# Dividing into the unknown

Students use area models to explore conventions of simplifying algebraic expressions involving division. They use dynamic and pictorial models before moving to abstract representations of division.

This lesson is based on students having had experience with using area models to represent numerical division.

## Visible learning

### Learning intentions

* To understand common conventions when writing and simplifying algebraic expressions involving division.
* To be able to use an area model to explain how algebraic expressions with division simplify.

### Success criteria

* I can interpret algebraic terms involving division.
* I can simplify algebraic expressions involving division.
* I can construct an area model to represent division between 2 algebraic terms.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* **generalises number properties to operate with algebraic expressions including expansion and factorisation MA4-ALG-C-01**

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Please use the associated PowerPoint *Dividing into the unknown* to display images in this lesson.

## Activity structure

### Warm up

This activity is designed to activate prior knowledge of the use of area models for division of numbers. Students demonstrating proficiency in this skill can move straight to the next section of the lesson.

1. Write the following divisions on the teacher board.
2. $15÷5=$
3. $20÷10=$
4. $120÷15=$
5. $108÷9=$
6. Have students attempt these individually without a calculator.
7. Use the Desmos graph ‘Area model division’ ([bit.ly/DesmosAMDivision](https://bit.ly/DesmosAMDivision)) to demonstrate the solution to the first problem using an area model. The teacher can adjust the problem using the sliders ‘Dividend’ and ‘Divisor.



Then form the area model by dragging the areas down until the total matches the dividend.



1. Have students represent the 3 remaining divisions using area models. Students can complete this by either drawing area models on grid paper or by using a device with internet and the Desmos graph ‘Area model division’ ([bit.ly/DesmosAMDivision](https://bit.ly/DesmosAMDivision)).

### Launch

1. Display or read out the following scenario to students.

‘Some money is shared equally among 3 children. How much will each child receive?’

1. In a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) students are to write an expression for this problem.

Variations from students may include $m÷3$ or $\frac{m}{3}$. Students may need to be prompted that they will need to use a pronumeral to represent the unknown amount of money.

1. Explain to students that the use of the pronumeral makes this an algebraic expression.
2. Add to the situation that we now know that the amount of money being shared was $36 and ask students to calculate the amount of money each child will receive.
3. Ask students to repeat the calculation if the amount of money is now $108.
4. Hand students copies of Appendix A ‘Algebraic expressions’ and instruct them to write an algebraic expression for each scenario by first identifying any unknowns and labelling them with a pronumeral.
5. Bring students together and call on random students to share their responses.

Appendix A includes problems where the divisor is unknown and needs to be represented by a pronumeral. Teachers should ensure this is discussed while reviewing results.

### Explore

#### Division conventions

1. Display the expressions below, also available on slide 2 of the *Dividing into the unknown* PowerPoint.
2. $x÷2=\frac{x}{2}$
3. $ab÷c=\frac{ab}{c}$
4. $\frac{y}{1}=y$
5. Inform students that these are ways of writing expressions in shorter ways.
6. Have students return to Appendix A and write as many expressions as they can using these simplified conventions.

#### Area models for division

1. Use slides 3–10 from the *Dividing into the unknown* PowerPoint for explicit teaching of using area models to simplify expressions involving division of algebraic terms.

The explicit teaching technique used in the associated PowerPoint is ‘Your turn.’ The first slide is a worked example which should be displayed for the students and then use the following steps.

1. Reveal the question to students and its solution.
2. Students read in silence.
3. Students individually think and explain to themselves what is happening in each step.
4. Students hold up a thumbs up to the teacher when they have finished reading and have some sort of understanding.
5. Think-Pair-Share. Students explain the solution to their partner.
6. In pairs students then answer the self-explanation questions.
7. Finally, randomly select students to share their answers with the whole class.
8. Hand students copies of Appendix B ‘Dividing algebraic terms’ and have them complete with peers, representing each expression as an area model.

The expressions in Appendix B apply variation theory ([variationtheory.com/introduction/](https://variationtheory.com/introduction/)) and increase in difficulty beyond the problems shown in the *Dividing into the unknown* PowerPoint. Solutions to Appendix B are available at the end of this document.

Students can be arranged into visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) and allowed to work at whiteboards if available ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)).

