# Euler’s number

Students explore the effect that increasing the frequency of compounding periods has on the overall interest. They then explore how the frequency of compounding interest relates to Euler’s number .

## Visible learning

### Learning intention

* To understand how increasing the compounding periods affects interest and depreciation.

### Success criteria

* I can calculate compound interest when compounded at different periods.
* I can describe how interest and depreciation change as compound frequency changes.
* I can explain how compound interest relates to Euler’s number .

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* solves financial problems involving compound interest and depreciation **MA5-FIN-C-02**
* identifies connections between algebraic and graphical representations of quadratic and exponential relationships in various contexts **MA5-NLI-C-01**

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Table 1: lesson summary

|  |  |  |  |
| --- | --- | --- | --- |
| Section | Summary of activity | Teaching strategies | Teaching points |
| Launch | Students discuss a scenario (slides 3 and 4 of the *Euler’s number* PowerPoint) where interest is calculated annually and then discuss the changes that occur if interest was calculated daily. | Pose-Pause-Pounce-BounceThink-Pair-ShareNotice and wonder | The purpose of this activity is to show students that interest is not always calculated per annum and is often calculated per day. |
| Explore | Would you rather scenarios on slide 7 of the PowerPoint are explored, calculated and used to generalise.Students vote on and discuss which investment is best and why. | Visibly random groups of 3Vertical non-permanent surfacesGallery walkPose-Pause-Pounce-Bounce | The purpose of this activity is for students to use calculated values to recognise that the interest changes if it is compounded more often. |
| Summarise | Students complete [Appendix A](#_Appendix_A_1) followed by a notice and wonder. PowerPoint slide 9 displays associated graphs where students confirm their thinking.Students complete a card sort [Appendix B](#_Appendix_B) or the alternate on Desmos. | Think-Pair-Share | The purpose of this activity is for students to practise using the compound interest formula and to become familiar with changing the time period and the interest rate. |
| Apply | Students explore the various version of $1 invested at 100% per annum, for one year, compounded … to discover Euler’s number. | Visibly random groups of 3Vertical non-permanent surfacesPose-Pause-Pounce-BounceNotice and wonder | The purpose of this activity is for students to develop Euler’s number by increasing the compounding periods. |

## Activity structure

Please use the associated PowerPoint *Euler’s number* to display images in this lesson.

### Launch

1. Write or display the following scenario on slide 3 of the PowerPoint *Euler’s number*:

Sam invested $20 000 in a savings account for 5 years, which pays interest at 6% per annum, compounded annually.

1. Use a questioning strategy such as Pose-Pause-Pounce-Bounce (PDF 557 KB] ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) to ask students if this is a realistic scenario of how money is invested. Why or why not? Prompting questions could include:
* Would you be happy only receiving interest once per year?
* Would a bank calculate interest annually?
* What would be some problems that occur if interest was calculated annually?
* How often do you think banks calculate interest?
1. Explain to students that most banks calculate interest daily but pay it at the end of each month. Edit the previous scenario or display slide 4 of the PowerPoint *Euler’s number* to now be:

Sam invested $20 000 in a savings account for 5 years, which pays interest at 6% per annum, compounded daily.

The following link from Westpac could be shared with students, explaining how Westpac calculates interest ([bit.ly/westpacinterestcalculated](https://bit.ly/westpacinterestcaclualted)).

1. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), ask students:
* How does this change affect the amount of interest earned?
* How many days are there in 5 years?
* What is 6% per annum as a daily rate?
* Could you use these values to calculate the future value of Sam’s investment compounded daily?

### Explore

1. Display slide 6 of the PowerPoint *Euler’s number*. Students are to compare the scenario compounded annually and compounded daily. Ask students what they notice and wonder ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) about the future value when compounded daily compared to compounded annually.

Students might notice that the rate and number of periods have been divided and multiplied by 365, respectively.

Students might wonder if compounded hourly would produce even more interest.

1. Assign students to visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) at vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)) around the classroom, with one marker between each group.
2. Write up the prompt below or display slide 7 of the PowerPoint *Euler’s number* to pose the following would you rather problems for groups to work on:

Would you rather invest at:

* an interest rate of 100% per annum compounded annually
* an interest rate of 90% per annum compounded bi-annually
* an interest rate of 80% per annum compounded quarterly
* an interest rate of 70% per annum compounded daily?
1. Allow students time in their groups of 3 to investigate the would you rather problems.

Students could test values, for example and years:

1. Once students have had time to engage with the task, have them complete a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) to observe how other groups approached the problem.
2. Have students return to their vertical non-permanent surface and conduct a finger vote ([bit.ly/hingepointstrategy](https://bit.ly/hingepointstrategy)), for students to vote on which investment they would rather.
3. Use a questioning strategy such as Pose-Pause-Pounce-Bounce to conduct a class discussion for students to share their reasoning, any generalisations they were able to make, and how they approached the problem.

