# Sphere’s the thing

Students show that the surface area of a sphere is 4 times the area of a circle with the same diameter through an exploration involving an orange. Students then explore a mix of routine and non-routine questions to apply the formulas for surface area and volume of spheres.

## Visible learning

### Learning intentions

* To be able to solve problems involving the surface area and volume of spheres.

### Success criteria

* I can calculate the volume of spheres.
* I can calculate the surface area of spheres.
* I can solve routine and non-routine problems involving spheres.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* applies knowledge of the surface area of right pyramids and cones, spheres and composite solids to solve problems **MA5-ARE-P-01**
* applies knowledge of the volume of right pyramids, cones and spheres to solve problems involving related composite solids **MA5-VOL-P-01**

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## Activity structure

### Launch

1. Display **screen 1** from the Desmos activity ‘What proportion of the Earth is covered by water?’ ([bit.ly/saearth](https://bit.ly/saearth)) for students to view. To do this, select **Student Preview** or select the first slide under the heading, **Screens**.

An alternative to the Desmos activity is to throw a globe of the Earth from student to student. Students call out whether their right thumb is on land or water when they catch the globe. Create a tally on the board of land or water.

1. With the image of the map of the world displayed for students to view, ask students to consider ‘What percentage of the Earth’s surface is water?’
2. Allow time for students to think independently. They can then share and discuss their prediction with a partner and then decide on an estimate as a pair.
3. Move to **screen 2** in the Desmos activity for the students to view.
4. Click on the button **New Location** and the point on the map should move to a new location. Record in the table if this point has landed in water or on land.
5. Repeat this 10 times and then use the table under the map to record the findings and make another prediction of what percentage of the earth’s surface is water.
6. Ask each pair to compare this to their own predictions before moving to **screen 3** and revealing that 71% of the Earth’s surface is water.
7. Ask pairs to share with the class how they came up with their original predictions, example questions could include:

* Did you picture the Earth as a sphere or a flat map?
* What do you think the net of a sphere would look like?
* Imagine a circle with the same diameter as the Earth, how many circles do you think would wrap around the Earth?

1. Explain to students that in today’s lesson they will explore how to calculate the surface area and volume of spheres so that they are equipped to solve problems involving spheres. It may be beneficial to conduct a brainstorm of where spheres are seen in real life, for example bearings, nature and design.

### Explore

#### Surface area

Students will perform an experiment to discover the formula for the surface area of a sphere. The video ‘Surface area of a sphere with cutie (0:37)’ ([bit.ly/orangepeelsphere](https://bit.ly/orangepeelsphere)) demonstrates the steps they will take.

##### Equipment

* One orange per group
* One sheet of paper per group
* One protractor and ruler per group

##### Method

1. Assign students into random groups of 3. Explain to students that they will be peeling an orange and placing the peel onto circles that are the same size as the orange.
2. Students should begin by drawing a circle that is equal in diameter to their orange. Issue each student with their orange and have them construct this circle. It is important that this circle is as close as possible to the diameter of the orange. They can do this by tracing around the orange, cutting the orange in half and tracing that circle or by using a ruler and protractor.
3. Each group is to start peeling the orange, placing the peel into the constructed circle. Once they fill one circle, they are to construct a new circle to continue until the orange has no peel left. If students have a significant amount of pith (white) showing they should be challenged to consider if the pith is part of the surface area. Students can peel smaller pieces to avoid excessive pith.
4. Students should end up with 4 complete circles filled with orange peel. Once groups are finished, they should do a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) to observe other groups’ results.
5. Students can be shown the ‘Surface area of a sphere with cutie (0:37)’ ([bit.ly/orangepeelsphere](https://bit.ly/orangepeelsphere)) demonstration the experiment they just did.
6. Have a class discussion about what this means. The orange was a sphere and the orange peel ended up fitting into 4 circles that had the same diameter as the sphere.
7. Clearly explain that the peel represents the surface area and revise the formula for the area of circle, . Then allow students to discuss with their partner what the formula for the surface area of a sphere is.
8. Reveal to students that the formula for the surface area of a sphere is .

#### Volume

Other than verifying that the formula works by pouring water from hemispheres into a cylinder of equal diameter and height (the video ‘Develop and apply the formula for volume of a sphere (4:47)’ ([bit.ly/derivingvolumeofsphere](https://bit.ly/derivingvolumeofsphere)) demonstrates the concept. Methods of proving the formula require algebra beyond the scope of Stage 5.

