# Difference of 2 squares

Students explore the difference of 2 squares by discovering an efficient way to square large numbers.

## Visible learning

This lesson incorporates Path content.

### Learning intention

* To know how to use the difference of 2 squares to expand binomial products.

### Success criteria

* I can expand binomial products.
* I can explain why a binomial product would result in a difference of 2 squares.
* I can identify when a binomial product will result in a difference of 2 squares.

Table 1: lesson summary

|  |  |  |  |
| --- | --- | --- | --- |
| Section | Summary of activity | Teaching strategies | Teaching points |
| Warm up | Students guess what number from [Appendix A.](#_Appendix_A) | Mini whiteboardsThink-Pair-SharePose-Pause-Pounce-Bounce | Teacher reads out clues for students to revise square numbers. |
| Launch | Students try to calculate $999^{2}$.  | Visibly random groups of 3Vertical non-permanent surfacesGallery walkPose-Pause-Pounce-BounceThink-Pair-Share | Students discuss what is the most efficient method.  |
| Explore | Students expand $(x+1)(x-1)$ and then discuss why it might be called the ‘difference of 2 squares’. Students explore the relationship by completing [Appendix B](#_Appendix_B). In pairs, students identify cards in [Appendix C](#_Appendix_C) that are a difference of 2 squares. | Think-Pair-SharePose-Pause-Pounce-BounceArea modelVariation Theory | Students explore how to quickly multiply 2 numbers that are in the form of difference of 2 squares. |
| Summarise | Students watch the video ‘Difference of Squares’ (1:17) ([bit.ly/differenceofsquaresvideo](https://bit.ly/differenceofsquaresvideo)) and write notes to their future forgetful selves and give peer feedback. Students then complete the Tarsia activity from Go Teach Maths website ‘Difference of 2 squares’ ([goteachmaths.co.uk/difference-of-two-squares/](https://schoolsnsw.sharepoint.com/sites/CurriculumReview655/Shared%20Documents/Work%20Stream%20-%20Secondary%20PL%20and%20Resources/Project%20-%20Mathematics%207-10/3.%20Draft%20documents/Year%2010%20units/Unit%2012%20-%20Investigating%20parabolas/goteachmaths.co.uk/difference-of-two-squares)). | Notes to future forgetful selvesTAG feedback | Students learn to identify binomial products written in the form of difference of 2 squares. |
| Apply | Students explore how to multiply large numbers using a difference of 2 squares. | Pose-Pause-Pounce-BounceVisibly random groupsVertical non-permanent surfacesAssessing and advancing questionsGallery walk | Students can choose the difficulty of numbers they wish to multiply. |

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* simplifies algebraic fractions with numerical denominators and expands algebraic expressions **MA5-ALG-C-01**
* simplifies algebraic fractions involving indices, and expands and factorises algebraic expressions **MA5-ALG-P-01** (Path)
* selects and applies appropriate algebraic techniques to operate with algebraic fractions, and expands, factorises and simplifies algebraic expressions **MA5-ALG-P-02** (Path)

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## Activity structure

Please use the associated PowerPoint *Difference of 2 squares* (DO2S PPT) to display images in this lesson.

### Warm up

This activity has been adapted from the NZ Maths resource ‘Square Xs’ (<https://meaningfulmaths.nt.edu.au/mmws/nz/resource/square-xs.html>).

1. Assign students into pairs and distribute a mini whiteboard ([bit.ly/miniwhiteboards](https://bit.ly/miniwhiteboards)) to each pair.
2. Read each set of clues from Appendix A ‘Square number detective’, giving time for students to respond after each clue in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)).
3. Using the Pose-Pause-Pounce-Bounce questioning strategy (PDF 557 KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)), ask students how you could find a number that when squared is 100, but is less than 5.

This activity is to remind students that when negative numbers are squared, they create a positive solution. If students are struggling to connect what number the solution could be, they may benefit from looking at a number line to advance their thinking to negative numbers.

1. Continuing in their pairs, students create their own square number puzzle.

To avoid using calculators, restrict the solutions to be between −12 and 12.

1. Students are to swap with another pair and attempt their square number puzzle.

### Launch

1. Assign students into visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) at vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)), and ask students, without a calculator, to try and find $999^{2}$.
2. Students then complete a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) to see how many strategies groups used to approach the problem.

Students could show a variety of strategies to approach the problem. Some may have used an area model, breaking up 999 into 900, 90 and 9 before adding the different parts together. Other students may have used a standard algorithm.

1. Using the Pose-Pause-Pounce-Bounce questioning strategy, ask students which method they thought was the most efficient and why.
2. State to the class that one method used to solve $999^{2}$ is called ‘the difference of 2 squares’. In a Think-Pair-Share, ask students what they think a difference of 2 squares might be.

