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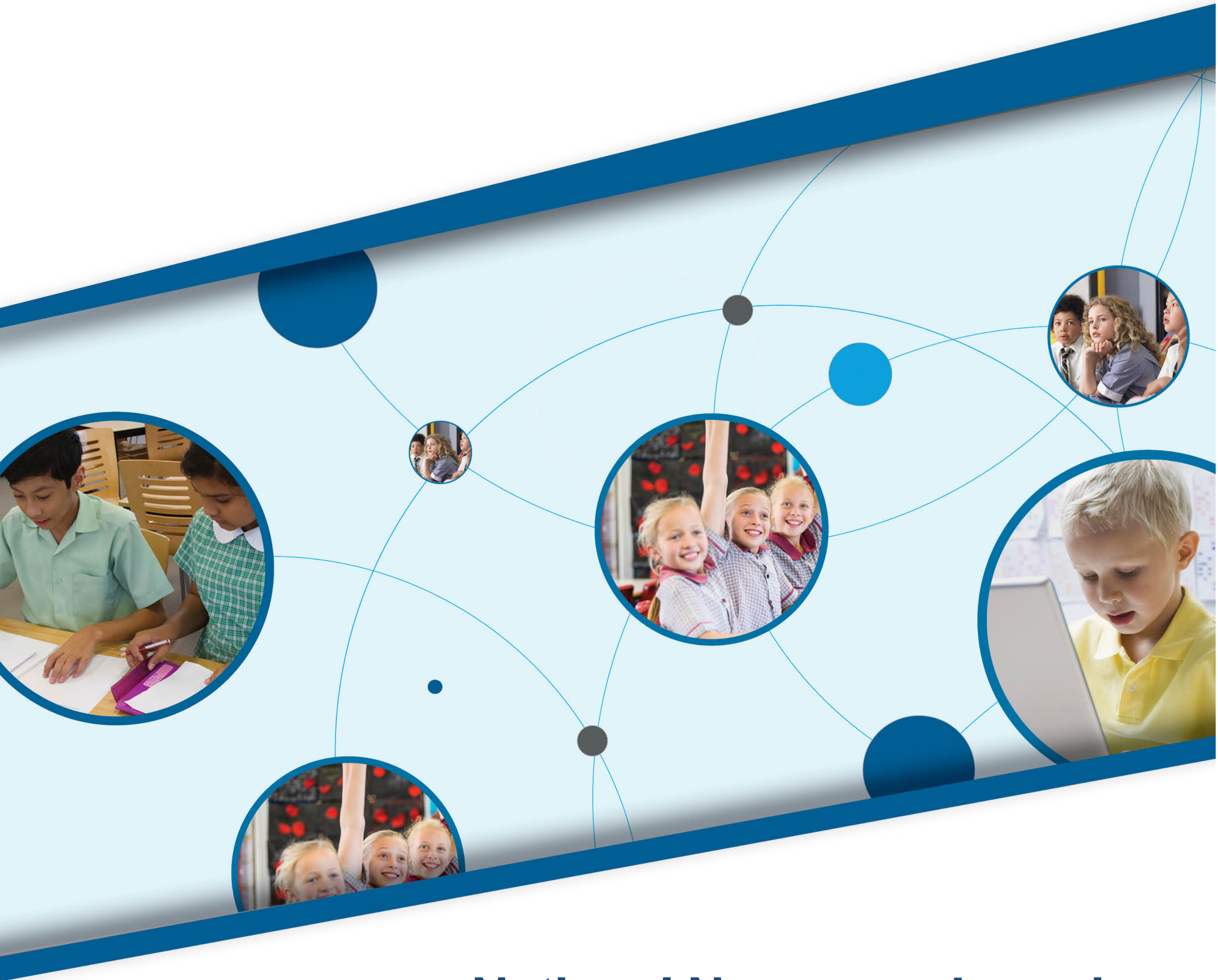

National Numeracy Learning Progression

Version 3.0

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AUSTRALIAN CURRICULUM, ASSESSMENT AND REPORTING AUTHORITY
Australian CURRICULUM
NAP NATIONAL ASSESSMENT PROGRAM
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Acknowledgements

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Contents

INTRODUCTION

National learning progressions

National learning progressions describe the skills, understandings and capabilities that students typically acquire as their proficiency increases in a particular aspect of the curriculum over time.

They describe the learning pathway(s) along which students typically progress in particular aspects of the curriculum regardless of age or year level, and are designed to help teachers ascertain the stage of learning reached, identify any gaps in skills and knowledge, and plan for the next step to progress learning.

National learning progressions and the Australian Curriculum

National learning progressions sit within the broader framework of the Australian Curriculum. They supplement and underpin the Australian Curriculum. They do not replace the Australian Curriculum.

The Australian Curriculum identifies what students need to learn; national learning progressions describe the learning pathway(s) along which students typically progress in particular aspects of the curriculum regardless of age or year level. Where learning progressions exist, they can help inform the refinement of the Australian Curriculum.

THE NATIONAL NUMERACY LEARNING PROGRESSION

What is numeracy?

