# Exponential Marbleslides

Students make use of Desmos’ Marbleslide activities to explore transformations with exponential equations.

Students will need at least one digital device per pair to interact with Desmos during this lesson.

## Visible learning

This lesson incorporates Path content.

### Learning intention

* To be able to graph exponential relationships in the form $y=k\left(a\right)^{x}+c$.

### Success criteria

* I can identify the y-intercept of an exponential graph.
* I can write the equation for an exponential graph's horizontal asymptote.
* I can describe how graphs in the form $y=k\left(a\right)^{x}+c$ differ from $y=a^{x}.$

## Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* identifies connections between algebraic and graphical representations of quadratic and exponential relationships in various contexts **MA5-NLI-C-01**
* identifies and compares features of parabolas and exponential curves in various contexts
**MA5-NLI-C-02**
* interprets and compares non-linear relationships and their transformations, both algebraically and graphically **MA5-NLI-P-01**

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Table 1: lesson summary

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| Section | Summary of activity | Teaching strategy | Teaching points |
| Launch | Display screens 2–4 of the Desmos activity ‘Exponential Marbleslides’ ([bit.ly/desmosexponslide](https://bit.ly/desmosexponslide)). Pairs respond to each screen by writing answers on mini whiteboards, teacher uses answers to facilitate class discussion. | Mini whiteboardsPose-Pause-Pounce-Bounce | Connect to prior learning of transformations of linear relationships. |
| Explore | Pairs sharing one digital device complete screens 5–13 of the Desmos activity. |  | Students interact with Desmos Marbleslide activities to explore how the values of $k$, $a$, and $c$ effect the graph of $y=k\left(a\right)^{x}+c$. |
| Summarise | Use slides 3–7 of the PowerPoint to teach graphing exponential relationships of the form $y=k\left(a\right)^{x}+c$.Display 5 exponential graphs using slides 8–17 of the PowerPoint. Pairs write the corresponding equation on mini whiteboards.Students write notes to their future forgetful selves. | Worked examples (your turn)Pose-Pause-Pounce-BounceNotes to future forgetful selves | Explicit teaching of graphing exponential relationships. Your turn problem and the use of mini whiteboards allow for assessment and opportunities to address gaps in knowledge or misconceptions. |
| Apply | Students play the Desmos activity ‘Polygraph: Exponentials’ ([bit.ly/exponentialpolygraph](https://bit.ly/exponentialpolygraph)). |  | Students practice communicating in reference to exponential graphs and their equations. |

## Activity structure

Please use the associated PowerPoint *Exponential Marbleslides* to display images in this lesson.

### Launch

1. Display screen 2 of the Desmos activity ‘Exponential Marbleslides’ ([bit.ly/desmosexponslide](https://bit.ly/desmosexponslide)) by selecting **Preview** then **Next >**.
2. Ask students to predict what will happen when you select **Launch**.
3. Select **Launch** to see the marbles drop from their starting point and pass through the star, resulting in ‘Success!’.
4. Assign students into pairs and provide each pair with a mini whiteboard and a marker ([bit.ly/miniwhiteboards](https://bit.ly/miniwhiteboards)).
5. Select **Next >** to display screen 3. Pairs discuss which value for $c$ would make the marbles pass through the star. Pairs write an answer on their mini whiteboards to hold up all at once for the teacher to see.
6. Choose several $c$ values that will fail before completing the slide.

Answers could be between $c=1.4$ and $c=2$.

1. Select **Next >** to display screen 4. Pairs discuss which of the 4 equations would make the marbles pass through the star. Pairs write an equation on their mini whiteboards to hold up all at once for the teacher to see.
2. Use a questioning strategy such as Pose-Pause-Pounce-Bounce (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) to facilitate a class discussion on what each graph might look like and why $y=2^{x}-3$ is the correct answer.

Launch all the incorrect graphs before launching the graph $y=2^{x}-3.$

### Explore

1. Before doing this activity, you will need to set up a Desmos classroom ([bit.ly/desmosclassroomstrategy](https://bit.ly/desmosclassroomstrategy)) and use the pacing feature to restrict the students to slides 5–13 of the Desmos activity ‘Exponential Marbleslides’ ([bit.ly/desmosexponslide](https://bit.ly/desmosexponslide)).
2. Assign one digital device between pairs of students and direct students to join the classroom activity.
3. Pairs work together to complete screens 5–13.
* Sample solutions for each screen are included in this learning episode.
* Screens 12 and 13 require limiting domains. Students could be restricted to screens 5–11 if teachers are wanting to avoid this.

### Summarise

1. Use slides 3–6 of the PowerPoint Exponential Marbleslides for explicit teaching of solving graphing exponential equations using the [Worked examples (your turn) method](https://education.nsw.gov.au/teaching-and-learning/curriculum/mathematics/planning-programming-and-assessing-mathematics-7-10/mathematics-7-10-units) ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)).
2. Use slide 7 of the PowerPoint to display the restrictions for exponential equations.
3. Use a questioning strategy such as Pose-Pause-Pounce-Bounce to facilitate a class discussion of the self-explanation prompts on slide 7.