If devices are available, students can be supported to represent their expressions using Polypad ([mathigon.org/polypad](https://mathigon.org/polypad)).

Appendix C ‘Constructing division area models using Polypad’ contains instructions of how to construct area models via Polypad. Appendix D ‘Saving your Polypad file’ contains instructions for students to save their Polypad solutions.

1. Display Figure 1 on the teacher screen, also available on slide 11 of the *Dividing into the unknown* PowerPoint.

Figure 1: $2x÷4$



1. Have students engage in a Think-Pair-Share to consider how we might answer this question, what it might look like as an area model and a simplified algebraic expression.
2. Conclude with students that to find a solution to $2x÷4$, we need to use $\frac{x}{2}$ tiles as shown below in Figure 2, also available on slide 12 of the *Dividing into the unknown* PowerPoint.

Figure 2: Area model solution



When reviewing Figure 2 as a solution, draw attention to the fact that the 4 ‘$\frac{x}{2}$’ tiles are the same as 2 ‘$x$’ tiles, or that $4×\frac{x}{2}=2x$.

1. Working in their pairs, instruct students to attempt to use area models to explain the result of the following 2 problems. These problems and their solutions can be displayed using slides 13 and 14 of the *Dividing into the unknown* PowerPoint.
2. $2x÷8$
3. $3y÷9$

### Summarise

1. Pose the following set of questions and have students write notes to their future forgetful selves ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)) on how to simplify each type of question. These can be displayed using slide 15 of the *Dividing into the unknown* PowerPoint.
2. $14x÷7$
3. $-9y÷3$
4. $16x÷4x$
5. $6xy÷2y$
6. $2x÷6$

### Apply

#### Algebra pyramids

1. Hand students a copy of Appendix E ‘Reversed algebra Pyramids’.
2. Students are to complete the worksheet individually or in pairs.
3. Have students engage in a Think-Pair-Share to discuss what they notice about the difference between finding answers up the pyramid and down the pyramid.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* Students who are less familiar with using algebra can be encouraged to use empty boxes to represent unknowns, at which point shifting to algebraic terms could be done gradually before continuing with the lesson.

**Explore**

* Students can be supported to develop their skills with area models by using physical algebra tiles or using the Polypad website. These methods both support introducing students naturally to the concept of multiplication with algebra by beginning with concrete representations.

**Apply**

* Students can be challenged in Appendix E to create an algebra pyramid with the least number of cells completed. Is there a maximum you can have empty while still having a single solution? Students can also be challenged to leave extra empty cells so that there is more than one solution and then explain how many solutions are possible and why.

### Suggested opportunities for assessment

**Launch**

* Appendix A supports teachers to gather information about students’ ability to express real situations as algebraic expressions, as well as their ability to identify relevant unknowns in written scenarios. Teachers should be prepared to support students with this skill.
* Appendix A also represents an opportunity to assess students’ ability with interpreting real scenarios as division. Teachers should be prepared to discuss why equal sharing and forming specifically sized groups are 2 important types of problems that both lead to division.

**Explore**

* Appendix B can be collected as evidence of a student’s ability to represent an algebraic expression as an area model and interpret the model to find a simplified solution to a division problem.

**Apply**

* Appendix E is evidence for teachers of the ability of students to see multiplication and division as inverse operations.

## **Appendix A**

### Algebraic expressions

Identify any unknowns and name them as pronumerals in each situation below. Then write an algebraic expression for the answer to the question. The first situation has an example expression completed for you.

|  |  |  |
| --- | --- | --- |
| Situation | Pronumerals | Algebraic expression |
| Some money is shared equally among 3 children. How much will each child receive? | $m$ is the amount of money.  | $$m÷3$$ |
| How many teams of 5 can be made from a crowd of people? | $p$ is the number of people in the crowd. |  |
| James has $30 and wants to buy boxes of chocolates for friends. How many boxes of chocolates can he buy? |  |  |
| There are large tables in a classroom for 24 students. How many students should sit at each table? |  |  |
| In a 60-minute lesson, how long will each student have to present their speech? |  |  |
| How many episodes of my favourite show can I watch in 2 hours? |  |  |

## Appendix B

### Dividing algebraic terms

For each expression in the table below, draw an area model representation and then write down a simplified expression. The first problem has been completed for you.