Numeracy is fundamental to a student's ability to learn at school and to engage productively in society.

In the Australian Curriculum, students become numerate as they develop the knowledge and skills to use mathematics confidently across learning areas at school and in their lives more broadly. The Australian Curriculum states:

> *Numeracy encompasses the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations. It involves students recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully* (ACARA, 2017).

What is the National Numeracy Learning Progression?

The National Numeracy Learning Progression describes the observable indicators of increasing complexity in the understanding of, and skills in, key numeracy concepts. The numeracy progression includes the elements of Number sense and algebra, Measurement and geometry and Statistics and probability. By providing a comprehensive view of numeracy learning and how it develops over time, the progression gives teachers a conceptual tool that can assist them to develop targeted teaching and learning programs for students who are working above or below year-level expectations.

Numeracy development influences student success in many areas of learning at school. The progression can be used to support students to successfully engage with the numeracy demands of the Foundation to Year 10 Australian Curriculum.

The progression does not advise schools on how to teach, plan, program, assess or report.

How is the National Numeracy Learning Progression structured?

Elements and sub-elements

The National Numeracy Learning Progression has three elements that reflect aspects of numeracy development necessary for successful learners of the F–10 Australian Curriculum and in everyday life. The three elements are:

- Number sense and algebra
- Measurement and geometry
- Statistics and probability.

Each element includes sub-elements that represent evidence-based aspects of numeracy development. There are eight sub-elements in Number sense and algebra, four in Measurement and geometry and two in Statistics and probability.

The diagram (Figure 1) represents the elements and sub-elements of the National Numeracy Learning Progression.

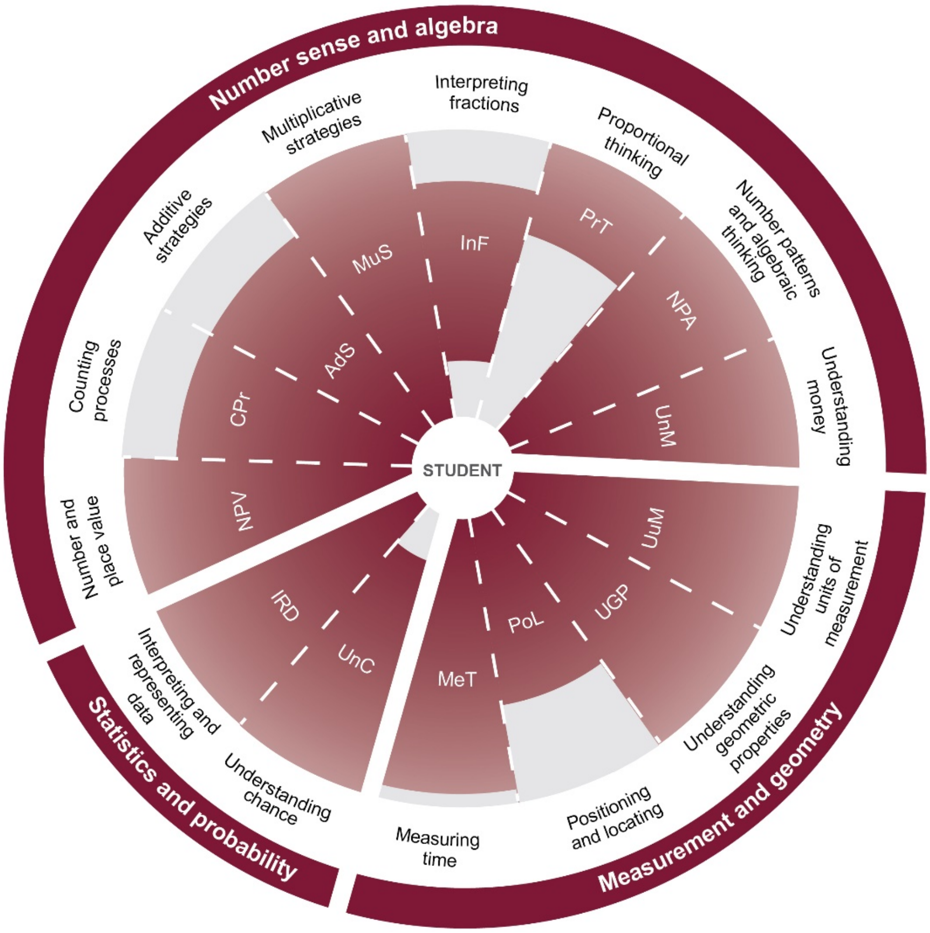

Figure 1. Elements and sub-elements of the National Numeracy Learning Progression

Levels and indicators

Within each sub-element, indicators are grouped together to form developmental levels. Each indicator describes what a student says, does or produces and begins with the stem ‘A student …’ as the subject of the sentence.

There are as many levels within each sub-element as can be supported by evidence. The listing of indicators within a level is non-hierarchical. Each level within a sub-element has one or more indicators and is more complex than the preceding level. The levels within each sub-element are named with a letter and number code that indicates the abbreviated name of the sub-element and the developmental level, in number order. For example, NPV4 indicates the sub-element of Number and place value at level 4.