1. If equipment is available, use water pouring methods to demonstrate how the volume of a sphere is formed. Alternatively show students the video ‘Develop and apply the formula for volume of a sphere (4:47)’ ([bit.ly/derivingvolumeofsphere](https://bit.ly/derivingvolumeofsphere)) which derives the formula.
2. Following this, explicitly show students the volume formula for a sphere is .
3. As a class, students can construct a Notice and Wonder list ([bit.ly/noticewonderstrategy](https://bit.ly/noticewonderstrategy)) about the formula for the volume of a sphere. Students might:

* Notice the similarities to the area of a circle formula.
* Notice that the power is 2 for a 2D circle and 3 for a 3D sphere
* Wonder where the fraction has come from.

### Summarise

1. Use the provided *Spheres the thing* PowerPoint for explicit teaching and worked examples to demonstrate how to apply the formulas for the volume and surface area of spheres.

### Apply

1. Randomly assign students into groups of 3 and distribute one set of questions from Appendix A ‘Spheres problems set’ per group. For example, one group will have ‘Spheres in architecture’, another will have ‘The solar system’. Students are to work on Vertical Non-Permanent Surfaces (VNPS) ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)) in their groups to solve the set of assigned problems.
2. When students have finished and can’t go any further with their problem, they are encouraged to move to another group and work on their problem.
3. At the conclusion, students should do a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) to view the solutions to all the problems.
4. Students can then complete the following question as an exit ticket to check for student understanding. This question involves students working backwards to discover the radius and has the added challenge of writing the volume and surface area in terms of .
5. Given that the volume of a sphere is , find its surface area in terms of .

## Assessment and differentiation

### Suggested opportunities for differentiation

**Explore**

* Students could be challenged to explore and explain the derivation of the formula for the volume of a sphere.

**Summary**

* Further composite solids can be provided to challenge students. For example, quarter spheres or spheres with volumes cut out.

**Apply**

* In the solar system questions, the answers are in scientific notation, this may require some review for some students.

### Suggested opportunities for assessment

* Collect student responses from Appendix A ‘Spheres problems set’ to check for student understanding.
* Monitor student conversations to check for any misconceptions that may need to be addressed.

## **Appendix A**

### Spheres problem sets

#### Spheres in architecture

1. Spaceship Earth, a spherical attraction at Walt Disney World's Epcot Centre, has a diameter of 50 metres. Calculate the surface area of the structure.



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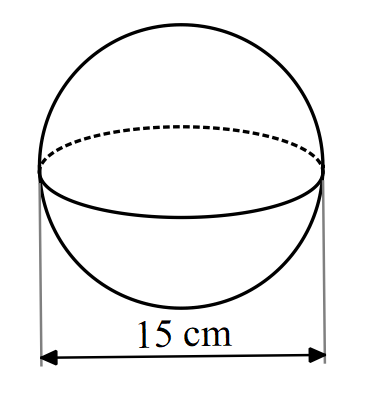
1. This statue of Atlas, carrying the globe on his shoulders can be found at Frankfurt Central Station in Germany.



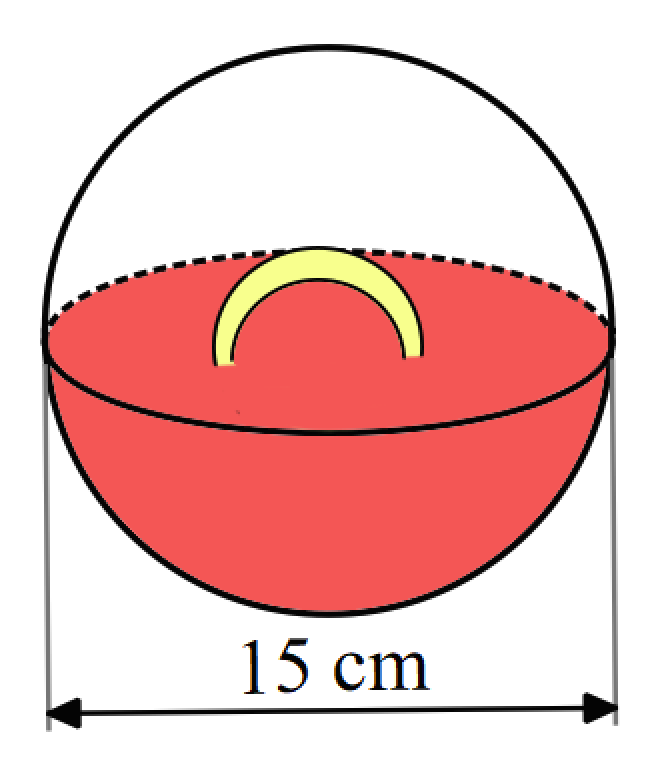
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Yindi has purchased a replica of the statue without the globe. They intend on 3D printing a sphere to fit in its place that can be used as a jewellery box.