|  |  |  |
| --- | --- | --- |
| Expression | Area model | Simplified expression |
| $$4x÷2$$ | An image from Polypad of an area model. There are 4 green 'x' tiles in a 2 across by 2 down grid, each vertically aligned. At the top of the grid are 2 orange 'one' tiles, separated from the grid by a horizontal black line. On the left are 2 blue 'x' tiles, vertically aligned, separated from the grid by a vertical black line.  | $$2x$$ |
| $$4x÷4$$ | An image from Polypad of an area model. There are 4 green 'x' tiles in a 4 across by 1 down grid, each vertically aligned. At the top of the grid are 4 orange 'one' tiles, separated from the grid by a horizontal black line. On the left is 1 blue 'x' tiles, vertically aligned, separated from the grid by a vertical black line.  |  |
| $$-4x÷4$$ |  |  |
| $$12x÷4$$ |  |  |
| $$\frac{12x}{6}$$ |  |  |
| $$12x÷6x$$ |  |  |
| $$\frac{12x^{2}}{6x}$$ |  |  |
| $$12xy÷6x$$ |  |  |
| $$-12xy÷6x$$ |  |  |

## Appendix C

### Constructing division area models using Polypad

The instructions below demonstrate how to construct an area model in Polypad ([mathigon.org/polypad](https://mathigon.org/polypad)) to represent the expression $6x÷3$.

1. Select **Algebra** and then **Algebra Tiles** on the left-hand side of your screen.



1. Select and drag 6 of the horizontal$x$ tiles into the main space to represent $6x$. Place these tiles to the side.



1. Select and drag 3 ‘one’ tiles into the main space to represent $3$.



1. Select and drag each of the $6x$ tiles into the multiplication space, under the 3 ‘one’ tiles to make a rectangle. Any tile can be turned by selecting and dragging the rotation handle, highlighted by the arrow in the diagram below.



1. To separate the divisor $3$ and the dividend $6x$, we can add a horizontal and vertical line.



1. Select where you would like to start your line and drag across to where you would like your line to end, as shown below.



1. Select appropriate $x$ tiles to fill the space on the left and complete the area model. The colour of these tiles can be changed to differentiate from the original $6x$ from the problem by selecting the tile and then selecting the colour palette in the bottom menu.



1. There are 2 $x$ tiles on the left-hand side, indicating that $6x÷3=2x$.

## Appendix D

### Saving your Polypad file

1. Students should ensure they are signed in using their NSW Department of Education gmail account, ending in @education.nsw.gov.au.



1. Select the file icon in the top left of your screen.



1. Select the words **Untitled Polypad** to give the graph a title.



1. Select **Save**. A link will be generated that can be shared with the teacher.



## Appendix E

### Reversed algebra pyramids

In the algebra pyramids below, 2 terms beside one another are multiplied to make the term above. Use the algebraic terms in the pyramids to fill in the blanks. The first one is an example to help you.

|  |  |
| --- | --- |
| An image containing a table in a pyramid like structure with 3 rows. In the bottom row there are 3 cells with algebraic terms in each. The first cell has the term x, the second has 2 and the third has 4x. In the second row there are two cells. The first cell has 2x, the second has 8x. In the top row there is one cell with 16 'x squared' in it. | An image containing a table in a pyramid like structure with 3 rows. In the bottom row there are 3 cells with the term 2 in the second, and the other 2 empty. In the second row there are 2 cells with 6 in the second cell and the other one is empty. In the top row is a single cell with the term 12x in it.  |
| An image containing a table in a pyramid like structure with 3 rows. In the bottom row there are 3 cells with the term 3 in the second, and the other 2 empty. In the second row there are 2 cells with 12x in the first and the other one is empty. In the top row is a single cell with the term 36 'x squared' in it.  | An image containing a table in a pyramid like structure with 3 rows. In the bottom row there are 3 cells with the term 5y in the first, and the other 2 empty. In the second row there are 2 cells with 10y in the first and the other one is empty. In the top row is a single cell with the term 140xy in it.  |
| An image containing a table in a pyramid like structure with 4 rows. In the bottom row there are 4 cells with the term 2 in the second cell, and the other 3 empty. In the second row there are 3 cells with 6 in the second cell and the other 2 empty. In the third row there are 2 cells with the term 24x in the first and the second one is empty. In the top row is a single cell with the term 1296 'x squared' in it.  | An image containing a table in a pyramid like structure with 4 rows. In the bottom row there are 4 cells with the term 4 in the third cell, and the other 3 empty. In the second row there are 3 cells with negative 4 in the second cell and the other 2 empty. In the third row there are 2 cells with the term 20y in the first and the second one is empty. In the top row is a single cell with the term negative 640xy in it.  |
| An image containing a table in a pyramid like structure with 4 rows. In the bottom row there are 4 cells with the term 6 in the third cell, and the other 3 empty. In the second row there are 3 cells with 48 in the second cell and the other 2 empty. In the third row there are 2 cells with the term 96x in the first and the second one is empty. In the top row is a single cell with the term 9216 'x squared' in it.  | An image containing a table in a pyramid like structure with 4 rows. In the bottom row there are 4 cells with the term 5x in the first, and the other 3 empty. In the second row there are 3 cells with 10xy in the first and the other 2 empty. In the third row there are 2 cells with the term 20 'x squared, y squared' in the first and the second one is empty. In the top row is a single cell with the term 120 'x to the power of 5, y cubed' in it.  |