Students may relate this to subtracting the area of one square from another square, or think of subtracting one square number from another.

### Explore

1. Ask students to expand the expression from slide 3 of the PowerPoint (DO2S PPT).

Students have explored how to expand a binomial product in Lesson 4 – areas for expansion.

1. Using the Pose-Pause-Pounce-Bounce questioning strategy, ask students:
* How can we explain why there are only 2 terms left after the binomial product is expanded and simplified?
* Why do you think the resulting expression from expanding the binomial product is called ‘the difference of 2 squares?’

After expanding, students should notice that the 2 terms with variable of $x$ cancel each other out, leaving the difference of 2 square numbers.

1. State to students that they are going to further explore the relationship between the binomial product and its expansion.
2. Distribute Appendix B ‘Expanding’ to each student to complete. This activity uses Variation Theory ([variationtheory.com/introduction/](https://variationtheory.com/introduction/)) to explore what a difference of 2 squares can look like.
3. Using the Pose-Pause-Pounce-Bounce questioning strategy, ask students:
* How can we identify if a binomial product will result in a difference of 2 squares?
* How could we move quickly from the binomial product to the simplified, expanded expression?
* What was different about question 7? Why?
1. Distribute Appendix C ‘Identifying the difference of 2 squares’ cut into cards. Working in pairs, ask students to identify if the cards contain an expression which is a difference of 2 squares or results in a difference of 2 squares when expanded. Students should justify their choices to each other.

### Summarise

1. Show students the video ‘Difference of Squares’ (1:17) ([bit.ly/differenceofsquaresvideo](https://bit.ly/differenceofsquaresvideo)).
2. Display slide 5 from the PowerPoint (DO2S PPT), which displays a screenshot from the previous video.
3. In a Think-Pair-Share, ask students to explain the visual representation to each other.
4. Students are to create notes to their future forgetful selves ([bit.ly/notestofutureself](https://bit.ly/notestofutureself)) on how to identify the difference of 2 squares and how the binomial product and the expanded expression are connected.
5. Each student is to swap their notes to their future forgetful self with another student to give peer feedback using the TAG proforma ([bit.ly/DLSpeerfeedback](https://bit.ly/DLSpeerfeedback)).
6. Students should return to their notes and modify them using the feedback given.
7. Distribute to each pair of students the Tarsia activity from the Go Teach Maths website ‘Difference of 2 squares’ ([goteachmaths.co.uk/difference-of-two-squares/](https://goteachmaths.co.uk/difference-of-two-squares/)). The activity can be found under the **Activities** heading.

### Apply

1. In new visibly random groups of 3, on vertical non-permanent surfaces, return to the Launch activity, which was to find $999^{2}$. Ask students to explain how they could use a difference of 2 squares to find the solution.
2. If students struggled to connect how to calculate $999^{2}$ using this method, use slides 7–12 from the PowerPoint (DO2S PPT)*,* which shows how to approach finding a solution to $999^{2}$ using the difference of 2 squares.
3. Using the Pose-Pause-Pounce-Bounce questioning strategy, ask students to respond to the questions on slide 12.
4. Continuing in their visibly random groups of 3, on vertical non-permanent surfaces, ask students to try and find the square of the numbers 9, 23 and 37 using the difference of 2 squares.
5. Ask students assessing and advancing questions ([bit.ly/supportingstrategies](https://bit.ly/supportingstrategies)) to further student thinking. Some examples are provided below.

Table 2: assessing and advancing questions

|  |  |
| --- | --- |
| Assessing questions | Advancing questions |
| What numbers would make the multiplication easy? | Is your answer smaller or larger than the actual answer? |
| What are some square numbers that are close to the number you are working with? | Can you draw a diagram to show your thinking so far? |
| What sized squares have you chosen to use in your calculation? | How do we get back to the original number from where we started? |

1. When students are confident in using the difference of 2 squares to find the square of a number, challenge them to find the squares of large numbers of their choice.
2. Using the Pose-Pause-Pounce-Bounce questioning strategy, ask students what numbers this method would be most useful for.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Warm up**

* **Enable students by providing a list of numbers for students to eliminate from after each clue is revealed.**
* **Challenge students by including solutions outside the range −12 to 12 or adding mathematical facts from other topics such as ‘The square of this number is a possible angle inside a triangle’.**

**Launch**

* Challenge students by asking them to find multiple ways to calculate $999^{2}$.
* Enable students by giving them a smaller number such as 99 or 29.

