# Factor it in

Students factorise algebraic expressions, including those with indices by determining the highest common factor (HCF).

Students will need at least one digital device per pair to interact with Graspable Math during this lesson.

## Visible learning

This lesson incorporates Path content.

### Learning intention

* To be able to factorise algebraic expressions.

### Success criteria

* I can identify common factors in algebraic expressions.
* I can recognise the highest common factor of terms, including those that are negative.
* I can recognise the highest common factor of terms, including those with indices.
* I can factorise expressions involving negative terms.
* I can factorise expressions involving indices.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* simplifies algebraic fractions involving indices, and expands and factorises algebraic expressions **MA5-ALG-P-01**

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Table 1: lesson summary

|  |  |  |  |
| --- | --- | --- | --- |
| Section | Summary of activity | Teaching strategy | Teaching points |
| Warm up | Students determine if the statement is always, sometimes or never true. | Think-Pair-Share | Identifies student misconceptions with indices. |
| Launch | Use slides 6 and 7 of the PowerPoint *Factor it in* for students to consider the dimensions of a rectangle. | Think-Pair-SharePose-Pause-Pounce-Bounce | Revise factors. |
| Explore | Students consider factors of and 12. They are prompted to consider negative factors and the highest factor.Use slide 9 to introduce the definition of ‘factorise’.Students access Graspable Math ([bit.ly/graspablemaths](https://bit.ly/graspablemaths)) to complete [Appendix A](#_Appendix_A) to recognise HCF in expressions and to factorise expressions.  | Think-Pair-Share | Recognise highest common factors. |
| Summarise | Students look at worked examples from slides 11–18 of the PowerPoint on factorising expressions.Students complete the banner task in [Appendix B](#_Appendix_C).Students complete four quadrant notes in [Appendix C](#_Appendix_D). | Worked examples (Your turn)Visibly random groups of 3Vertical non-permanent surfacesFour quadrant notes  | Explicit teaching of factorising expressions.  |
| Apply | Students complete variation task in [Appendix D](#_Appendix_D_1).Students complete an open middle problem in [Appendix E](#_Appendix_E_1) and complete an exit ticket. | Variation theoryExit ticket | Students apply their knowledge. |

## Activity structure

Please use the associated PowerPoint *Factor it in* to display images in this lesson.

### Warm up

1. Display slide 3 from the PowerPoint Factor it in.
2. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), ask students to consider if the statement would be always true, sometimes true or never true.

This statement is only true when , as that makes both sides of the equation equal zero. Slide 4 from the PowerPoint can be used to show a visual representation of why the 2 expressions are not always equal.

### Launch

1. Display slide 6 from the PowerPoint which shows Figure 1.

Figure 1: area of a rectangle



1. Using a Think-Pair-Share ask students to discuss what the dimensions of the rectangle could be.
2. Using the Pose-Pause-Pounce-Bounce questioning strategy (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) ask students how we can write expressions for the area of the rectangle calculation.

Students should realise that the area can be expressed as 4 × 4 + 4 × 2 which is equivalent to
4 × (4 + 2) (or any of the other relevant combinations).

1. Display slide 7 from the PowerPoint which shows Figure 2.

Figure 2: area of another rectangle



1. Working in the same pairs, students are to discuss what the dimensions of this rectangle could be and how they can express the area calculation.
2. Using the Pose-Pause-Pounce-Bounce questioning strategy, ask students to share their answers, focusing on how they knew what lengths the height of the rectangle could be

Students should identify that the height of the rectangle must be a common factor of both 18 and 12.

### Explore

1. In a Think-Pair-Share, ask students to write down all the factors of and 12. From those lists ask students to determine the highest common factor.
2. Students will most likely only have considered positive factors. Ask students to consider whether -2 would be a factor of either expression.

The factors of are as well as the negative factors of each. The factors of 12 are 1, 2, 3, 4, 6 and 12, as well as the negative factors of each. The highest common factor between and 12 is 6.

1. Show slide 9 from the PowerPoint *Factor it in* to demonstrate the meaning of the term factorised.

Teachers should tell students that the process of removing a common factor out of an expression and placing it out the front of the remaining expression in brackets is called ‘factorisation’. It is the opposite process to expanding brackets.

