# Line it up

This lesson will explore the features that define lines and linear relationships as parallel or perpendicular to one another and explore ways to determine this both graphically and algebraically.

Students will need at least one digital device per pair to interact with Desmos during this lesson.

## Visible learning

This unit contains optional Path content.

### Learning intentions

* To understand what makes linear relationships parallel or perpendicular.

### Success criteria

* I can identify and define the features of parallel lines in equations and graphs.
* I can identify and define the features of perpendicular lines in equations and graphs.
* I can write equations of lines that are parallel and perpendicular to other lines.
* I can justify whether 2 lines are parallel or perpendicular.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* determines the midpoint, gradient and length of an interval, and graphs linear relationships, with and without digital tools **MA5-LIN-C-01**
* graphs and interprets linear relationships using the gradient/slope-intercept form   
  **MA5-LIN-C-02**
* describes and applies transformations, the midpoint, gradient/slope and distance formulas, and equations of lines to solve problems **MA5-LIN-P-01**

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## Activity structure

Please use the associated PowerPoint *Line it up* to display images in this lesson.

### Warm-up

1. Provide students with a copy of Appendix A ‘Defining a line’.
2. Once students have had the opportunity to attempt to match the terms and definitions, initiate a class discussion to randomly have students share their solutions.
3. Before moving on, ensure students have been provided with the correct solutions by displaying the Sample Solutions found towards the end of this document.

### Launch

1. Use slide 2 of the associated PowerPoint *Line it up* to display the image of the optical illusion ‘The Café Wall’ for your students.

The original image was created with all horizontal and vertical lines, however to better serve the purpose of this task the image has been rotated and cropped according to the guidelines stipulated under the CC BY-SA 3.0 license (<https://creativecommons.org/licenses/by-sa/3.0/>).

1. Ask students to complete a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)) considering the following prompts:

* Are there any straight lines in this image?
* Are there any perpendicular lines in this image?
* Are there any parallel lines in this image?

1. Initiate a whole-class discussion to collate students’ responses. Randomly select students to answer each question ensuring, as close as possible, that all pairs have an opportunity to share at least once.
2. Inform students that this is an optical illusion, and every line segment utilised in its design is parallel to half of the total number of lines drawn, and every line segment is perpendicular to the other half of every line segment drawn.

If the warm-up was not undertaken, remind students about the terms parallel and perpendicular in relation to angles.

1. Pose the following question to students: ‘This image convincingly tricks the brain into thinking that these lines are not perpendicular and parallel, so how can we prove that this is just an optical illusion?’

### Explore

1. Students will need to be allocated at least one device capable of accessing the internet between every 2 students.
2. Assign the Desmos activity ‘Parallel and Perpendicular Lines’ ([bit.ly/parallel\_perpendicular](https://bit.ly/parallel_perpendicular)) for your students to work their way through.
3. Make sure students cannot progress beyond screen 5 by using the pacing feature embedded into the application.

If you would like support in better understanding how to utilise Desmos’s Classroom Activity Builder, please refer to ‘Using Activity Builder by Desmos to engage students during class - Olga Bulakh & Sarah Coombs (5:33)’ (<https://www.youtube.com/embed/_HCEORKpDxk?t=295s>). If you skip ahead to 4:44 you will find a brief overview of how to pause and pace students.

1. Once students have engaged with the desmos screens, use Pose-Pause-Pounce-Bounce question strategy [PDF 200KB] ([bit.ly/pausepouncebouncestrategy](https://bit.ly/pausepouncebouncestrategy)) to initiate a class discussion asking the question: ‘Why were people able to more accurately create parallel lines when the grid was visible?’

Ensure students are guided to considering the following in their responses:

* the grid squares help to copy the pattern made as the line passes through them
* the values shown on the axes make it easier to check the relative changes in values
* references to calculating the slope or gradient.

1. Once students have had an opportunity to discuss these observations and intuitions, use the pacing feature and allow them to progress through to screen 9 of the Desmos classroom activity.
2. Once all students have had the opportunity to engage with the screens to this point initiate a further class discussion, utilising Pose-Pause-Pounce-Bounce questioning, and have students respond to the following statement:

Using an equation is the most accurate way to ensure that lines will be parallel when graphed. Provide reasoning to communicate why you agree or disagree with this statement.

Ensure that the class discussion is guided towards the following conclusions:

* If the gradient (slope) of a line is identical to that of another line, they will always be parallel.
* Using the gradient-intercept format allows for this to be checked quickly and simply just by comparing the respective values given to in each line.
* The lines do not require graphing to determine if they are parallel, which is quick, efficient and does not rely on any estimating, guessing or trial and error.