In all of the sub-elements, subheadings have been included to assist teachers by grouping indicators into particular categories of skills that develop within that level or over a number of levels.

The amount of time it takes a student to progress through each level is not specified because students progress in numeracy development at different rates.

The levels do not describe equal intervals of time in a student's learning. They are designed to indicate the order in which students typically acquire the mathematical knowledge and skills necessary to be numerate.

The amount of detail in any level or sub-element is not an indication of importance. A single indicator at a more advanced level in the progression may rely on a substantial number of indicators being evident in earlier levels.

The diagram (Figure 2) shows the various components included in the progression.

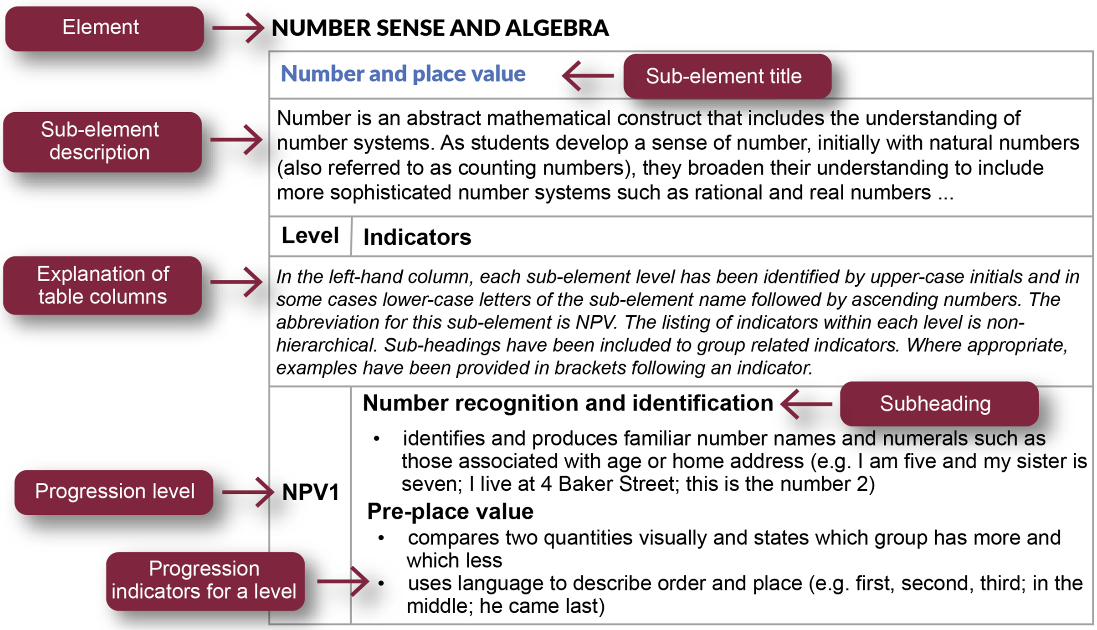

Figure 2. Annotated example of a numeracy sub-element

How is the National Numeracy Learning Progression related to the Australian Curriculum?

The skills and understandings required to be numerate are explicit in the Australian Curriculum: Mathematics. Students need opportunities to recognise that mathematics is constantly used outside the mathematics classroom and that numerate people apply mathematical skills in a wide range of familiar and unfamiliar situations.

Applying mathematical skills and knowledge across the curriculum can enrich the study of other learning areas and helps to develop a broader and deeper understanding of numeracy. It is essential that the mathematical ideas with which students interact are relevant to their lives.

Australian Curriculum: Mathematics

> *The Australian Curriculum: Mathematics provides students with essential mathematical skills and knowledge in number and algebra, measurement and geometry, and statistics and probability … Mathematics is composed of multiple but interrelated and interdependent concepts and systems which students apply beyond the mathematics classroom …* (Australian Curriculum: Mathematics, Rationale 2017)

The Australian Curriculum: Mathematics sets teaching expectations for mathematics learning at each year level, providing carefully paced, in-depth study of critical mathematical skills and concepts. The curriculum focuses on developing the mathematical proficiencies of understanding, fluency, reasoning and problem solving. Numeracy involves the application of these proficiencies to authentic contexts both familiar and unfamiliar.

The National Numeracy Learning Progression helps teachers to develop fine-grain understandings of student numeracy development in the Australian Curriculum: Mathematics, especially in the early years. The progression has not been designed as a checklist and does not replace the Australian Curriculum: Mathematics. Each sub-element has been mapped to the year-level expectations set by the Australian Curriculum: Mathematics.

Other Australian Curriculum learning areas

This National Numeracy Learning Progression is designed to assist schools and teachers in all learning areas to support their students to successfully engage with the numeracy demands of the F–10 Australian Curriculum.

Advice is included on identifying the numeracy demands of each subject in the Australian Curriculum. This advice will assist teachers to identify opportunities to support students' numeracy development and to provide meaningful contexts for the application of numeracy skills.

How can the National Numeracy Learning Progression be used?

The National Numeracy Learning Progression can be used at a whole school, team or individual teacher level. The progression provides maximum student learning benefits when supported by professional learning and collaboration between teachers. Further advice on how to maximise the benefits of the progression is available on the progressions home page.

