.

## Visible learning

This lesson incorporates Path content.

### Learning intention

* To understand how to rationalise the denominators of fractions.

### Success criteria

* I can estimate the magnitude of a surd.
* I can explain what to multiply a fraction by to get an integer denominator.
* I can rationalise fractions that have surds in their denominators.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* **describes and performs operations with surds and fractional indices (Path)   
  MA5-IND-P-02**

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Table 1: lesson summary

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| --- | --- | --- | --- |
| Section | Summary of activity | Teaching strategies | Teaching points |
| **Launch** | Students estimate where numbers that involve surds are on number lines in [Appendix A](#_Appendix_A). | Visibly random groups of 3  Vertical non-permanent surfaces  Variation Theory  Pose-Pause-Pounce-Bounce | Students should discover that it is hard to accurately estimate the position of a number with a surd is in the denominator. |
| **Explore** | Students create equivalent fractions for rational numbers, and decimals and link this to surds. Students explain why it is called rationalising the denominator and explore the law before completing [Appendix B](#_Appendix_B). | Think-Pair-Share  Pose-Pause-Pounce-Bounce  Turn and talk  Variation Theory | Students connect their prior knowledge to explore how to rationalise denominators using the law . |
| **Summarise** | Students create their own notes in [Appendix C](#_Appendix_C_1) before returning to Appendix A to rationalise any other denominators. | Four quadrant notes | Students consolidate and apply their knowledge. |
| **Apply** | Students rationalise the denominator of a set of 5 surds, , , , and and discuss when it is best to rationalise or simplify surds first. | Visibly random groups of 3  Vertical non-permanent surfaces  Pose-Pause-Pounce-Bounce | Students apply their knowledge of surds to problems and carefully decide whether to simplify the expression first or rationalise it before proceeding. |

## Activity structure

### Launch

1. Assign students to visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) at vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)).
2. Distribute Appendix A ‘Number lines’ on A3 paper in plastic pockets to each group. The Appendix uses Variation Theory ([variationtheory.com/introduction](https://variationtheory.com/introduction/)) and asks students to estimate where they would place values on a number line.

Students should not use a calculator to complete this activity as a calculator still only provides an estimate of the actual value.

1. Have students compare their placement of each value on the number line with the groups near them.
2. Use the Pose-Pause-Pounce-Bounce questioning strategy (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) to initiate a sharing of ideas and reasoning of how students placed the values on the number lines.

Students estimated the size of surds in Stage 4 – Unit 15 – exploring circles and Lesson 1 – simplifying surds of this unit.

It is expected that students will struggle to accurately plot fractions with surds in their denominators and that leads us to the aim of the lesson.

1. State to students that in this lesson we will explore how to convert fractions with irrational denominators into numbers that we can more easily estimate the size of.

### Explore

1. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), ask students how they would find equivalent fractions for .

Students should connect that to create equivalent fractions, we multiply the fraction by equivalent fractions of 1 such as and so on.

1. Write the fraction for students to see and in another Think-Pair-Share ask students:

* When is a fraction considered to be in its simplest form?
* What would we multiply this fraction by to convert it into its simplest form?

Students should recognise that for a fraction to be in its simplest form it should have an integer denominator. To change this fraction into its simplest form we would need to multiply by , or a range of other values to result in .

1. Write the fraction for students to see and in another Think-Pair-Share ask students:

* What would we multiply this fraction by to convert it into its simplest form?
* If the denominator was instead, what would we multiply by?

1. Use the Pose-Pause-Pounce-Bounce questioning strategy to ask students:

* What would we multiply by to get a whole number?

1. Show students the law . In a turn and talk ([bit.ly/classroomtalkmoves](https://bit.ly/classroomtalkmoves)) ask students to explain why these expressions are all equivalent.
2. Initiate a sharing of responses using the Pose-Pause-Pounce-Bounce questioning strategy.

Students may explain that the square root and squaring operations are inverses of each other, returning you to your original value.

1. State to students that creating an equivalent fraction where there is no longer a surd in the denominator is called ‘rationalising the denominator’.
2. In a Think-Pair-Share ask students:

* Why is it called rationalising the denominator?
* How does the law help us to rationalise the denominator?

A surd is an irrational number, so by rationalising it, we convert the denominator into a rational number. The law helps us to determine what to multiply by to create an equivalent fraction with a rational denominator.