1. Allow students to progress through to and including screen 11, completing the tasks as outlined.
2. Ensure all students have had the opportunity to submit responses to these screens before allowing them to progress through to screen 15.
3. Once all students have had the opportunity to engage with the screens to this point initiate a further class discussion, utilising Pose-Pause-Pounce-Bounce questioning, and have students respond to the following statement:

Using an equation is the most accurate way to ensure that lines will be perpendicular when graphed. Provide reasoning to communicate why you agree or disagree with this statement.

Ensure that the class discussion is guided towards the following conclusions:

* If the gradients (slopes) of 2 lines are multiplied, and the result is negative one, they will always be perpendicular.
* Using can help to quickly determine if the gradients are negative reciprocals of one another.
* Using the gradient-intercept format allows for this to be checked relatively quickly and simply if each is written or converted to a proper or improper fraction.
* The lines do not require graphing to determine if they are perpendicular, which is quick, efficient and does not rely on any estimating, guessing or trial and error.

1. Allow students to progress through to and including screen 19, completing the tasks as outlined.

### Summarise

1. Use slides 3–10 in the accompanying PowerPoint *Line it up* to explicitly teach students the concepts, properties, and relationships between perpendicular and parallel lines.

The explicit teaching technique used in the associated PowerPoint is ‘Your turn.’ The first slide is a worked example which should be displayed for the students and then use the following steps.

1. Reveal the question to students and its solution.
2. Students read in silence.
3. Students individually think and explain to themselves what is happening in each step.
4. Students hold up a thumbs up to the teacher when they have finished reading and have some sort of understanding.
5. Think-Pair-Share. Students explain the solution to their partner.
6. In pairs students then answer the self-explanation questions.
7. Finally, randomly select students to share their answers with the whole class.
8. Using the Desmos activity ‘Café wall askew’ ([bit.ly/cafewallaskew](https://bit.ly/cafewallaskew)) have students create straight lines to test if the image of the café wall shows only parallel and perpendicular lines, or not.

Students may need to be reminded to consider where the lines in the image appear to create intercepts with the - and -axes, and how this could help in establishing equations for appropriate linear relationships.

1. Students are to complete notes to their forgetful future self ([bit.ly/notesstrategy](https://bit.ly/notesstrategy)) on finding parallel and perpendicular gradients.

### Apply

1. On vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)), in visible random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)), students are to complete Appendix B ‘Finish that quadrilateral’. This activity has students writing linear equations to finish trapeziums and rectangles.
2. Students are to do a gallery walk. Ask them to compare their answers with others in the classroom and note what is the same and what is different.
3. Using Pose-Pause-Pounce-Bounce, ask students if all the answers around the classroom are correct. How do they know? How could they check they have met all the properties?

The teacher can interleave knowledge of properties of quadrilaterals from Stage 4, as well as finding distance from *Unit 2 – Working with triangles* for students to prove they have the correct shape.

1. Students are to return to their boards in their groups of 3 to prove they have made rectangles.
2. Prompt students to see if they can make any other shapes such as right-angled triangles or other quadrilaterals.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Explore**

* Students requiring additional support may need to be explicitly directed to focus their attention on the similarities between the gradients of the linear relationships being compared.
* For students requiring additional challenges, examples of perpendicular lines where neither equation involves a gradient with an integer value should be utilised.

**Summarise**

* For students requiring additional support they may need to be reminded to consider where the lines in the image appear to create intercepts with the x- and y-axes, and how intercepts can be used to create equations for linear relationships.

**Apply**

* To only include core content, students can identify what lines are parallel and perpendicular, but do not need to justify why.

### Suggested opportunities for assessment

**Warm up**

* Student responses may be utilised as a component for formative assessment of current levels of knowledge and understanding related to the properties of parallel and perpendicular lines.

**Launch**

* Students’ responses to the question for consideration (point 5) could be recorded as a pre-test to understand the level of prior knowledge related to parallel and perpendicular lines.

**Explore**

* The discussion questions could be utilised as a hinge question to assess if students’ have an adequate understanding of the concepts being addressed for parallel and perpendicular lines respectively.

**Summarise**

* Observe student responses, record if students accurately represent all the parallel lines with equal gradients, and that their perpendicular lines have a gradient that is the negative reciprocal.

**Apply**

* Observe student responses, record if students accurately represent all the parallel lines with equal gradients, and that the perpendicular lines have a gradient that is the negative reciprocal.

