# Free falling

Students explore exponential relationships through an experiment and question what occurs as $x$ becomes extremely small or large.

During this lesson students will need at least one digital device per pair to interact with Desmos.

## Visible learning

### Learning intention

* To understand what happens to an exponential relationship as $x$ gets really large.

### Success criteria

* I can use an exponential relationship to model a real-world scenario.
* I can explain what happens to the $y$-values as the $x$-values increase in an exponential relationship.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* identifies connections between algebraic and graphical representations of quadratic and exponential relationships in various contexts **MA5-NLI-C-01**
* identifies and compares features of parabolas and exponential curves in various contexts **MA5-NLI-C-02**

[Mathematics K–10 Syllabus](https://curriculum.nsw.edu.au/learning-areas/mathematics/mathematics-k-10-2022/overview) © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2022.

Table 1: lesson summary

|  |  |  |  |
| --- | --- | --- | --- |
| Section | Summary of activity | Teaching strategy | Teaching points  |
| Launch | Students work through the ‘Falling rocks’ task by Dan Meyer ([bit.ly/fallingrockslaunch](https://bit.ly/fallingrockslaunch)) to estimate the depth of the chasm. | Think-Pair-Share | The purpose of the launch is for students to use their instincts and current knowledge to make an estimate. |
| Explore  | Students investigate ‘Does a rock fall at a constant rate’ by performing a ball drop experiment. Students use the Desmos Ball drop template ([bit.ly/balldropgraphtemplate](https://bit.ly/balldropgraphtemplate)) to graph their results and determine an equation that represents their graph. Students then explore what would happen if the ball dropped forever. | Visibly random groups of 3Pose-Pause-Pounce-BounceThink-Pair-Share | The purpose of this activity is for students to develop their own results and test them using an equation. The students are provided with a scenario to analyse what happens when $x $gets very large. |
| Summarise | Students write a script describing what happens as $x$ becomes very large in the equation $y=2^{x}$. Students then expand on their script by using the graph $y=2^{-x}$. | ScriptwritingTAG feedback | The purpose of this activity is for students to be able to explain the features of an exponential graph. |
| Apply | Students complete the Desmos activity ‘Game, Set, Flat’ ([bit.ly/DesmosGameSetFlat](https://bit.ly/DesmosGameSetFlat)). |   | The purpose here is for students to apply their knowledge of exponential relationships to make a better estimate and to discuss the exponential model’s limitations. |

## Activity structure

### Launch

1. Navigate to the ‘Falling rocks’ task by Dan Meyer ([bit.ly/fallingrockslaunch](https://bit.ly/fallingrockslaunch)).
2. Select the video under **Act one** to download and play ‘Falling rocks’ (0:49) (MOV 9.1 MB).
3. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)), ask students to estimate how deep they think the chasm is.

Teachers can prompt students by asking them to choose values they think would be too high or too low.

1. Select the video under **Act two** to download and play ‘Falling rocks’ (0:14) (MOV 2.8 MB).
2. Continuing in their pairs, ask students to discuss if this new information changes their estimate of the depth of the chasm.
3. Ask students to determine their final estimate and collect all responses on the board. This will be referenced later in the lesson.

### Explore

During this section, students will work in visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) to conduct an experiment. Distribute the following equipment to each group.

#### Equipment per group

* A measuring tape
* A coloured table tennis ball or handball
* Digital device
* A stopwatch

#### Method

1. Ask students to drop their balls from 5 different heights and record in the Desmos graph ‘Ball drop template’ ([bit.ly/balldropgraphtemplate](https://bit.ly/balldropgraphtemplate)) how long it takes for the ball to hit the ground).

Students graph time in seconds on the $x$-axis and height in centimetres on the $y$-axis.

Although students are changing the height they drop the ball, we are using this as our y-value as it will help to connect the activity back to the Launch scenario. Students may benefit from discussing this as a class.

The axes provided in the Desmos graph may need to be modified depending on the height from which students dropped their balls. A sample solution has been provided in the Desmos graph ‘Ball drop sample solution’ ([bit.ly/balldropsamplesolution](https://bit.ly/balldropsamplesolution)).

Students may benefit from dropping the ball from the same height multiple times for accuracy.

1. In their groups, ask students to discuss ‘Which relationship could be used to describe the data from our experiment and why?’
2. Challenge each group to use their knowledge of exponential equations and transformations to find an equation for their graph of the data from their experiment.

Students should recognise that this graph could be exponential. The best model for the distance a ball drops is a parabola, though it is assumed students have not explored parabolas before this learning episode.

Students were first introduced to exponential equations in Lesson 4 – invasive species and transformations in Lesson 5 – exponential marble slides of Unit 11 – applying exponentials.

1. Ask each group to display their screen to the class. Students are to conduct a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) to observe other groups’ work noting the similarities and differences between the equations they found.

Alternatively, teachers can collate each of the students’ equations in their own Desmos calculator. Students should have different graphs as each group will have different data.