1. Yindi designed a sphere with a diameter of 15 cm as shown below. If they print the hollow sphere, will 800 of plastic be enough material?

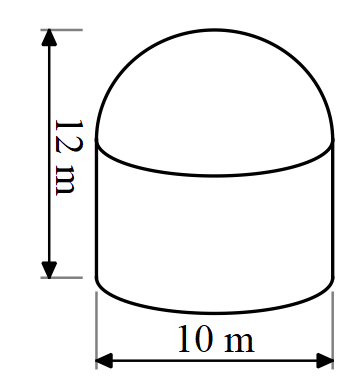


1. Yindi intends on filling the bottom hemisphere with foam that they can sit a ring inside the sphere, as shown below.



How many cubic centimetres of foam should Yindi buy?

1. A silo is built by placing a dome roof on top of a cylinder as shown in the diagram below.



1. Find the capacity of the silo in litres.
2. The exterior of the silo is to be painted, find the surface area that will be painted.

#### The solar system

1. The circumference of Earth measured around the equator is 40 075.017 km. What is the surface area of Earth, assuming it is spherical.
2. The planet Neptune has a radius of 24 622 km and Jupiter has a radius of 69 911 km. How many times bigger is the surface area of Jupiter than Neptune?
3. The diameter of the moon is approximately one-fourth of the diameter of the Earth. What percentage of the volume of the Earth is the volume of the moon?

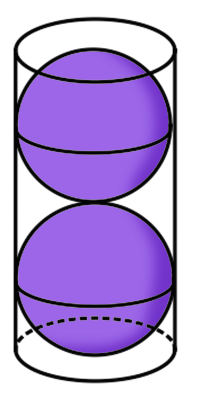
#### Tennis balls

Round all answers to a consistent degree of accuracy.

1. Some tennis balls are sold in cylinders of 3 balls that fit perfectly inside so that the height of the 3 spheres is equal to the height of the cylinder, and each ball touches the cylinder wall.

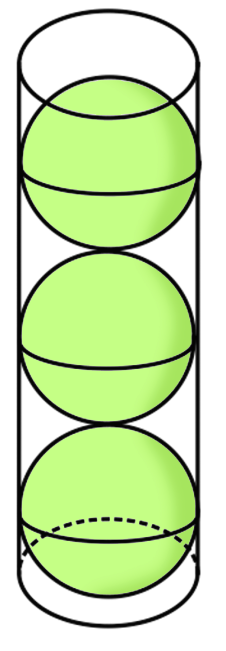
Will the height of a cylinder that holds 3 tennis balls be greater than or less than its circumference?

1. Two identical spherical balls fit exactly inside a cylinder as shown below.



Given that the circumference of each ball is 20 cm, find the surface area of the cylinder.

1. Three tennis balls of diameter 8 cm fit perfectly inside a tube so that the height of the 3 spheres is equal to the height of the tube, and each ball touches the cylinder wall.



1. Find the volume of the tube.
2. Find the empty space in the tube.
3. Calculate the percentage of the tube that is not filled. Round your answer to the nearest whole percent.

## **Sample solution**

### **Appendix A – spheres problem sets**

#### Spheres in architecture

Therefore, yes 800 cm2 of plastic is enough material.

#### The solar system

How many times bigger?

Therefore the surface area of Jupiter is approximately 8 times larger than the surface area of Neptune.

#### Tennis balls

The height of the cylinder will be less than the circumference since

1. Given that the circumference of each ball is 20 cm, find the surface area of the cylinder.

Finding the diameter:

Finding the height of the cylinder:

Finding the surface area of the cylinder:

1. Find the volume of the tube.
2. Find the empty space in the tube.

## References

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