## Sample solutions

### Appendix A – algebraic expressions

|  |  |  |
| --- | --- | --- |
| Situation | Pronumerals | Algebraic expression |
| Some money is shared equally among 3 children. How much will each child receive? | $m$ is the amount of money.  | $$m÷3$$ |
| How many teams of 5 can be made from a crowd of people? | $p$ is the number of people in the crowd.  | $$p÷5$$ |
| James has $30 and wants to buy boxes of chocolates for friends. How many boxes of chocolates can he buy? | $c$ is the cost of a box of chocolates. | $$30÷c$$ |
| There are large tables in a classroom for 24 students. How many students should sit at each table? | $t$ is the number of tables.  | $$24÷t$$ |
| In a 60-minute lesson, how long will each student have to present their speech? | $s$ is the number of students in the class.  | $$60÷s$$ |
| How many episodes of my favourite show can I watch in 2 hours? | $E$ is the length in time of an episode of my favourite show.  | $$120÷t$$ |

### Appendix B – dividing algebraic terms

|  |  |  |
| --- | --- | --- |
| Expression | Area model | Simplified expression |
| $$4x÷2$$ | An image from Polypad of an area model. There are 4 green 'x' tiles in a 2 across by 2 down grid, each vertically aligned. At the top of the grid are 2 orange 'one' tiles, separated from the grid by a horizontal black line. On the left are 2 blue 'x' tiles, vertically aligned, separated from the grid by a vertical black line.  | $$2x$$ |
| $$4x÷4$$ | An image from Polypad of an area model. There are 4 green 'x' tiles in a 4 across by 1 down grid, each vertically aligned. At the top of the grid are 4 orange 'one' tiles, separated from the grid by a horizontal black line. On the left is 1 blue 'x' tiles, vertically aligned, separated from the grid by a vertical black line.  | $$x$$ |
| $$-4x÷4$$ | An image from Polypad of an area model. There are 4 red 'negative x' tiles in a 4 across by 1 down grid, each vertically aligned. At the top of the grid are 4 orange 'one' tiles, separated from the grid by a horizontal black line. On the left is 1 red 'negative x' tiles, vertically aligned, separated from the grid by a vertical black line.  | $$-x$$ |
| $$12x÷4$$ | An image from Polypad of an area model. There are 12 green 'x' tiles in a 4 across by 3 down grid, each vertically aligned. At the top of the grid are 4 orange 'one' tiles, separated from the grid by a horizontal black line. On the left are 3 blue 'x' tiles, vertically aligned, separated from the grid by a vertical black line.  | $$3x$$ |
| $$\frac{12x}{6}$$ | An image from Polypad of an area model. There are 12 green 'x' tiles in a 6 across by 2 down grid, each vertically aligned. At the top of the grid are 6 orange 'one' tiles, separated from the grid by a horizontal black line. On the left are 2 blue 'x' tiles, vertically aligned, separated from the grid by a vertical black line.  | $$2x$$ |
| $$12x÷6x$$ | An image from Polypad of an area model. There are 12 green 'x' tiles in a 6 across by 2 down grid, each horizontally aligned. At the top of the grid are 6 blue 'x' tiles, separated from the grid by a horizontal black line. On the left are 2 orange 'one' tiles, separated from the grid by a vertical black line.  | $$2$$ |
| $$\frac{12x^{2}}{6x}$$ | An image from Polypad of an area model. There are 12 square blue 'x squared' tiles in a 6 across by 2 down grid. At the top of the grid are 6 green 'x' tiles, separated from the grid by a horizontal black line. On the left are 2 orange 'x' tiles, vertically aligned, separated from the grid by a vertical black line.  | $$2x$$ |
| $$12xy÷6x$$ | An image from Polypad of an area model. There are 12 purple 'xy' tiles in a 6 across by 2 down grid. At the top of the grid are 6 green 'x' tiles, separated from the grid by a horizontal black line. On the left are 2 aqua 'y' tiles, vertically aligned, separated from the grid by a vertical black line.  | $$2y$$ |
| $$-12xy÷6x$$ | An image from Polypad of an area model. There are 12 red 'negative xy' tiles in a 6 across by 2 down grid. At the top of the grid are 6 green 'x' tiles, separated from the grid by a horizontal black line. On the left are 2 red 'negative y' tiles, vertically aligned, separated from the grid by a vertical black line.  | $$-2y$$ |