The NESA glossary defines factorising as expressing a number or algebraic expression as a product. It is preferable in mathematics to factorise using the highest common factor as it creates the simplest version of an expression.

1. Distribute a copy of Appendix A ‘Graspable math’ to each student.
2. Working in pairs, students are to use devices to access the Graspable Math activity ‘Algebraic factorisation’ ([bit.ly/graspablemaths](https://bit.ly/graspablemaths)). Students will be prompted to enter their name before entering the task.
3. Students record the expression from Graspable Math, the highest common factor and its factorised form as they work through the activity. Students should try to work out the highest common factor for the expression before dragging terms across to find the solution.

For help with using Graspable Math with a class, use the tutorials on the webpage ‘Get Started’ ([activities.graspablemath.com/teacher/get-started](https://activities.graspablemath.com/teacher/get-started)).

Students select one term from the expression (for example, 12) and drag it over the top of the The colour of the term that is moving changes colour. This process should produce the factorised version of the expression. When the expression is fully factorised, the screen will turn green.

The expressions must be written using mathematical conventions. Coefficients must be before variables and variables should be in alphabetical order. Graspable Math only recognises the mathematical convention where the factor sits out the front of the expression, although it doesn’t necessarily need to (example, the formula for finding the nth term in an arithmetic series) so for the purposes of this exercise, the highest common factor should be in front of the brackets.

### Summarise

1. Use slides 11–18 from the PowerPoint *Factor it in* for explicit teaching of factorising expressions with indices and positive and negative coefficients using the [worked examples (Your turn) method (DOCX 420 KB)](https://education.nsw.gov.au/content/dam/main-education/documents/teaching-and-learning/curriculum/mathematics/mathematics-s4-supporting-strategies-worked-examples-your-turn.docx).
2. Working in visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) on vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)), students are to complete a series of banner questions. A list of questions for teachers to select questions from is in Appendix B ‘Banner questions’.
3. Instruct each group to draw a horizontal line approximately 10 cm from the top of their vertical non-permanent surfaces to create a banner, as shown in Figure 3.

Figure 3: banner



1. Ask groups to write the following problem on their banner:
2. Explain to students that only the question will go in the banner, all their working and the answer will go underneath the banner. Groups can then begin working on solving the problem in the space underneath their banner.

Actively monitor groups for student misconceptions. Many groups may **incorrectly** state the final answer as being , where their working could look as follows:

If this occurs, have the group go and talk to a group that has the correct answer. Don’t tell the groups who is correct, just ask them to discuss their differing answers. The correct answer is Students could also have correctly answered or . This could be an opportunity to talk to the group about different representations of the same expression.

1. When a group has found the correct answer, students can ask the teacher for another question from the list in Appendix B ‘Banner questions’ or they can ‘steal’ questions from another group’s banner.

Students should be encouraged to ‘steal’ questions from other groups in a banner task. This enables students to continue to work without the continual intervention by the teacher, allowing the teacher to spend time observing and assessing student progress. Students do not have to complete all of the questions listed in Appendix B. It is a list for the teacher to select from.

Challenging questions have been included in Appendix B ‘Banner questions’ to extend students who have mastered the easier content. It may be necessary to use assessing and advancing questions to assist these students with their thinking.

Table 2: assessing and advancing questions

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| Assessing questions | Advancing questions |
| * What terms have you factorised so far?
 | * Could you factorise parts of the expression?
 |
| * What are the factors that you have found for each term in the expression?
 | * Could you consider an expression as a common factor?
 |
| * What is the highest common factor that you have found?
 | * Could you consider changing the sign in front of your factor?
 |

1. By continuing to work in their same visibly random groups of 3, students are to complete [four quadrant notes (DOCX 319 KB)](https://education.nsw.gov.au/content/dam/main-education/documents/teaching-and-learning/curriculum/mathematics/mathematics-s4-supporting-strategies-four-quadrant-notes.docx) from Appendix C ‘Four quadrant notes’. After reviewing other groups’ four quadrant notes, students should then write their version of four quadrant notes in their workbook.

### Apply

1. Distribute a copy of Appendix D ’Variation task’ to each student.
2. Working in pairs, students are to complete the task which uses variation theory ([variationtheory.com/introduction/](https://variationtheory.com/introduction/)).
3. Working in visibly random groups of 3 at a vertical non-permanent surface, students are to complete Appendix E ‘Open middle problems’.