The progression can be used to identify the numeracy performance of individual students within and across the 14 sub-elements. In any class there may be a wide range of student abilities. Individual students may not neatly fit within a particular level of the progression and may straddle two or more levels within a progression. While the progression provides a logical sequence, not all students will progress through every level in a uniform manner.

When making decisions about a student's numeracy development, teachers select relevant indicators. It is important to remember indicators at a level are not a prescriptive list and the progression is not designed to be used as a checklist. Teacher judgements about student numeracy capability should be based on a range of learning experiences. Rich tasks, observations and investigations from other learning areas may provide suitable evidence of a student's numeracy capability; for example, interpreting and representing data collected during a science experiment, reading and interpreting maps during a HASS lesson and pattern recognition skills applied in Design and Technologies.

Teachers can use the progression to support the development of targeted teaching and learning programs and to set clearer learning goals for individual students. For example, teaching decisions can be based on judgements about student capability that relate to a single indicator rather than all indicators at a level.

NUMBER SENSE AND ALGEBRA

Number and place value

Number is an abstract mathematical construct that includes the understanding of numbers and number systems. As students develop a sense of number, initially with natural numbers (also referred to as counting numbers), they broaden their understanding to include more sophisticated number systems such as rationals and real numbers.

This sub-element describes how a student becomes increasingly able to recognise, read, represent, order and interpret numbers within our place value number system, expressed in different ways. It outlines key understandings needed to process, communicate and interpret quantitative information in a variety of contexts.

Cardinal numbers are used to quantify collections, construct matching collections, measure an attribute or to assign a value. They can be represented by a collection, a diagram, a word or a symbol (numeral) and they are central to quantitative thinking. Ordinal numbers do not show a quantity but rather position and order. Ordinality and cardinality are two important components of building number sense. Numerals provide a conventional representation of a cardinal number and allow us to communicate and operate with numbers efficiently. Numerals can also be used as labels that do not involve cardinal or ordinal properties.

Place value relies on understanding the relationship between digits in a numeral, which then enables the cardinal value to be represented in multiple ways. The place value system relies on students having a strong sense of the number ten and utilises both additive and multiplicative properties. That is, the quantity represented by a numeral can be expressed additively as the sum of the values represented by its individual digits (e.g. $326 = 3 \times 100 + 2 \times 10 + 6 \times 1 = 300 + 20 + 6$) or multiplicatively using the relationship between the places (e.g. $326 = 10 \times 32.6$; $326 = 100 \times 3.26$; $326 = 3260 \div 10$)

There are several well documented misconceptions associated with the learning of decimal place value such as thinking the size of decimal numbers relates to the number of digits after the decimal place or the decimal point separates two sets of whole numbers. The introduction of integers can also see some students regress in their understanding of decimals. Students need to understand the multiplicative relationship between place values is consistent across the decimal point. This sub-element underpins the development of number sense, and the learning associated with measuring attributes and quantifying data. Place value is also evident in the learning of metric units in the sub-element *Understanding units of measurement*.

Some students will demonstrate the skills of *Number and place value* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Number and place value

Level	Indicators
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is NPV. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
NPV1	**Numeral recognition and identification** • identifies and produces familiar number names and numerals such as those associated with age or home address, but may not distinguish whether they refer to a quantity, an ordinal position or a label (e.g. I am five and my sister is seven; I wear the number 7 jumper; I live at 4 Baker Street; this is the number 2) **Pre-place value** • compares two collections visually and states which group has more items and which group has less • instantly recognises collections up to three without needing to count • uses language to describe order and place (e.g. understands 'who wants to go first?'; in the middle; 'who was the last person to read this book?')
NPV2	**Numeral recognition and identification** • identifies and names numerals in the range of 1–10 (e.g. when asked 'which is three?' points to the numeral 3; when shown the numeral 5, says 'that's five') • matches a quantity of items in a collection to the correct number name or numeral in the range of 1–10 (e.g. when shown the numeral 5 and asked to 'go and collect this many items', gathers five items) • identifies standard number configurations such as on a standard dice or dominos or in other arrangements up to six, using subitising (e.g. moves a counter the correct number of places on a board game based on the roll of a dice; recognises a collection of five items by perceptually subitising 3 and 2) **Developing place value** • orders numerals to at least 10 (e.g. using number cards, places the numerals 1–10 in the correct order) • indicates the larger or smaller of two numerals in the range from 1 to 10 (e.g. when shown the numerals 6 and 3, identifies 3 as representing the smaller amount) • identifies smaller collections within collections to ten • demonstrates that one ten is the same as ten ones (e.g. using concrete manipulatives such as ten frames and bundles of ten)