1. In pairs, have students complete Appendix B ‘Rationalising the denominator’ which uses Variation Theory to build the complexity of questions.
2. Use the Pose-Pause-Pounce-Bounce questioning strategy to ask students:

* When we had a simplified surd in our denominator, what did you choose to multiply it by and why?
* Could you multiply by a smaller surd value?

Students may notice that surds such as and could be simplified first and rationalised by multiplying by or.

### Summarise

1. By returning to their visibly random groups of 3, students are to complete four quadrant notes to their future forgetful selves ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)) using Appendix C ‘Four quadrant notes’.
2. Have students return to their groups of 3 and the number lines in Appendix A. They are to rationalise the fractions with irrational denominators to more accurately plot the values on their number lines.

### Apply

1. Students will continue to work in their groups of 3 on vertical non-permanent surfaces.
2. Write the following numbers so all students can see: , , , and .
3. Ask students to rationalise and simplify each of the surds in 2 different ways:

* by first rationalising the denominator and then simplifying
* by simplifying, rationalising the denominator and simplifying again.

1. Use the Pose-Pause-Pounce-Bounce questioning strategy to ask students which strategy was the most efficient and why.

The benefits of simplifying first are you reduce the complexity of the calculations by working with smaller numbers.

The benefits of rationalising first are you only simplify the fraction once to get the simplest solution.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* Students from non-English speaking backgrounds can access this task as it is visually based.
* Students use plastic pockets to encourage participation as their answer can be erased and changed at any point.
* During discussion, teachers can encourage their students to think about their decisions when plotting numbers on the number lines.

**Explore**

* All students will be able to identify equivalent fractions, particularly for a half, but may show their understanding at different depths.Students may benefit from revisiting equivalent fractions from Stage 4 to connect their learning to surds.
* Students may benefit from seeing equivalent fractions using visual representations to better their understanding.
* Challenge students to consider what fractions we cannot create an integer denominator for.
* Challenge students to consider what we would multiply binomial denominators by to get a rational denominator such as or .
* Students should be challenged to make connections with prior knowledge they have of index laws when explaining why .
* As students are working in pairs in Appendix B it provides the opportunity for students with additional needs to work in peer-assisted environments.
* Students may benefit from additional modelling or guided practice, such as Worked examples (Your turn) ([bit.ly/supportingstrategies](https://education.nsw.gov.au/teaching-and-learning/curriculum/mathematics/planning-programming-and-assessing-mathematics-7-10/mathematics-7-10-units#:~:text=.-,Supporting%20strategies,-These%20strategies%20are)) to enable them to complete Appendix B.
* Appendix B can be modified to include binomial numerators such as .

**Summarise**

* Students should be challenged to include an example that involves a binomial in the numerator.

**Apply**

* The numbers in the Apply can be modified to meet the challenge level of your students such as simpler surds in the denominators and numerators without surds.
* Challenge students to justify when it is best to simplify or rationalise first.

### Suggested opportunities for assessment

**Launch**

* Students will demonstrate their Working mathematically skills in discussions and justifications.
* When placed in groups of 3, students provide and receive peer feedback on their understanding.

**Explore**

* Monitor responses in class discussions to check for student understanding of equivalent fractions for formative assessment.
* Collect Appendix B as evidence of learning rationalising the denominator.
* Student responses to the provided prompts in the learning episode could be collected as a work sample for assessment, such as what to multiply by when surd in the form is our denominator.

**Summarise**

* Review students’ notes to future forgetful selves for the understanding of rationalising the denominator of fractions in the form .
* Collect Appendix A as evidence of learning of rationalising the denominator.

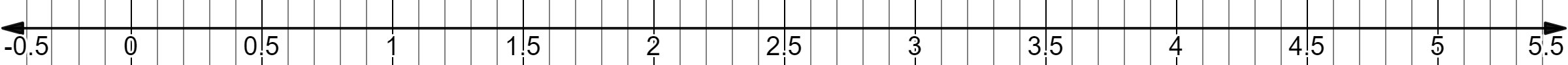
**Apply**

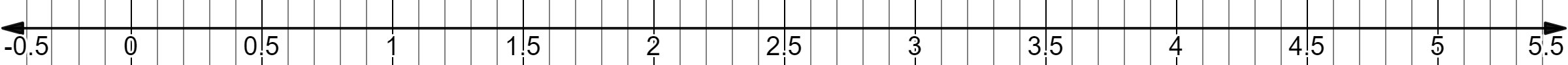
* Teacher could facilitate class discussions and observe students’ reasoning and justification in response to which strategy was most efficient in each type of question.