### Summarise

1. Students are to complete the faded worked examples ([bit.ly/fadedexamplesstrategy](https://bit.ly/fadedexamplesstrategy)) in Appendix A ‘Faded worked examples’. These problems have students calculate the future value of an investment and the salvage value of an asset with varying compounding periods.

The faded worked examples show one way that students could set out their working, by adjusting the rate and periods within the formula. Students could instead adjust the rate and periods before entering these values into the formula. If doing so, they either need to be wary of rounding or use the memory function in the calculator to avoid rounding errors.

1. Once students have completed the faded worked examples, ask them what they notice and wonder about the interest gained and depreciation lost as compounding becomes more frequent.

Students should notice that as the frequency of compounding periods increases, the interest earned on the investment increases, however, the depreciation lost on the laptop decreases.

Students might also notice that the magnitude of the difference between interest and depreciation value reduces with each increase in frequency.

1. Display the graphs of , , and on slide 9 of the PowerPoint.

There are 3 graphs on the slide, however, you may need to zoom in on the graphs as they approach to see the difference between and .
The purpose of showing the graphs is for students to understand that there is a significant difference between compounding annually and monthly, but a negligible difference between compounding monthly and daily.

1. In a Think-Pair-Share, ask students how the graphs support their understanding of increasing the compounding periods.

Students may notice that the graph increases faster with more compounding periods.

1. Print, cut and distribute one copy of Appendix B ‘Card sort’ to pairs of students.

Teachers could choose to provide each set of cards, interest and depreciation separately or together. The cards are matched with their solution in Appendix B. They should be cut prior to being distributed to students.

1. Pairs work together to match each scenario with its corresponding formula. Once finished, pairs should check that they have the same answers as a neighbouring pair.

Alternatively, you can complete this activity online. Before doing this activity, you will need to set up a Desmos classroom ([bit.ly/managingdesmosclassroom](https://bit.ly/managingdesmosclassroom)) and assign the activity ‘Compound interest and depreciation card sort’ ([bit.ly/desmosinterestdeprec](https://bit.ly/desmosinterestdeprec)). Distribute one digital device between pairs of students, and direct students to join the activity. Pairs work together to match each scenario with its corresponding formula.

### Apply

1. Students return to their vertical non-permanent surfaces in their earlier assigned groups of 3.
2. Pose the following scenario to students, which is also on slide 11 of the PowerPoint:

$1 invested at 100% per annum, for one year, compounded …

1. Starting with compounding annually, ask students to compare as many variations as possible of compounding periods that they can think of, within one year. Recording the results for each compounding period.

For example, compounded bi-annually, quarterly, monthly, weekly, daily, hourly, every minute, every second or every millisecond.

1. Once students have had a chance to try a variety of compounding periods, students conduct a gallery walk to compare answers from other groups and observe how they have recorded their results.
2. Use a questioning strategy such as Pose-Pause-Pounce-Bounce to ask students what they notice and wonder about their results.

Students may notice that as the compounding periods become more frequent, the future value approaches 2.71828182846.

1. Ask a student from each group to type into a calculator then press **=**.

On most calculators can be entered by pressing **ALPHA** then **EXP**.

1. Show the graph of as it approaches , on slide 12 of the PowerPoint *Euler’s number*.
2. Explain to students that is known as Euler’s number (pronounced oy-luh). The number was first discovered by Jacob Bernoulli in 1683 when performing the same calculations that you did with compound interest. Leonhard Euler gave its name in 1731, as it was found to occur in many natural settings, including probability, statistics, engineering, biology, thermodynamics and physics.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* Students could be provided alternate compounding periods to compare in the launch to make the calculations simpler or more difficult.
* Students may need assistance with entering these calculations into their calculators.
* Regardless of readiness to engage with content, all students should be able and encouraged to add to class discussions by linking interest to a real-world context.

**Explore**

* **Students could verify their answers throughout the learning episode using an online compound interest calculator (**[bit.ly/investorcalc](https://bit.ly/investorcalc)**).**
* **The would you rather problem has a low floor and high ceiling. Students could test just one present value and number of years to compare each scenario or high readiness students could attempt to generalise which scenario is best given variable present values and numbers of years.**
* **Low readiness students could be assigned a role such as ‘knowledge mobiliser’, where they observe the values other groups are using and report back to their own group, to allow them to contribute without having to take the lead in their own group.**
* **If many students are not confident changing compound frequency within the formula, further instruction on how to use the formula could be required.**

**Summarise**

* Students who require further assistance could complete faded worked examples with support then complete a similar set of problems with different values to receive more guided support before completing the activity independently.
* Students could be challenged to compare the difference in interest earned or charged between each time period and explain why some are larger than others.
* Students could mix printed interest and depreciation cards to create further challenge in the matching activity.