Students can access the Graspable Math ‘whiteboard’ function ([bit.ly/graspable\_whiteboard](https://bit.ly/graspable_whiteboard)) to assist in solving the problems in Appendix E.

There are multiple ways that students can attempt these questions. All problems have multiple answers and students should aim to find as many answers as they can.

1. Students are to complete an exit ticket ([bit.ly/exitticketstrategy](https://bit.ly/exitticketstrategy)) by responding to the prompt, ’Explain why hasn’t been completely factorised.’

## Assessment and differentiation

### Suggested opportunities for differentiation

**Warm up**

* The teacher could suggest substituting different values of and observe what the results are when looking at the differences between the 2 expressions.

**Launch**

* Students could use grid paper or geoboards to help them to determine the dimensions of the rectangles.
* Students could use algebra tiles to create the areas and determine the dimensions of the rectangles.

**Explore**

* Students could again use algebra tiles or grid paper to help them to determine the factors of the numbers.
* When completing Appendix A ‘Graspable math’ students can reset the expression back to the original form if they overcomplicate the factorisation process. Graspable Math only accepts answers in conventional algebraic form.
* Graspable Math determines the highest common factor, so students can focus on the idea of factorisation.

**Summarise**

* Banner questions do not need to be given in order. Teachers can choose a question at the appropriate level of readiness. A separate list of Challenge banner questions in Appendix B can be used for students at that level of readiness.
* During the banner task students could observe other students’ work for hints to move forward in the task. Groups of 3 work together to support each other’s learning and understanding.
* Assessing and advancing questions have been provided to allow teachers to assist students without just telling them the answer.

**Apply**

* The variation task gets progressively more challenging as the list goes on. The questions towards the end of the list challenge students, but the easier questions support students to understand the process.
* **Appendix E ‘Open middle problems’ has multiple solutions. Students may only find one solution whereas students needing extension could find multiple solutions, or specifically only look for the fully factorised solutions.**

### Suggested opportunities for assessment

**Warm up**

* Discussions from the Think-Pair-Share could provide formative assessment on student understanding of indices.

**Launch**

* Discussions from the Think-Pair-Share could provide formative assessment information on student understanding of factors.

**Explore**

* Observe student understanding of factorisation through the completion of the Graspable Math activity.
* Appendix A ‘Graspable math’ could be collected as evidence of student understanding of finding factors, calculating highest common factors, and using the HCF to factorise an expression.

**Summarise**

* Observe student achievement and understanding through conversations in the banner task.
* Students’ four quadrant notes can be used to demonstrate understanding.

**Apply**

* Appendix D ‘Variation task’ could be collected as evidence of learning and understanding.
* Collect the exit ticket for formative assessment purposes.

## Appendix A

### Graspable math

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| Expanded expression | Highest Common Factor | Factorised expression |
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## Appendix B

### Banner questions

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2.

#### Challenge banner questions

## Appendix C

### Four quadrant notes

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| **Example 1****Factorise** | **Example 2****Factorise**  |
| **Things to remember** | **Example 3**  |

## Appendix D

### Variation task

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| Question | Solution |
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## Appendix E

### Open middle problems

1. Place the numbers 0–9 into the boxes to make the statement correct. Each digit may only be used once. There are 2 solutions to this problem.
2. Place the numbers 0–9 into the boxes to make the statement correct. Each digit may only be used once. There are 35 solutions to this problem.
3. If is the result of using the distributive property, find all possible combinations that could have resulted. Which one is using the highest common factor? There are 8 solutions to this problem.

## Sample solutions

### Appendix B – banner questions

#### Challenge banner questions

### Appendix C – four quadrant notes

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| --- | --- |
| **Example 1****Factorise** | **Example 2****Factorise**  |
| **Things to remember.*** **Look for the highest common factor in all terms of the expression.**
 | **Example 3****Factorise** |

### Appendix D – variation task

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| Question | Solution |
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### Appendix E – Open middle problems

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| --- | --- | --- |
| Problem 1 | Problem 2 | Problem 3 |
|  |  |  |

## References

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