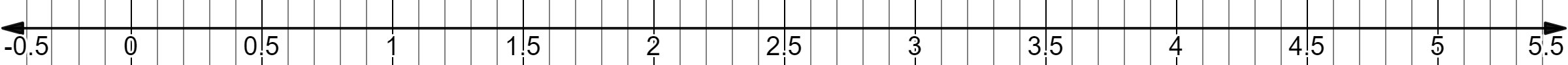
## Appendix A

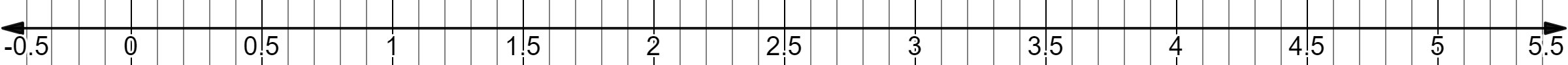
### Number lines

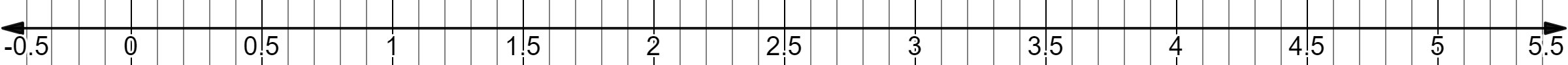
Estimate where each of the values would be placed on the following number lines.

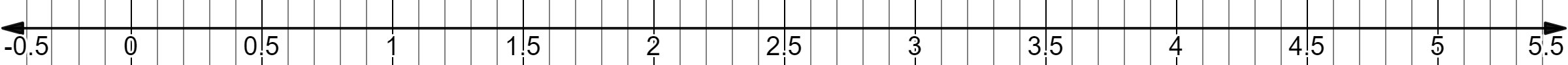


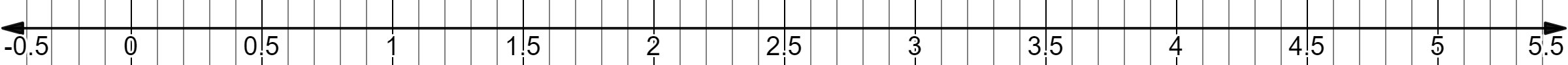


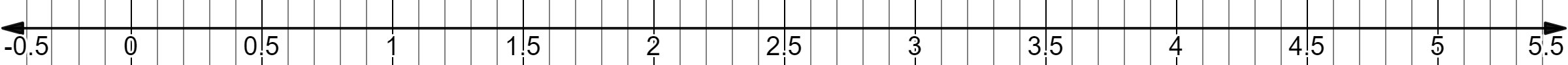












## Appendix B

### Rationalising the denominator

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This Appendix has been created from the Variation Theory activity ‘Surds: Rationalising the denominator’ ([bit.ly/rationalise](https://bit.ly/rationalise))

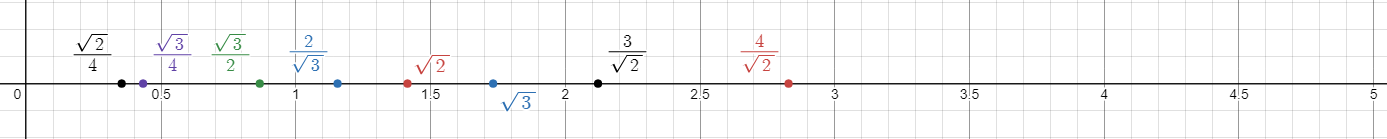
## Appendix C

### Four quadrant notes

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| --- | --- |
| **Example 1**  Rationalise the denominator of . | **Example 2**  Rationalise the denominator of . |
| **Notes to future forgetful self** | **Example 3** |

## Sample solutions

### Appendix A – number lines



### Appendix B – rationalising the denominator

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### Appendix C – four quadrant notes

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| --- | --- |
| **Example 1**  Rationalise the denominator of . | **Example 2**  Rationalise the denominator of . |

### Apply – When should I simplify?

|  |  |  |
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| Question | Simplify first | Rationalise first |
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For question 3 some students may recognise you can also simplify into by breaking the into .

## References

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Prior J (15 June 2018) [‘Rationalising the denominator’](https://variationtheory.com/2018/06/15/rationalising-the-denominator/), Variation Theory, accessed 8 January 2025.

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