# Inventing units of measurement

Students create a unique unit of measurement with a strip of paper and use it to measure lengths of common objects. Students fold their strip of paper to create fractions to aid more accurate measurement, develop relationships between equivalent fractions and begin representing these fractions as decimals.

## Visible learning

### Learning intentions

* To understand the usefulness of fractions in describing what is between whole number units.
* To be able to convert common fractions into decimals.

### Success criteria

* I can use fractions and decimals to describe lengths that are not exact, whole units.
* I can compare the size of fractions.
* I can convert fractions to decimals.

### Syllabus outcomes

A student

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* represents and operates with fractions, decimals and percentages to solve problems **MA4-FRC-C-01**

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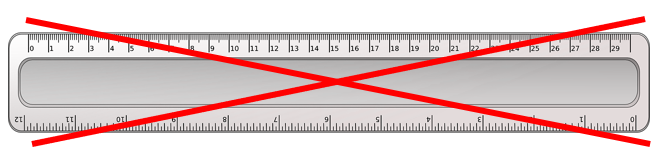
## Activity structure

Please use the associated PowerPoint *Inventing units of measurement* to display images in this lesson.

### Launch

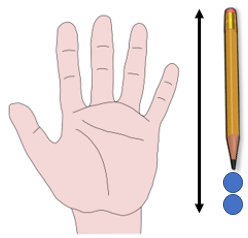
1. Show students Figure 1 and introduce the premise that we are in a world before rulers and measurement have been invented.

Figure 1 – there are no more rulers



1. Students are to find a common object to describe the length of their hand. Is their hand as long as a pencil? A book? A packet of chips? All answers should be encouraged and accepted, including parts of objects, such as ‘my hand is half the length of my desk’.
2. Display Figure 2. Ask students to consider why we wouldn’t measure a hand with a pencil plus 2 smarties. Establish that once we have chosen to measure length in pencils, we should be counting how many pencils the length of something is.

Figure 2 – a hand that is longer than a pencil



### Explore

This activity is adapted from Mindset Mathematics Grade 4 (Boaler, Munson, Williams).

#### Creating a unit of measurement

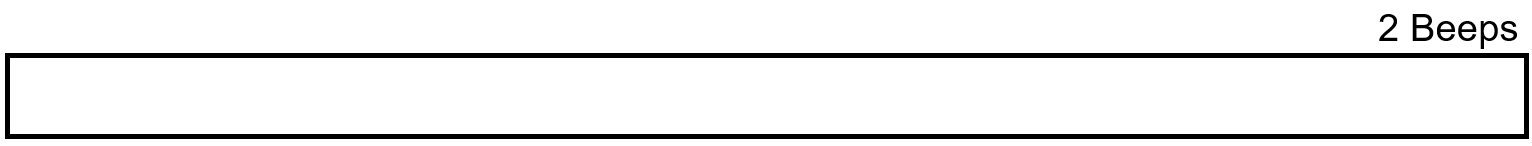
##### Equipment

* Paper strips of varying lengths, one per student.

##### Method

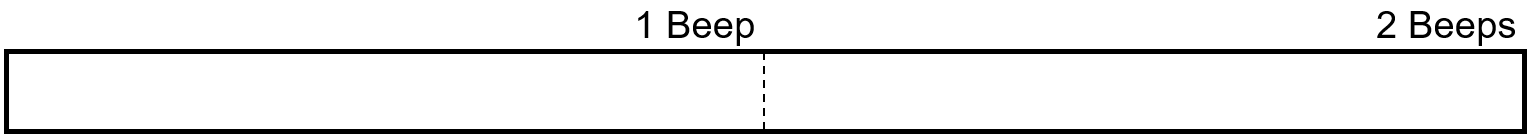
1. Students are each given strips of paper of different lengths, ideally between 1–2 metres long.
2. State that the strip of paper represents 2 units of their own unique measurement of length. Students can make up a name for their unit of measurement. We have chosen to name ours ‘beeps’.

Figure 3 – unfolded strip of paper



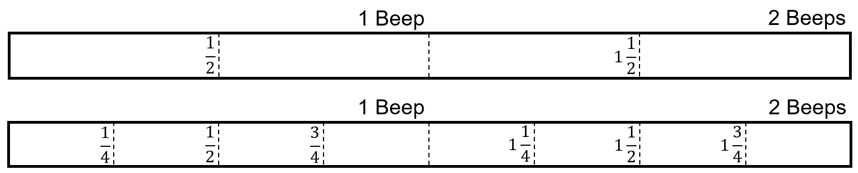
1. Students should fold their paper in half to indicate 1 unit of their measurement.