 $a>0$ because if $a<0$ the graph is not continuous. For example, let $a=-2$; $(-2)^{\frac{1}{2}}$ does not exist (can’t square root a negative number), $\left(-2\right)^{1}=-2, \left(-2\right)^{2}=4, \left(-2\right)^{3}=-8$.

If $a=1$ then the equation would be $1^{x}$. One to any power is one.

1. Use slides 8–17 of the PowerPoint to show 5 exponential graphs. For each graph, pairs of students write down on mini whiteboards the equation they believe the graph represents. Ask students to raise their whiteboards and facilitate a class discussion using the prompts below:
* Why did you choose that equation?
* Which equation did you rule out first? Why?
* How does $c$ affect the asymptote of $y=a^{x}+c$?
* How does $k$ affect the asymptote of $y=k\left(a\right)^{x}$?

Graph 3 on slides 12–13 has 2 solutions: $y=10^{-x}$ is equivalent to $y=(\frac{1}{10})^{x}$. Challenge students to explain why, using the property $a^{-1}=\frac{1}{a}$. Graph 5 on slides 16–17 also has 2 solutions.

1. Have students write notes to their future forgetful self ([bit.ly/notestofutureself](https://bit.ly/notestofutureself)) explaining how the values of $k$, $a$, and $c$ affect the graph of $y=ka^{x}+c$.

### Apply

1. Before beginning this activity, you will need to set up a Desmos classroom for the Desmos activity ‘Polygraph: Exponentials’ ([bit.ly/exponentialpolygraph](https://bit.ly/exponentialpolygraph)).
2. With one digital device between pairs of students, direct students to join the Desmos classroom activity.

Challenge students to ask questions about the equation of each graph. For example, is your exponent positive? Is your $c$ value less than 1?

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* Students discuss answers in a Think-Pair-Share, allowing them to communicate their reasoning and test their thinking with a partner. Low readiness students can bounce off their partner’s ideas if they are not confident suggesting their own.
* There are a range of possible answers for screen 3, reducing the accuracy required by students.

**Explore**

* To challenge students, set the Desmos Classroom activity ‘Marbleslide challenge set’ ([bit.ly/marbleslideset1](https://bit.ly/marbleslideset1)), to complete further challenges that require the use of various types of graphs.
* Use pacing to restrict the screens students have access to if wanting to work through each screen as a whole class.
* Provide students with graph paper and have them design their own Marbleslide puzzles. These could be placed in plastic pockets and drawn on with markers to make reusable puzzles that can be shared with peers.

**Summarise**

* Graph 3 on slides 12–13 has 2 solutions as $y=10^{-x} $is equivalent to $y=\frac{1}{10}^{x}.$ Challenge students to explain why using the property $a^{-1}=\frac{1}{a}$. Graph 5 on slides 16–17 also has 2 solutions.
* Low readiness students may not be confident providing answers when discussing the graphs on slides 8–17, in this case, they should be asked to agree or disagree with another student’s statements.
* Student progress can be monitored using Desmos’ teacher dashboard. This can be used to provide hints or extensions as students work through the Marbleslide activity. For example, challenging students to come up with an alternative equation to solve a problem.

**Apply**

* Challenge students to focus on asking questions that reference the equation of exponential graphs.
* Lower readiness students could ask questions about the appearance of the graphs with informal language, for example, ‘Does your graph curve upwards?’.

### Suggested opportunities for assessment

**Launch**

* Mini whiteboards allow for whole class assessment.
* Look for students to demonstrate prior knowledge of how transformations affect linear functions.

**Explore**

* **Students’ progress can be monitored in real-time or after this learning episode using Desmos’ teacher dashboard. Individual feedback can also be given using the dashboard.**

**Summary**

* **Students’ responses to the ’Your turn’ problem should be observed to ensure they can sketch the graph of an exponential equation. Additional examples could be required.**
* **Mini whiteboards allow for whole class assessment.**
* **Use class discussions to draw out misconceptions and observe students’ reasoning.**

**Apply**

* **A question-and-answer log is created as students interact with the Desmos Polygraph activity. Observe students’ questions and challenge them to use correct terminology that focuses on the equation of each graph.**

## Sample solutions

### Desmos activity ‘Exponential Marbleslides’

**Screen 5:** $y=2^{x}+4$

**Screen 7:** $y=2^{-x}-3$

**Screen 8:** $y=2^{-x}-10$

**Screen 9:** $y=2\left(2\right)^{x}$, $k=2$

**Screen 10:** $y=1.1^{x}+2$

**Screen 11:** $y=3^{-x}+1$

**Screen 12:** $y=0.9^{x}+10 \left\{x<6\right\}, y=1.1^{x}+7 \left\{x>-5\right\}, y=0.9^{x}+4 \left\{x<6\right\}, =1.1^{x}+1 \left\{x>-5\right\}$

## References

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