Number and place value	
NPV3	**Numeral recognition and identification** • identifies and names numerals up to 20 (e.g. when shown the numbers 4, 17, 9 and 16 and asked, 'which is 16?', points to the number 16 or when shown the numeral 17 says its correct name) • identifies the 1–9 repeating sequence in the writing of teen numerals • identifies a whole quantity as the result of recognising smaller quantities up to 20 (e.g. uses part, part, whole knowledge of numbers to solve problems) **Developing place value** • orders numbers from 1–20 (e.g. determines the largest number from a group of numbers in the range from 1 to 20; students are allocated a number between 1 and 20 and asked to arrange themselves in numerical order) • reads, writes, models and describes teen numbers as ten and some more (e.g. 16 is ten and 6 more; using ten frames)
NPV4	**Numeral recognition and identification** • identifies, models and names numerals up to and beyond 100 (e.g. is shown the numerals 70, 38, 56 and 26 and when asked 'which is 38?', identifies the numeral 38) • identifies the 1-9 repeating sequence, both in and between the decade numerals (e.g. using hundreds charts or vertical number lists) • identifies zero as both a number and a placeholder for reading and writing larger numerals, denoted by the numeral 0 **Place value** • uses knowledge of place value to order numerals within the range of 0 to at least 100 (e.g. locates the numeral 21 on a number line between 20 and 22; re-orders a set of numerals from smallest to largest) • models, represents, orders and renames two-digit numbers as counts of tens and ones (e.g. 68 is 6 tens and 8 ones, 68 ones, or 60 + 8; uses concrete materials such as bundles of ten straws or base ten blocks)
NPV5	**Numeral recognition and identification** • identifies and names a numeral from a range of numerals up to 1000 (e.g. is shown the numerals 70, 318, 576 and 276 and when asked 'which is 276?', identifies 276) **Place value** • orders and flexibly regroups three-digit numbers according to their place value (e.g. 247 is 2 hundreds, 4 tens and 7 ones or 2 hundreds and 47 ones or 24 tens and 7 ones) • applies an understanding of zero in place value notation when reading numerals that include internal zeros (e.g. says 807 as eight hundred and seven or 80 tens and seven ones, not eighty and seven)

Number and place value	
NPV6	**Numeral recognition and identification** • identifies, reads and writes numerals beyond 1000 applying knowledge of place value, including numerals that contain a zero (e.g. student reads 1345 as one thousand, three hundred and forty-five; student reads one thousand and fifteen and writes as 1015) **Place value** • flexibly partitions numbers by their place value into thousands, hundreds, tens and ones • estimates and rounds whole numbers to the nearest ten or nearest hundred (e.g. pencils come in a pack of ten, estimate the number of packs required for 127 Year 6 students; to check the reasonableness of their solution to the computation 212 + 195, student rounds both numbers to 200) • represents and names tenths as one out of 10 equal parts of a whole (e.g. uses a bar model to represent the whole and its parts; uses a straw that has been cut into ten equal pieces to demonstrate that one piece is one-tenth of a whole straw and two pieces are two-tenths of the whole straw) • represents and names one-tenth as its decimal equivalent 0.1, zero point one • extends the place value system to tenths
NPV7	**Numeral recognition and identification** • identifies, reads and writes numerals, beyond four digits in length, with spacing after every three digits (e.g. 10 204, 25 000 000; 12 230.25; reads 152 450 as 'one hundred and fifty-two thousand four hundred and fifty') • identifies, reads and writes decimals to one and two decimal places **Place value** • estimates and rounds whole numbers to the nearest ten thousand, thousand etc. (e.g. estimates the crowd numbers at a football match; says that the $9863 raised at a charity event was close to ten thousand dollars) • explains that the place value names for decimal numbers relate to the ones place value • explains and demonstrates that the place value system extends beyond tenths to hundredths, thousandths … • models, represents, compares and orders decimals up to 2 decimal places (e.g. constructs a number line to include decimal values between 0 and 1, when asked 'which is larger 0.19 or 0.2?' responds '0.2') • rounds decimals to the nearest whole number in order to estimate answers (e.g. estimates the length of material needed by rounding up the measurement to the nearest whole number)

Number and place value	
NPV8	**Numeral recognition and identification** • identifies, reads and writes decimal numbers applying knowledge of the place value periods of tenths, hundredths and thousandths and beyond **Place value** • compares the size of decimals including whole numbers and decimals expressed to different number of places (e.g. selects 0.35 as the largest from the set 0.2, 0.125, 0.35; explains that 2 is larger than 1.845) • describes the multiplicative relationship between the adjacent positions in place value for decimals (e.g. understands that 0.2 is 10 times as large as 0.02 and that 100 times 0.005 is 0.5) • compares and orders decimals greater than 1 including those expressed to an unequal number of places (e.g. compares the heights of students in the class that are expressed in metres such as 1.50 m is shorter than 1.52 m; correctly orders 1.4, 1.375 and 2 from largest to smallest) • rounds decimals to one and two decimal places for a purpose
NPV9	**Numeral recognition and identification** • reads, represents and uses negative numbers in computation (e.g. explains that the temperature −10 °C is colder than the temperature −2.5 °C; recognises that negative numbers are less than zero; locates −12 on a number line) **Place value** • identifies that negative numbers are integers that represent both size and direction (e.g. uses a number line to model, position and order negative numbers; uses negative numbers in financial contexts such as to model an overdrawn account) • understands that multiplying and dividing numerals by 10, 100, 1000 changes the positional value of the numeral (e.g. explains that 100 times 0.125 is 12.5 because each digit value in 0.125 is multiplied by 100, so 100 x 0.1 is 10, 100 x 0.02 is 2 and 100 x 0.005 is 0.5) • rounds decimals to a specified number of decimal places for a purpose (e.g. the mean distance thrown in a school javelin competition was rounded to two decimal places; if the percentage profit was calculated as 12.467921% the student rounds the calculation to 12.5%)
NPV10	**Numeral recognition and identification** • identifies, reads and interprets very large numbers and very small numbers (e.g. reads that the world population is estimated to be seven billion and interprets this to mean 7 000 000 000 or 7×10^9; interprets the approximate mass of protons and neutrons as 1.67×10^{-24} g)