1. Ask students to justify why their graph or another group’s graph is a good representation of the time it takes for a ball to drop.
2. Revisit the launch and ask students to use their model to predict the depth of the chasm. Ask them to compare this to their initial estimate.
3. Pose the question: What do you expect to happen if the chasm is never-ending?
4. Allow students time to manipulate their Desmos graphs to investigate.

Students can click and drag on the screen to view different parts of a graph or zoom in and out.

1. Ask students to use their model to estimate the height from which the rock was dropped. They can use either their graph or equation.
2. Demonstrate to students, using the Desmos calculator, how as the time increases, the depth increases to become a very large number.

### Summarise

1. Students are to write a script outlining a conversation between 2 people. One person is trying to explain to the other what happens to the graph of $y=2^{x}$ as $x $becomes very large.
2. Have students perform each script in pairs and provide peer feedback using the TAG feedback strategy ([bit.ly/TAGstrategy](https://bit.ly/TAGstrategy)).
3. Students continue their script using the sentence starter ‘I wonder whether it is the same for the graph $y=2^{-x}$?’

### Apply

1. Instruct students to navigate to the Desmos activity ‘Game, Set, Flat’ ([bit.ly/DesmosGameSetFlat](https://bit.ly/DesmosGameSetFlat)) and work through the activity.

Before doing this activity, you will need to set up a Desmos classroom ([bit.ly/managingdesmosclassroom](https://bit.ly/managingdesmosclassroom)). The activity includes percentage calculations, which may need to be revised.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* As this activity has no correct answers and is subject to opinion, all students should be able to make an attempt.

**Explore**

* Students may benefit from discussing as a whole class why we are using the height as our y-value and the time as the x-value.
* Students may benefit from being reminded of the equation $y=k\left(a\right)^{x}+c$ and the resultant transformations.

**Summarise**

* The graph $y=2^{x} $could be changed to another graph or simply the graph of $y=\left(a\right)^{x}$.
* Enable students by providing a script generated using AI and ask students to act out the script with a partner.
* Students could continue the script for both graphs explaining what occurs when$ x$ becomes very small and why.

**Apply**

* The interactive nature of the Desmos activity allows students to retry or repeat steps to aid understanding.

### Suggested opportunities for assessment

**Launch**

* Students will demonstrate their Working mathematically skills in their discussions and justifications.

**Explore**

* Collect students’ graphs as evidence of graphing exponentials and using transformations.
* Teachers could ask students to explain how they found their equation to check their understanding of transformations of exponential graphs related to their equation.

**Summarise**

* The teacher could collect students’ scripts to assess the correct use of language and knowledge of exponential functions.

**Apply**

* Review students’ responses in the optional Desmos classroom activity to check their understanding of an exponential relationship.

## References

This resource contains NSW Curriculum and syllabus content. The NSW Curriculum is developed by the NSW Education Standards Authority. This content is prepared by NESA for and on behalf of the Crown in right of the State of New South Wales. The material is protected by Crown copyright.

Please refer to the NESA Copyright Disclaimer for more information <https://educationstandards.nsw.edu.au/wps/portal/nesa/mini-footer/copyright>.

NESA holds the only official and up-to-date versions of the NSW Curriculum and syllabus documents. Please visit the NSW Education Standards Authority (NESA) website <https://educationstandards.nsw.edu.au/> and the NSW Curriculum website <https://curriculum.nsw.edu.au/>.

[Mathematics K–10 Syllabus](https://curriculum.nsw.edu.au/learning-areas/mathematics/mathematics-k-10-2022/overview) © NSW Education Standards Authority (NESA) for and on behalf of the Crown in right of the State of New South Wales, 2022.

**© State of New South Wales (Department of Education), 2024**

The copyright material published in this resource is subject to the *Copyright Act 1968* (Cth) and is owned by the NSW Department of Education or, where indicated, by a party other than the NSW Department of Education (third-party material).

Copyright material available in this resource and owned by the NSW Department of Education is licensed under a [Creative Commons Attribution 4.0 International (CC BY 4.0) license](https://creativecommons.org/licenses/by/4.0/).



This license allows you to share and adapt the material for any purpose, even commercially.

Attribution should be given to © State of New South Wales (Department of Education), 2024.

Material in this resource not available under a Creative Commons license:

* the NSW Department of Education logo, other logos and trademark-protected material
* material owned by a third party that has been reproduced with permission. You will need to obtain permission from the third party to reuse its material.

**Links to third-party material and websites**

Please note that the provided (reading/viewing material/list/links/texts) are a suggestion only and implies no endorsement, by the New South Wales Department of Education, of any author, publisher, or book title. School principals and teachers are best placed to assess the suitability of resources that would complement the curriculum and reflect the needs and interests of their students.

If you use the links provided in this document to access a third-party's website, you acknowledge that the terms of use, including licence terms set out on the third-party's website apply to the use which may be made of the materials on that third-party website or where permitted by the *Copyright Act 1968* (Cth). The department accepts no responsibility for content on third-party websites.