Figure 4 – paper strip folded once



1. Students are to use their strip of paper to measure objects around the room. If a required object is not exactly 1 or 2 units of the student's measure, they need to continue to fold their paper strip to make half, quarter or more precise units as shown below.

Figure 5 – paper strips folded to quarters



1. Students could potentially measure pens, their pencil case, their desk width, the height of their chair, the width of the door, the height of the whiteboard, or any other classroom furniture.
2. Bring students together to engage in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) around the following questions:
3. What was the smallest fraction used?
4. What was the largest fraction used?
5. How many folds do you need to do before you have a finished measuring device?
6. What equivalent fractions can you see on your measuring device?

#### Comparing fractions

1. Display for students a strip of paper folded to show halves. Identify that the position of one half is one section or crease from the left and that on this strip, there are 4 sections.

Figure 6 – the position of one half

An image of a long strip of paper, labelled as being 2 "Beeps". The strip of paper is folded in half, and the halfway point is labelled as 1 "Beep".
There are also folds equally spaced to the left and right of the centre fold. The one on the left is labelled as one half, and the one to the right is labelled as one and one half. 

There is an arrow indicating the location of the half. 

Alternatively, all students can be handed another strip of paper and label up to halves together.

1. Fold the paper as per the previous instructions so that quarters are shown.
2. Focus in on one half, showing it is equivalent to 2 quarters, displaying Figure 7.

Figure 7 – comparing halves and quarters

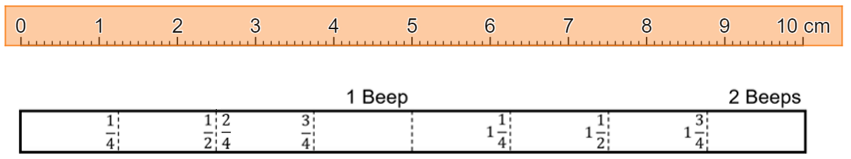
An image of a long strip of paper, labelled as being 2 "Beeps". The strip of paper is folded in half, and the halfway point is labelled as 1 "Beep".
There are also folds equally spaced to the left and right of the centre fold. The one on the left is labelled as one half, and the one to the right is labelled as one and one half. 

There is a second strip with four additional folds equally spaced between the existing 3 and the ends of the paper strip, labelled from left to right as one quarter, three quarters, one and one quarter, and one and three quarters. 

There is an arrow drawing attention to the half. 

1. Use a Pause-Pose-Pounce-Bounce question strategy ([PDF 200KB] [bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)) to ask students what has happened to the paper to make the fraction turn into . Lead students to the conclusions that:
2. The number of sections on the entire strip of paper has doubled. Emphasise the sections up to 1 and how this shows the denominator having doubled.
3. The number of sections or creases from the left before you get to the position of has also doubled. Connect this to the numerator having doubled.
4. Display Figure 8 to students.

Figure 8 – paper strip versus a ruler



1. Have students engage in a Think-Pair-Share around the following questions:

* What is different about your strip of paper and a common measuring device such as a metre ruler?
* What is the same between these 2 tools?
* Which one do you think is easier to use?
* How many folds do you need to do to the paper before it becomes the same as the ruler?

It is important here that students conclude that our common measuring tools are divided into tenths using decimals and that this will never be the case for our measuring strips, no matter how many folds we complete.

### Summarise

#### Converting fractions into decimals

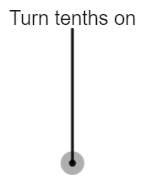
##### Visual representations

1. Display the Desmos graph *‘Fractions to decimals’* ([bit.ly/DesmosFtD](https://bit.ly/DesmosFtD)) on the teacher screen.

Supporting images can also be displayed using the *Inventing units of measurement* PowerPoint file, slides 9–12.

1. Use a Pause-Pose-Pounce-Bounce question strategy ([PDF 200KB] [bit.ly/pausepouncebounce](https://bit.ly/pausepouncebounce)) to ask students what we need to do to the number line to find the location of decimal numbers, leading students to the conclusion that we need to divide each interval into 10 equal parts, tenths.
2. Turn on the tenths using the slider on the screen, to determine whether tenths can represent the fraction . Conclude with students that while it is clearly between 0.2 and 0.3, tenths is not accurate enough to display .

Figure 9 – tenths slider



Many students will know the decimal value of , so teachers should challenge students to justify why using the images they see on the screen.

1. Turn on the hundredths using the slider on the screen.
2. Have students silently write down, on mini whiteboards ([bit.ly/miniwhiteboards](https://bit.ly/miniwhiteboards)) if available, and hold up the decimal value that they believe is shown to be.
3. Change the denominator to 8 using the denominator slider, as shown below.

