# Revisiting the mean

Students solve problems involving the mean and finding unknown values.

## Visible learning

### Learning intention

* To be able to solve problems using the mean.

### Success criteria

* I can calculate the mean of a dataset.
* I can use the mean to determine unknown values in a dataset.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* compares and analyses datasets using summary statistics and graphical representations **MA5-DAT-C-01**

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## Activity structure

### Launch

1. Assign random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) and position groups at vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)) around the room.
2. Pose the following problem to students. This problem is called the ‘averages problem’.

Consider a list of all the ways you could take 4 different digits from 1 to 9 and arrange them to make the sum of two 2-digit numbers.

Some sums might appear more than once. For example, 93 + 41 and 91 + 43 have the same answer.

Problem: What is the mean of the answers to all the different possible arrangements?

1. As groups work on the problem, the teacher can move about the room asking assessing and advancing questions where appropriate.

Assessing questions draw out students’ thinking about a problem and what methods they have tried so far. Advancing questions are intended to help move student’s thinking forward toward the lesson goals. We want to draw their attention to something they may not have noticed or considered yet.

Examples of assessing and advancing questions might include:

Table 1 – assessing and advancing questions

|  |  |
| --- | --- |
| Assessing questions | Advancing questions |
| What is the task asking you to do? | Which answers appear to be the most repeated? |
| What have you noticed so far? | Can you organise your thinking to make sure you have all the solutions? |
| How are you recording your thinking? | Could you generalise your approach to find the mean of 6 distinct digits from 1 to 9 arranged to make the sum of 2- or 3-digit numbers? |

1. Select non-volunteer students to share how their group approached the problem.

The intent behind the Launch is on how students approach the problem, not on finding the solution.

### Explore

1. Assign pairs. Print and distribute a copy of Appendix A ‘Mean patterns’ to each pair.
2. Students are to first estimate and then calculate the mean for each dataset.

Allow for a variety of approaches, such as trial and error, using a calculator or solving linear equations. An alternative option is provided in Appendix A to provide students with faded examples utilising substituting into a formula.

1. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), students are to investigate how close their estimate was to the calculation of the mean. Prompting questions could include, but are not limited to:

* Did you see any patterns?
* How did each dataset change?
* What is the pattern between the numbers within a dataset?
* How did the changes affect the mean?

1. Distribute Appendix B ‘Mean puzzle’ to each pair. Students now need to find the missing value(s) from the datasets given the mean. Again, they need to first estimate what they think the missing value(s) are before calculating them.

Allow for a variety of approaches, such as trial and error, using a calculator or solving linear equations. An alternative option has been provided in Appendix B to provide students with faded examples if solving linear equations.

1. In a Think-Pair-Share, students are to investigate how close their estimate was to the calculation of the mean. Prompting questions could include, but are not limited to:

* How did each dataset change between rows?
* What is the pattern between the numbers within a dataset?
* How did this help in finding the missing values?

1. Students will now create their own mean puzzles using Appendix B ‘Mean puzzle DIY’. The mean for each dataset must be an integer solution. The shaded cells should not be filled, so the first row would be a dataset of 4 values. The last row is left for student choice.
2. Once students have completed their puzzles, have them swap with a partner to solve another person’s puzzle.
3. Provide an opportunity for students to discuss with their partner which puzzles were easiest to solve, and which caused them problems, and why.

### Summarise

1. Place a copy of Appendix C ‘Four quadrant notes’, printed on A3 paper, in reusable plastic sleeves and use adhesive putty to stick the plastic sleeves up around the room.
2. Assign new random groups of 3 and allocate each group a plastic sheet to work at. Provide each group with one whiteboard marker and cloth.
3. Groups are to discuss and work through the Four quadrant notes as described below:

Fill in the blanks: students choose numbers to fill in the blanks, completing the worked example. Students may choose numbers that don’t result in a mean of 10 and upon completing the worked example, will need to revisit their numbers chosen.

Example 1: students fill in the blanks as done previously. Students now complete the worked example.

Example 2: students repeat the worked example process, this time choosing the number of values, the pronumeral used and the mean.

Things to remember: students write down anything they think would be important for their future forgetful selves to remember.