**Explore**

* The activity asking what students think a difference of 2 squares might be, has no correct answers and is subject to opinion. All students should be able to make an attempt at answering the question.
* To enable students to engage in class discussions, provide them with a completed solution using an area model for slide 3 from the PowerPoint (DO2S PPT)*.*

**Summarise**

* Visual representations are used to assist students with their understanding of the relationship between the binomial product and expanded expression in a difference of 2 squares.
* Students may benefit from using the cards in Appendix C ‘Identifying the difference of 2 squares’ as examples and non-examples for their notes to their future forgetful selves.
* To enable students, modify the Tarsia activity to only include monic expressions.

**Apply**

* Students may benefit from revisiting factors to know that multiplying a number such as 20 is the same as multiplying by 10 and then 2, or vice versa.

### Suggested opportunities for assessment

**Warm up**

* Monitor responses to check for student understanding of square numbers.

**Launch**

* **Students working at vertical non-permanent surfaces allows the teacher to assess student progress and provide support where appropriate.**

**Explore**

* Students will demonstrate their Working mathematically skills in discussions and justifications.
* Collect Appendix B ‘Expanding’ as evidence of learning towards expanding binomial products.

**Summarise**

* Students provide peer feedback when reviewing each other’s notes to future forgetful selves.
* Collect Appendix C as evidence of learning for identifying the difference of 2 squares.

**Apply**

* Student responses to the provided prompts could be collected as a work sample for assessment.

## Appendix A

### Square number detective

1. I'm thinking of a 2-digit number, which is a perfect square.
It's an odd number less than 50.
The sum of the digits is 13.
What is the number?
2. I'm thinking of a number.
It’s square is a 3-digit odd number.
The sum of the digits is 2.
What is the number?
3. I'm thinking of a 2-digit number.
When I multiply its digits, I get zero.
The square is 100.
It's a number less than 5.
What is your answer?

## Appendix B

### Expanding

#### Question set 1

1. $(x+2)(x-2)$
2. $\left(x-2\right)\left(x+2\right)$
3. $(2-x)(2+x)$
4. $\left(2+x\right)\left(2-x\right)$
5. $\left(x+2\right)\left(2-x\right)$
6. $(x-2)(2+x)$
7. $(x+2)(x+2)$

#### Question set 2

1. $(x+2)(x-2)$
2. $(x+3)(x-3)$
3. $(x+4)(x-4)$
4. $(3x+4)(3x-4)$
5. $(5x+4)(5x-4)$

## Appendix C

### Identifying the difference of 2 squares

|  |  |
| --- | --- |
| $$x^{2}-25$$ | $$(2x-5)(5x+2)$$ |
| $$(x+5)(x-25)$$ | $$4x^{2}-25x$$ |
| $$(2-5x)(2x+5)$$ | $$(x+5)(x-5)$$ |
| $$4x^{2}-25$$ | $$(2-5x)(2+5x)$$ |
| $$(2x-5)(2x+5)$$ | $$x^{2}+25$$ |
| $$x^{2}-4x+4$$ | $$4-25x^{2}$$ |

## Sample solutions

### Appendix A – square number detective

1. 49
2. 11
3. −10

### Appendix B – expanding

#### Question set 1

1. $\left(x+2\right)\left(x-2\right)=x^{2}-4$
2. $\left(x-2\right)\left(x+2\right)=x^{2}-4$
3. $\left(2-x\right)\left(2+x\right)=4-x^{2}$
4. $\left(2+x\right)\left(2-x\right)=4-x^{2}$
5. $\left(x+2\right)\left(2-x\right)=2x-x^{2}+4-2x=4-x^{2}$
6. $\left(x-2\right)\left(2+x\right)=2x+x^{2}-4-2x=x^{2}-4$
7. $\left(x+2\right)\left(x+2\right)=x^{2}+4x+4$

#### Question set 2

1. $\left(x+2\right)\left(x-2\right)=x^{2}-4$
2. $\left(x+3\right)\left(x-3\right)=x^{2}-9$
3. $\left(x+4\right)\left(x-4\right)=x^{2}-16$
4. $\left(3x+4\right)\left(3x-4\right)=9x^{2}-16$
5. $\left(5x+4\right)\left(5x-4\right)=25x^{2}-16$

### Appendix C – identifying the difference of 2 squares

|  |  |
| --- | --- |
| Difference of 2 squares | Other |
| $$(x+5)(x-5)$$ | $$(x+5)(x-25)$$ |
| $$(2x-5)(2x+5)$$ | $$(2x-5)(5x+2)$$ |
| $$(2-5x)(2+5x)$$ | $$(2-5x)(2x+5)$$ |
| $$x^{2}-25$$ | $$x^{2}+25$$ |
| $$4x^{2}-25$$ | $$4x^{2}-25x$$ |
| $$4-25x^{2}$$ | $$x^{2}-4x+4$$ |

## References

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