Figure 11 – denominator slider



1. Have students attempt to describe the decimal value of , using mini whiteboards.
2. Conclude with the class that the decimal value of cannot be described perfectly using only tenths and hundredths.
3. Have students attempt Appendix A ‘Describing fractions with tenths and hundredths’ by constructing a list of fractions from the fraction wall that can be described using decimals up to tenths and hundredths.

### Apply

1. Make sure students have their original strip of measuring tape.
2. Hand students a new strip of paper of the same length.
3. Give students access to the Desmos graph *Fractions to decimals* ([bit.ly/DesmosFtD)](http://bit.ly/DesmosFtD).
4. Using the steps from the *Your turn* PowerPoint and the Desmos visual representations, have students copy their existing fractional measurements from their original strip of measuring tape across to their new tape, representing them as decimals, as demonstrated in the figure below.

Figure 12 – measuring strips

An image of a long strip of paper, labelled as being 2 "Beeps". The strip of paper is folded in half, and the halfway point is labelled as 1 "Beep".
There are also folds equally spaced to the left and right of the centre fold. The one on the left is labelled as one half, and the one to the right is labelled as one and one half. There are also folds equally spaced between the existing 3 and the ends of the paper strip, labelled from left to right as one quarter, three quarters, one and one quarter, and one and three quarters. There is an additional strip of paper aligned to the first one, with labels of 0.25, 0.5, 0.75, 1, 1.25, 1.5 and 1.75 from left to right.

1. Have students attempt to fill in common decimal measurements, such as tenths and hundredths, on their new measuring device, based on the measurements already placed on the strip.
2. Students can then use their measuring device to reattempt the measurements they completed during the Explore activity.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Explore**

* **Using the physical measuring strip allows students to develop concepts of fractions in a concrete way.**
* **Students can be challenged early to consider which fractions are available using the folding method and to invent methods of their own to obtain other fractions of their measurement, such as one tenth.**

**Summarise**

* **Multiple visual representations, in this case the use of number lines and the constructed measuring tape, should support students to verify comparisons between fractions and decimals.**

**Apply**

* **Students can be challenged to construct their entire measuring strip using decimals.**

### Suggested opportunities for assessment

**Explore**

* **Observe and record students’ ability to use a measurement device effectively to determine the length of common objects.**
* **Students’ measuring strips can be collected as evidence of their ability to interpret fractions of a whole, as well as improper fractions between 1 and 2.**