1. Conduct a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) for students to see how other groups completed the examples.
2. Students will then complete their own 4 quadrant notes in their workbooks.

Teachers may want to print an A4- or A5-sized version of Appendix C ‘4 quadrant notes’ for students to complete and glue into their workbooks.

1. Print and distribute Appendix D ‘Mild, medium, spicy’ to each group. Groups work together through at least 3 questions of their choice.

### Apply

1. Either display or print and distribute Appendix E ‘Using the mean to make decisions’ to each student.
2. Students complete the questions, showing their reasoning on mini whiteboards or vertical non-permanent surfaces.
3. Facilitate a class discussion, asking students to consider when understanding the mean could be useful in their lives.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* Revise levelling strategy for mean in Lesson 2 – [what is normal?](https://education.nsw.gov.au/content/dam/main-education/teaching-and-learning/curriculum/mathematics/media/documents/mathematics-s4-unit-2-lesson-02-what-is-normal.docx) (DOCX 613 KB) of Year 7 Unit 2 – making decisions.
* Students could find the median, mode and range of the sums.

**Explore**

* Students could use visual representations, such as blocks, to model each of the datasets to help them visualise the missing value.
* Students could be extended into solving linear equations by using the alternative mean patterns and mean puzzle.

**Apply**

* Students could investigate local sports ladders to calculate average goals or points per game, how teams are ranked and how many points or goals a team needed to be moved up a place on the ladder.
* Students may benefit from being given a scaffold to determine the required values.

### Suggested opportunities for assessment

**Explore**

* Teachers can collect student responses and working out for the mean patterns and mean puzzle worksheets in the appendices.
* Teachers can assess student understanding through whole-class discussions, following student completion of the mean patterns and mean puzzle activities in the appendices.

**Summarise**

* Teachers could choose to use the Appendix D ‘Mild, medium, spicy’ questions as part of an exit ticket process, supporting students to reflect on the learning intention and success criteria.

**Apply**

* Teachers could choose to collect student responses to Appendix E ‘Using the mean to make decisions’ questions to check for understanding.

## Appendix A

### Mean patterns

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Datasets |  |  |  |  | Estimated mean | Actual mean () |
| 3 | 6 | 8 | 9 | 14 |  |  |
| 2 | 5 | 7 | 8 | 13 |  |  |
| 4 | 7 | 9 | 10 | 15 |  |  |
| 5 | 8 | 10 | 11 | 16 |  |  |
| 6 | 12 | 16 | 18 | 28 |  |  |
| 9 | 18 | 24 | 27 | 42 |  |  |

### Mean patterns – alternative

|  |  |  |
| --- | --- | --- |
| First estimate and then find the mean of 3, 6, 8, 9, 14. | First estimate and then find the mean of 2, 5, 7, 8, 13. | First estimate and then find the mean of 4, 7, 9, 10, 15. |
| Estimate = | Estimate = | Estimate = |
|  |  |  |
|  |  |  |
|  |  |  |

|  |  |  |
| --- | --- | --- |
| First estimate and then find the mean of 5, 8, 10, 11, 16. | First estimate and then find the mean of 6, 12, 16, 18, 28. | First estimate and then find the mean of 9, 18, 24, 27, 42. |
| Estimate = | Estimate = | Estimate = |
|  |  |  |
|  |  |  |
|  |  |  |

## Appendix B

### Mean puzzle

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Datasets** |  |  |  |  | **Mean** | **Estimate missing value(s)** | **Actual missing value** |
| 2 | 4 | 7 | 9 |  | 7 |  |  |
| 3 | 5 |  | 10 | 14 | 8 |  |  |
| 1 |  |  | 8 | 12 | 6 | , | , |
|  |  | 14 | 18 | 26 | 14 | , |  |
|  | 9 | 12 |  | 18 | 12 | , |  |

### Mean puzzle DIY

Place integer values into unshaded cells. The shaded cells need to be left blank for another student to complete.