Number and place value	
	Place value • compares and orders very large numbers and very small numbers (e.g. understands the relative size of very large time scales such as a millennium) • relates place value parts to indices (e.g. 1000 is 100 times larger than 10, and that is why $10 \times 10^2 = 10^3$ and why 10^3 divided by 10 is equal to 10^2) • expresses numbers in scientific notation (e.g. when calculating the distance of the earth from the sun uses 1.5×10^8 as an approximation)

Counting processes

Counting processes form the basis for developing number sense, place value relationships, additive and multiplicative thinking.

This sub-element describes how a student becomes increasingly able to count both verbally, through the stable order of a counting sequence, and perceptually through counting collections. It is important that students connect the last number spoken in a counting sequence to the total quantity for that collection, developing cardinality.

Knowing number names and learning the sequential order by rote, through the use of nursery rhymes, songs or children’s literature, is useful in the early development of counting processes. However, students need to develop an understanding that counting is used to determine ‘how many’ in a collection regardless of the order, appearance or arrangement of items in the collection. To count a collection accurately, they must learn to assign each successive counting number to an item in the collection and ensure that every item is counted once and only once. As students make the link between counting ‘how many’ and the quantities represented by numbers, they begin to understand cardinality and the purpose of counting.

As students progress from learning to count using one-to-one correspondence they develop counting processes they can then apply beyond the tangible. Learning the counting processes is central to the development of Number sense and algebraic thinking.

This sub-element forms the prior learning to support the sub-elements *Additive strategies* and *Multiplicative strategies*. The strategies themselves form an integral part of numeracy and are foundational to the sub-element *Understanding units of measurement* and the element of *Statistics and probability*.

Some students will demonstrate the skills of *Counting processes* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	Indicators

Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is CPr.

The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.

Counting processes	
CPr1	**Counting sequences** • identifies number words when reciting counting rhymes or when asked to count (e.g. holds up three fingers to represent three little ducks) **Pre-counting** • substitutes small collections of objects, typically up to three items
CPr2	**Counting sequences** • counts in stable counting order from one within a known number range (e.g. engages with counting in nursery rhymes, songs and children's literature) **Perceptual counting** • conceptually subitises a collection up to 5 (e.g. recognises a collection of five items as a result of perceptually subitising smaller parts such as 3 and 2) • counts a small number of items typically less than 4 • engages in basic counting during play-based activities such as cooking or shopping (e.g. places 3 bananas in a shopping basket one at a time and says '1, 2, 3')
CPr3	**Counting sequences** • counts forward by one using the full counting sequence to determine the number before or after a given number, within the range of 1–10 (e.g. when asked what number comes after 6, student needs to count from 1 in sequence up to 7 then says 'it's 7'; when asked what number comes before 6 student needs to count from 1, 1-2-3-4-5-6 and responds 'its 5') **Perceptual counting** • matches the count to objects, using one-to-one correspondence (e.g. counts visible or orderly items by ones; may use objects, tally marks, bead strings, sounds or fingers to count; identifies that two sirens means it is lunchtime) • determines that the last number said in a count names the quantity or total of that collection (e.g. when asked 'how many' after they have counted the collection, repeats the last number in the count and indicates that it refers to the number of items in the collection)
CPr4	**Counting sequences** • uses knowledge of the counting sequence to determine the next number or previous number from a number in the range 1–10 (e.g. when asked what number comes directly after 8, students immediately respond 'nine' without needing to count from one) • continues a count starting from a number other than 1