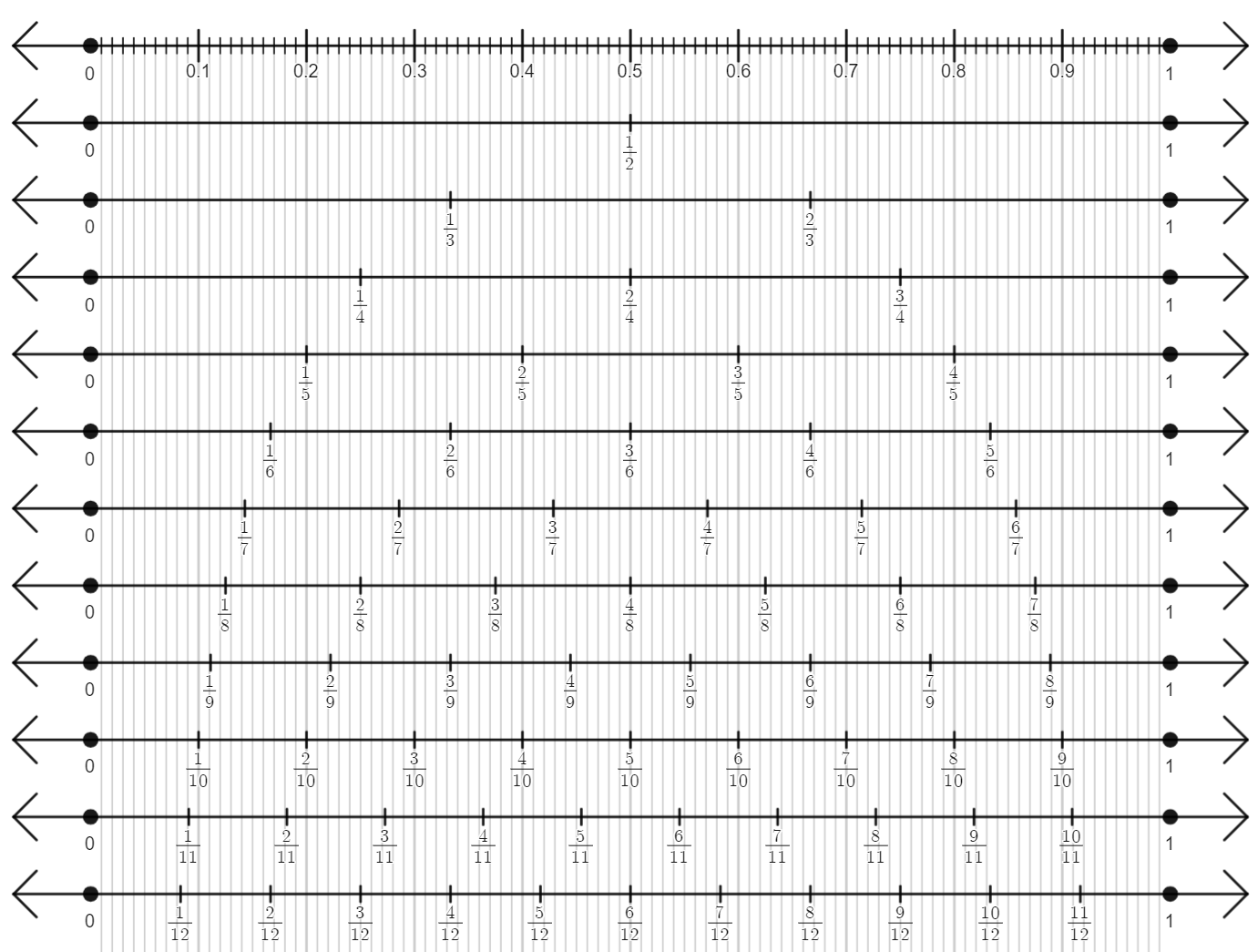
**Summarise and Apply**

* **Appendix A can be collected to assess students’ ability to visually interpret fractions as decimals.**

## Appendix A

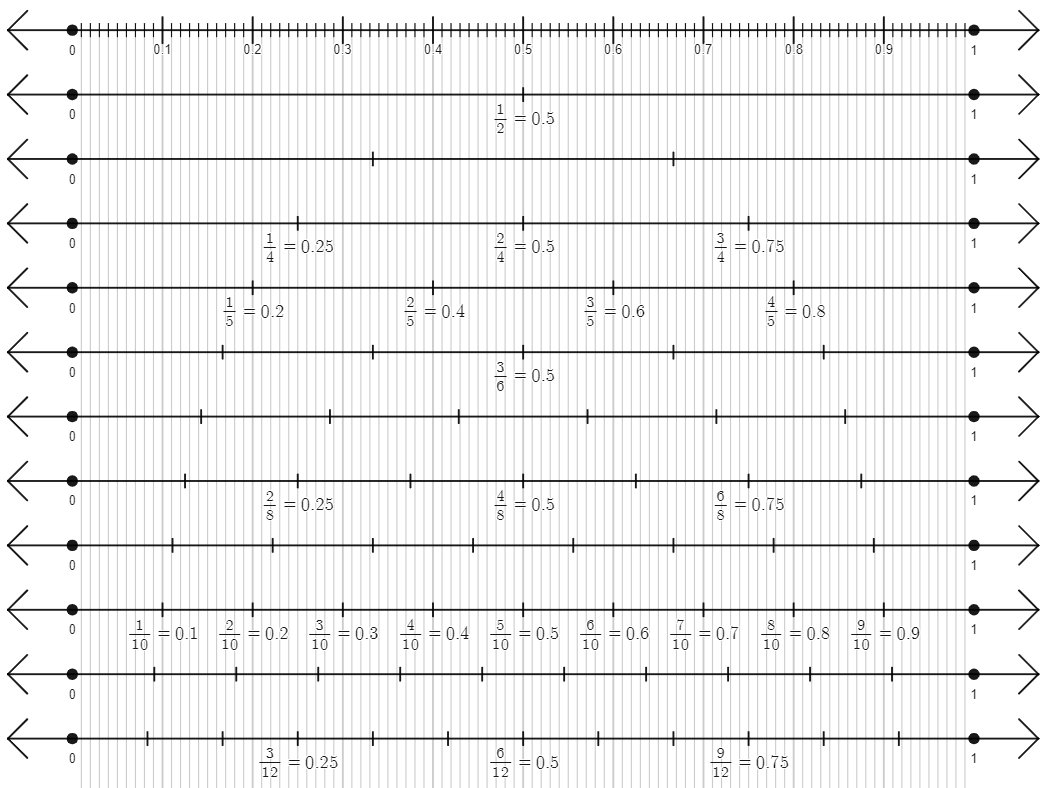
### Describing fractions with tenths and hundredths

By using the vertical lines behind this fraction wall and the decimal values on the number line at the top, write a list of all the fractions on this wall that can be described by tenths and hundredths.



## Appendix A – Sample solutions

### Describing fractions with tenths and hundredths



## ****References****

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