## Appendix A

### Defining a line

Table 1 – jumbled terms and definitions for components of lines

|  |  |  |
| --- | --- | --- |
| Term | Correct definition | Jumbled definition |
| angle | Formed by 2 straight lines meeting at a common endpoint, called the vertex. | The slope of a line, often referred to as . |
| degree |  | Where lines cross over (where they have a common point). |
| gradient |  | Two lines or line segments that intersect at a 90° angle (a right angle). |
| intersection |  | Two lines that have no points of intersection. |
| parallel |  | ~~Formed by 2 straight lines meeting at a common endpoint, called the vertex.~~ |
| perpendicular |  | A unit for measuring an angle. |

1. Using the definitions provided in the ‘Jumbled definition’ column, match these with the correct term by re-writing them on the correct row in the ‘Correct definition’ column.
2. Cross out the printed definition once you have re-written it in the correct spot.
3. The first example has been completed for you.

## Appendix B

### Finish that quadrilateral

#### Trapezium

A trapezium has one set of parallel lines.

1. The linear equations below create a trapezium between the 4 points of intersection.

Identify the equations that create parallel lines. Justify why.

1. Given the equations below, write a linear equation that would create a trapezium between the 4 points of intersection. Justify your answer.

An image of a cartesian plane, focused around the origin, displaying three distinct lines, one red, one blue and one black.
The red line displays an equation of y equals negative x plus four.
The blue line shows an equation of y equals negative two x plus 3.
The black line shows an equation of y equals three x minus one.
The image was generated with permission, using Desmos's online graphing calculator.

Image created using [Desmos](https://www.desmos.com/?lang=en) and is licensed under the [Desmos Terms of Service](https://www.desmos.com/terms?lang=en).

1. Given the equations below, write 2 linear equations that would create a trapezium between the 4 points of intersection. Justify your answer.

An image of a cartesian plane, focused around the origin, displaying two distinct lines, one blue and one black.
The blue line shows an equation of y equals negative two x plus 3.
The black line shows an equation of y equals three x minus one.
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1. Write 4 linear equations that create a trapezium between the 4 points of intersection.
2. Check your solution by graphing on a Cartesian plane.

#### Rectangle

A rectangle has 2 set of parallel lines that intersect perpendicular to one another.

1. The linear equations below create a rectangle.

Identify the equations are perpendicular to one another. Justify why.

1. Given the equations below, write a linear equation that would create a rectangle between the 4 points of intersection. Justify your answer.

An image of a cartesian plane, focused around the origin, displaying three distinct lines, one red, one blue and one green.
The red line displays an equation of y equals three quarters x plus 2.
The blue line shows an equation of y equals negative four thirds x plus one.
The green line shows an equation of y equals three quarters x minus two.
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1. Given the equations below, write 2 linear equations that would create a rectangle between the 4 points of intersection. Justify your answer.

An image of a cartesian plane, focused around the origin, displaying two distinct lines, one red and one blue.
The red line displays an equation of y equals five x plus two.
The blue line shows an equation of y equals five x minus 7.
The image was generated with permission, using Desmos's online graphing calculator.

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1. Write 4 linear equations that create a rectangle between the 4 points of intersection.
2. Check your solution by graphing on a Cartesian plane.

## Sample solutions

### Appendix A – defining a line

|  |  |  |
| --- | --- | --- |
| Term | Correct definition | Jumbled definition |
| angle | Formed by 2 straight lines meeting at a common endpoint, called the vertex. | ~~The slope of a line, often referred to as .~~ |
| degree | A unit for measuring an angle. | ~~Where lines cross over (where they have a common point).~~ |
| gradient | The slope of a line, often referred to as . | ~~Two lines or line segments that intersect at a 90° angle (a right angle).~~ |
| intersection | Where lines cross over (where they have a common point). | ~~Two lines that have no points of intersection.~~ |
| parallel | Two lines that have no points of intersection. | ~~Formed by 2 straight lines meeting at a common endpoint, called the vertex.~~ |
| perpendicular | Two lines or line segments that intersect at a 90° angle (a right angle). | ~~A unit for measuring an angle.~~ |

### Appendix B – finish that quadrilateral

#### Trapezium

1. and make parallel lines as they have the same gradient.
2. I needed an equation that would be parallel to . To do this I need the same gradient. I want it to the right of as if I go too far to the left, I might not make a trapezium. Therefore, my linear equation is .
3. I need 2 lines that are parallel to each other but decreased from left to right. This means I need 2 lines with the same gradient which are negative. Therefore, I chose the lines and .
4. The equations and enclose a trapezium.

A cartesian plane focused around the origin displaying four distinct lines: one blue; one red; one green; and, one purple.
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#### Rectangle

1. and

This means that the lines with these gradients are perpendicular to each other.

1. We need a line that is perpendicular to or parallel to . So, I picked the linear equation .
2. We need 2 lines that have gradients perpendicular to . Perpendicular gradients multiply to give negative 1. So, and

Therefore, I picked the equations and

1. The equations , , and make a rectangle.

A cartesian plane focused around the origin displaying four distinct lines: one blue; one red; one green; and, one purple.
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