Example: the first row would be a dataset of 4 values and the mean. The last row has been left open for you to choose which ones you would like to leave blank.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Datasets** |  |  |  |  | **Mean** | **Estimate missing value(s)** | **Actual missing value** |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

### Mean puzzle – alternative

|  |  |  |
| --- | --- | --- |
| First estimate the missing value(s) and then find the exact value(s) for this dataset 2, 4, 7, 9, | First estimate the missing value(s) and then find the exact value(s) for this dataset 3, 5, , 10, 14 | First estimate the missing value(s) and then find the exact value(s) for this dataset 1, , , 8, 12 |
| Estimate = | Estimate = | Estimate = |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

|  |  |
| --- | --- |
| First estimate the missing value(s) and then find the exact value(s) for this dataset , 14, 18, 26 | First estimate the missing value(s) and then find the exact value(s) for this dataset , 9, 12, , 18 |
| Estimate = | Estimate = |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Appendix C

### Four quadrant notes

|  |  |
| --- | --- |
| **Fill in the blanks**  Find the value of the pronumeral.  5, 5, 5, 9, | **Example 1**  Find the value of the pronumeral. |
| **Things to remember** | **Example 2**  Find the value of the pronumeral. |

## Appendix D

### Mild, medium, spicy questions

#### Mild

1. 9, , 6
2. 4, 20, 16,
3. 0, 5, , 1

#### Medium

1. 2.4, , 5.13
2. 10.11, 18.4, 8, , 19.6
3. 34.2, 44.9, 25.6, 38.3,

#### Spicy

1. , 15, 13 ,
2. 19, , 4, 12, 4,
3. 40, 39, 44, 40, 3

## Appendix E

### Using the mean to make decisions

1. Sam receives the following scores on their math tests: 78, 93, 82, 95. What score do they need on the next test to have an average mark of 90?
2. Sam receives the following scores on their science tests: 48, 56, 24. What scores do they need on their next 2 tests to have an average mark of at least 50?
3. In the event of a tie, an amateur football league decides which teams make the finals based on their average points scored per game.

|  |  |  |
| --- | --- | --- |
| Team | Games played | Average points |
| Griffith Goannas | 12 | 98 |
| Deniliquin Dogs | 11 | 95 |

How many points do the Deniliquin Dogs need to score in their final game to finish ahead of the Griffith Goannas?

## Sample solutions

### Appendix A – mean patterns

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Datasets** |  |  |  |  | **Estimated mean** | **Actual mean ()** |
| 3 | 6 | 8 | 9 | 14 |  | 8 |
| 2 | 5 | 7 | 8 | 13 |  | 7 |
| 4 | 7 | 9 | 10 | 15 |  | 9 |
| 5 | 8 | 10 | 11 | 16 |  | 10 |
| 6 | 12 | 16 | 18 | 28 |  | 16 |
| 9 | 18 | 24 | 27 | 42 |  | 24 |

Patterns to notice:

* Second dataset: all values one less than the first. Mean becomes one less.
* Third dataset: all values one more than the first. Mean becomes one more.
* Fourth dataset: all values 2 more than the first. Mean becomes 2 more.
* Fifth dataset: all values are doubled. The mean is doubled.
* Sixth dataset: all values are tripled. Mean is tripled.

### Appendix A – mean patterns – alternative

7, 9, 10, 16, 24

### Appendix B – mean puzzle

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Datasets** |  |  |  |  | **Mean** | **Estimate missing value(s)** | **Actual missing value** |
| 2 | 4 | 7 | 9 |  | 7 |  | 13 |
| 3 | 5 |  | 10 | 14 | 8 |  | 8 |
| 1 |  |  | 8 | 12 | 6 | , | 3,  6 |
|  |  | 14 | 18 | 26 | 14 | , | 8 |
|  | 9 | 12 |  | 18 | 12 | , | 14 |

Patterns to notice:

* Second dataset: all values one more than the first.
* Third dataset: all values one less than the first.
* Fourth dataset: all values are double the first.
* Fifth dataset: all values are 5 more than the first.

### Appendix B – mean puzzle – alternative

13, 8, 3, 4, 7

### Appendix D – mild, medium, spicy questions

#### Mild

#### Medium

#### Spicy

### Appendix E – using the mean to make decisions

1. 102
2. 61
3. 131

## References

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