Counting processes	
	Perceptual counting • interprets the count independently of the type of objects being counted (e.g. a quantity of five counters is the same quantity as five basketball courts) • counts a collection, keeping track of items that have been counted and those that haven't been counted yet to ensure they are only counted exactly once (e.g. when asked to count a pile of blocks, they move each block to the side as it is counted)
CPr5	**Counting sequences** • uses knowledge of the counting sequence to determine the next number or previous number from any starting point within the range 1–100 **Perceptual counting** • matches known numerals to collections of up to 20, counting items using a one-to-one correspondence • uses zero to denote when no objects are present (e.g. when asked 'how many cards have you got?' and has no cards left, says 'zero') • counts objects in a collection independent of the order, appearance or arrangement (e.g. understands that counting seven people in a row from left to right, is the same as counting them from right to left)
CPr6	**Counting sequences** • continues counting from any number forwards and backwards beyond 100 using knowledge of place value • counts in sequence by twos and fives starting at zero (e.g. counts items using number rhymes '2, 4, 6, 8 Mary's at the cottage gate …'; skip counts in fives as '5, 10, 15, 20') • counts in sequence forwards and backwards by tens on the decade up to 100 **Perceptual counting** • counts items in groups of twos, fives and tens (e.g. e.g. counts a quantity of 10-cent pieces as 10, 20, 30 … to give the total value of the coins; counts the number of students by twos when lined up in pairs)
CPr7	**Counting sequences** • counts in sequence forwards and backwards by tens or fives off the decade to 100 (e.g. 2, 12, 22 … or 8, 13, 18, 23) **Perceptual counting** • counts large quantities in groups or multiples (e.g. grouping items into piles of ten, then counts the piles, adding on the residual to quantify the whole collection) • estimates the number of items to count to assist with determining group sizes (e.g. decides that counting in twos would not be the most efficient

Counting processes	
	counting strategy based on a quick estimate of the quantity and decides instead to use groups of ten)
CPr8	**Counting sequences** • counts forwards and backwards from any rational number • applies counting processes flexibly to count in rational numbers (e.g. counts in thirds such as $\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2 …; counts backwards by 0.3 starting from four 4, 3.7, 3.4, 3.1 …) • counts backwards from zero understanding that the count can be extended in the negative direction (e.g. 0, -1, -2, -3, -4) **Abstract counting** • applies counting processes to any collection beyond the tangible (e.g. systematically counts the number of possible outcomes of an event; applies a frequency count)

Additive strategies

This sub-element describes how a student becomes increasingly able to think additively, represent a wide range of additive situations and choose and use additive computational strategies for different purposes.

The transition from counting by one to more flexible methods of dealing with the nature of change to a quantity, where numbers are treated as the sum of their parts, is essential for students to become fluent users of number. Rather than only focusing on the speed of producing correct answers, an emphasis on attending to the relation of given numbers to sums and differences is needed for flexibility.

Additive strategies apply equally to subtraction, and the ability to understand the relationship between addition and subtraction is essential as students progress towards generalised arithmetic and algebraic understandings. As students learn to recognise, represent and solve problems involving additive situations, they learn to choose and use additive strategies appropriate to the situation.

Students learn to recognise real world situations that can be represented and solved additively. Initially they may use objects, drawing and diagrams to represent additive situations. Later they learn to represent additive problems with numbers and symbols and to efficiently apply various strategies appropriate to the additive situation.

Rounding and estimation can be used to determine whether solutions to addition or subtraction computations are reasonable.

Some students will demonstrate the skills of *Additive strategies* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	**Indicators**
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is AdS. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
AdS1	**Emergent strategies** • describes the effects of 'adding to' and 'taking away' from a collection of objects • combines two groups of objects and attempts to determine the total
	Perceptual strategies • represents additive situations involving a small number of items with objects, drawings and diagrams

Additive strategies	
AdS2	• counts all items to determine the total of two groups (e.g. when told 'I have three red bottle tops in this pile and two blue bottle tops in this pile how many do I have all together?' student counts each bottle top 'one, two, three' then 'four, five' responding 'five') • counts or changes a quantity by adding to or taking from a quantity using concrete materials or fingers • combines two or more objects to form collections up to 10
AdS3	**Figurative (imagined units)** • solves additive tasks involving two concealed collections of items by visualising the numbers, then counts from one to determine the total (e.g. student can construct a mental image of five and of three but when asked to combine to give a total, will count from one and may use head gestures to keep track of the count)
AdS4	**Counting on (by ones)** • uses a range of counting strategies to solve addition problems such as counting-up-to and counting-up-from (e.g. to solve 'I have seven apples, I want ten. How many more do I need?' counts the number of apples needed to increase the quantity from seven to ten; uses a counting on strategy to calculate 6 + 3, says '6, 7, 8, 9 it's 9'; to solve 6 + ? = 9, says '6 ... 7, 8, 9 it's 3') • uses the additive property of zero, that a number will not change in value when zero is added to or taken away from it (e.g. when asked what is 5 + 0 the student responds 'five')
AdS5	**Counting back (by ones)** • uses a range of counting strategies to solve subtraction problems such as counting-down-from, counting-down-to (e.g. to solve 'Mia had ten cupcakes. She gave three cupcakes away, how many cupcakes does Mia have left?' she counts back from ten, '9, 8, 7 I have 7 left'; to solve 9 take away something equals 6, responds 9 ... 8, 7, 6 ... It's 3)
AdS6	**Flexible strategies with combinations to 10** • describes subtraction as the difference between numbers rather than taking away using diagrams and a range of representations (e.g. using a number line to model 8 – 3 as the difference between 8 and 3) • uses a range of strategies to add or subtract two or more numbers within the range of 1-20 (e.g. bridging to 10; near doubles; adding the same to both numbers 7 + 8 = 15 because double 8 is 16 and 7 is one less than 8; 8 + 6 = 14 because 8 + 2 = 10 and 4 more is 14; 15 – 8 = 7 because I can add 2 to both to give 17 – 10 = 7) • uses knowledge of part-part-whole number construction to partition a whole number into parts to solve addition problems (e.g. to solve 6 + ? = 13, says '6 plus 4 makes 10, and 3 more … so it's 7')