**Apply**

* **Students could explore applications of Euler’s number , to enhance their engagement with subsequent learning episodes.**
* **Students could be provided with a table of values with values already filled to provide further scaffolding.**

### Suggested opportunities for assessment

**Launch**

* Students’ responses within the Think-Pair-Share should be observed to ensure students are able to convert between units of time and have a sound understanding of compound interest.

**Explore**

* **Students’ discussions and responses to the would you rather problem should be used to assess their confidence with the compound interest formula with varying compound frequencies.**
* **The finger vote activity allows for formative assessment of the entire class. Use this as an opportunity to draw out and address any misconceptions.**

**Summarise**

* Students’ faded worked examples could be collected as evidence of learning.
* If using the Desmos classroom activity, teachers can use the teacher dashboard to observe student progress and provide feedback in real time or after the lesson.

**Apply**

* **The Apply acts as a final opportunity to assess students’ ability to calculate compound interest with a variety of compounding periods.**

###

## Appendix A

### Faded worked examples

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Calculate the interest earned on $25 000 invested at 4.5% per annum, compounded quarterly for 10 years. | Calculate the interest earned on $25 000 invested at 4.5% per annum, compounded monthly for 10 years. | Calculate the interest earned on $25 000 invested at 4.5% per annum, compounded weekly for 10 years. | Calculate the interest earned on $25 000 invested at 4.5% per annum, compounded daily for 10 years. | Calculate the interest earned on $25 000 invested at 4.5% per annum, compounded hourly for 10 years. |
| Interest |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Calculate the salvage value of a laptop initially worth $1999, depreciating at a rate of 5% per annum, compounded annually for 6 years. | Calculate the salvage value of a laptop initially worth $1999, depreciating at a rate of 5% per annum, compounded bi-annually for 6 years. | Calculate the salvage value of a laptop initially worth $1999, depreciating at a rate of 5% per annum, compounded quarterly for 6 years. | Calculate the salvage value of a laptop initially worth $1999, depreciating at a rate of 5% per annum, compounded monthly for 6 years. | Calculate the salvage value of a laptop initially worth $1999, depreciating at a rate of 5% per annum, compounded daily for 6 years. |
|  |  |  |  |  |

## Appendix B

### Card sort

|  |  |
| --- | --- |
| You invested $5000 at 6% per annum compounded annually for 5 years. |  |
| Your 2-year investment of $5000 earns 2.7% per annum and is compounded annually. |  |
| You invested $5000 at 4.2% per annum which is compounded monthly for 7 years. |  |
| Your 5-year investment of $5000 earns 6% per annum and is compounded monthly. |  |
| You borrowed $5000 at 5.5% per annum which is compounded weekly for 2 years. |  |
| Your investment earned interest at a rate of 5% per annum compounded annually for 3 years, after which its value is $5000. |  |

|  |  |
| --- | --- |
| An asset has an initial value of $35 000 and depreciates at a rate of 5% per annum, compounded annually for 6 years. |  |
| An asset has an initial value of $35 000 and depreciates at a rate of 2% per annum compounded annually for 10 years. |  |
| An asset has an initial value of $35 000 and depreciates at a rate of 12% per annum compounded monthly for 3 years. |  |
| An asset has an initial value of $35 000 and depreciates at a rate of 6.5% per annum compounded weekly for 5 years. |  |
| Your stock valued at $35 000 loses value at a rate of 18% per annum compounded monthly for 3 months. |  |
| Your motorbike has an initial value of $38 000 and depreciates at a rate compounded annually for 6 years, resulting in a salvage value of $35 000. |  |

## Sample solutions

### Appendix A – faded worked examples

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Calculate the interest earned on $25 000 invested at 4.5% per annum, compounded quarterly for 10 years. | Calculate the interest earned on $25 000 invested at 4.5% per annum, compounded monthly for 10 years. | Calculate the interest earned on $25 000 invested at 4.5% per annum, compounded weekly for 10 years. | Calculate the interest earned on $25 000 invested at 4.5% per annum, compounded daily for 10 years. | Calculate the interest earned on $25 000 invested at 4.5% per annum, compounded hourly for 10 years. |
|  |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Calculate the salvage value of a laptop initially worth $1999, depreciating at a rate of 5% per annum, compounded annually for 6 years. | Calculate the salvage value of a laptop initially worth $1999, depreciating at a rate of 5% per annum, compounded bi-annually for 6 years. | Calculate the salvage value of a laptop initially worth $1999, depreciating at a rate of 5% per annum, compounded quarterly for 6 years. | Calculate the salvage value of a laptop initially worth $1999, depreciating at a rate of 5% per annum, compounded monthly for 6 years. | Calculate the salvage value of a laptop initially worth $1999, depreciating at a rate of 5% per annum, compounded daily for 6 years. |
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## References

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