### Appendix E – reversed algebra pyramids

|  |  |  |
| --- | --- | --- |
| An image containing a table in a pyramid like structure with 3 rows. In the bottom row there are 3 cells with algebraic terms in each. The first cell has the term 'x', the second has 2 and the third has 3. In the second row there are two cells. The first cell has the term 2x, the second cell has the term 6. The top row has one cell which has the term 12x in it. | An image containing a table in a pyramid like structure with 3 rows. In the bottom row there are 3 cells with algebraic terms in each. The first cell has the term '4x', the second has 3 and the third has x. In the second row there are two cells. The first cell has the term 12x, the second cell has the term 3x. The top row has one cell which has the term 36 'x squared in it. | An image containing a table in a pyramid like structure with 3 rows. In the bottom row there are 3 cells with algebraic terms in each. The first cell has the term 5y, the second has 2 and the third has 7x. In the second row there are two cells. The first cell has the term 10y, the second cell has the term 14x. The top row has one cell which has the term 140xy in it. |
| An image containing a table in a pyramid like structure with 4 rows. In the bottom row there are 4 cells with algebraic terms in each. The first cell has the term 2x, the second has 2, the third has 3 and the fourth has 3x. In the second row there are 3 cells. The first cell has the term 4x, the second cell has the term 6 and the third cell has the term 9x. The third row has 2 cells. The first cell has the term 24x and the second cell has the term 54x. The top row has one cell which has the term 1296 'x squared' in it. | An image containing a table in a pyramid like structure with 4 rows. In the bottom row there are 4 cells with algebraic terms in each. The first cell has the term 5y, the second has negative 1, the third has 4 and the fourth has 4x. In the second row there are 3 cells. The first cell has the term negative 5y, the second cell has the term negative 4 and the third cell has the term 8x. The third row has 2 cells. The first cell has the term 20y and the second cell has the term negative 32x. The top row has one cell which has the term negative 640xy in it. | An image containing a table in a pyramid like structure with 4 rows. In the bottom row there are 4 cells with algebraic terms in each. The first cell has the term x/4, the second has 8, the third has 6 and the fourth has x/3. In the second row there are 3 cells. The first cell has the term 2x, the second cell has the term 48 and the third cell has the term 2x. The third row has 2 cells. The first cell has the term 96x and the second cell has the term 96x. The top row has one cell which has the term 9216 'x squared' in it. |
| An image containing a table in a pyramid like structure with 4 rows. In the bottom row there are 4 cells with algebraic terms in each. The first cell has the term 5x, the second has 2y, the third has x and the fourth has 3x. In the second row there are 3 cells. The first cell has the term 10xy, the second cell has the term 2xy and the third cell has the term 3 'x squared'. The third row has 2 cells. The first cell has the term 20 'x squared, y squared' and the second cell has the term 6 'x cubed, y'. The top row has one cell which has the term 120 'x to the power of 5, y cubed' in it. |  |  |

## References

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