Additive strategies	
	• represents additive situations using number sentences and part-part-whole diagrams including when different parts or the whole are unknown (e.g. uses the number sentence 8 – 3 = 5 to represent the problem 'I had 8 pencils. I gave 3 to Max. I now have 5 remaining'; matches the number sentence 4 + ? = 9 to the problem, 'I have 9 cups and only 4 saucers, how many more saucers do I need?')
AdS7	**Flexible strategies with two-digit numbers** • chooses from a range of known strategies to solve additive problems involving two-digit numbers (e.g. uses place value knowledge, known facts and part-part-whole number knowledge to solve problems like 24 + 8 + 13, partitioning 8 as 6 and 2 more, then combining 24 and 6 to rename it as 30 before combining it with 13 to make 43, and then combining the remaining 2 to find 45 …; adding the same to both numbers 47 – 38 = 49 - 40) • identifies that the same combinations and partitions to ten are repeated within each decade (e.g. knowing that 8 + 2 = 10, they know 18 + 2 = 20 and 28 + 2 = 30 etc.) • identifies addition as associative and commutative but subtraction is neither • applies the commutative and associative properties of addition to simplify mental computation (e.g. to calculate 23 + 9 + 7 adds 23 to 7 to get 30, then adds 9 to give 39) • applies inverse relationship of addition and subtraction to solve problems and uses the inverse relationship to justify an answer (e.g. when solving 23 – 16 chooses to use addition 16 + ? = 23) • represents a wide range of additive problem situations involving two-digit numbers using appropriate addition and subtraction number sentences
AdS8	**Flexible strategies with three-digit numbers and beyond** • uses place value, standard and non-standard partitioning, trading or exchanging of units to mentally add and subtract numbers with three or more digits (e.g. to add 250 and 457, partitions 250 into 2 hundreds and 5 tens, says 457 plus 2 hundreds is 657, plus 5 tens is 707; to add 184 and 270 partitions into 150 + 34 + 250 + 20 = 400 + 34+ 20 = 454) • chooses and uses strategies including algorithms and technology to efficiently solve additive problems • uses estimation to determine the reasonableness of the solution to an additive problem (e.g. when asked to add 249 and 437 says '250 + 440 is 690') • represents a wide range of familiar real-world additive situations involving large numbers as standard number sentences explaining their reasoning
	Flexible strategies with fractions and decimals • uses knowledge of place value and how to partition numbers in different ways to make the calculation easier to add and subtract decimals with up to three decimal places

Additive strategies	
AdS9	• identifies and justifies the need for a common denominator when solving additive problems involving fractions with related denominators • represents a wide range of familiar real-world additive situations involving decimals and common fractions as standard number sentence, explaining their reasoning
AdS10	**Flexible strategies with rational numbers** • uses knowledge of equivalent fractions, multiplicative thinking and how to partition fractional numbers to make calculations easier when adding and subtracting fractions with different denominators • solves additive problems involving the addition and subtraction of rational numbers including fractions with unrelated denominators and integers • chooses and uses appropriate strategies to solve multiple-step problems involving rational numbers

Multiplicative strategies

This sub-element describes how a student becomes increasingly able to think multiplicatively and use multiplicative strategies in computation to solve problems related to a range of multiplicative situations. The coordination of units multiplicatively involves using the values of one unit applied to each of the units of the other, the multiplier. This process of coordinating units is equally relevant to problems of division.

As students move through their development of number sense, they develop counting principles and additive strategies that may lead into thinking and working multiplicatively. Students initially encounter multiples in counting rhymes, patterns and later in skip counting. They investigate multiples through the use of concrete materials and more abstract representations such as number lines, arrays and hundreds charts.

Students are introduced to division through equal sharing and equal grouping situations. In sharing collections of objects equally among their peers and creating equal groups, students begin to develop a sense of equivalence and fairness and understand that the division symbol can represent a quantity divided into equal groups. As students progress to gaining a better understanding of the concept of division, they also begin to explore the relationship between division and multiplication using arrays and area models.

As students develop an understanding of multiplication and division and can think multiplicatively, they also develop fluency with the operations of multiplication and division, through the learning of basic multiplication and related division facts. The gradual introduction to symbology through number sentences, algorithms and the use of technology, allows students to further strengthen their capacity to represent and solve multiplicative problems efficiently, extending to include very large and very small numbers, rational numbers and variables.

Multiplicative strategies are used in the sub-elements *Proportional thinking* and *Interpreting fractions* and the elements of *Measurement and geometry* and *Statistics and probability*.

Some students will demonstrate the skills of *Multiplicative strategies* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	Indicators

Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is MuS.

The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.

Multiplicative strategies	
MuS1	**Forming equal groups** • shares collections equally by dealing (e.g. distributing all items one-to-one until they are exhausted, checking that the final groups are equal) • makes equal groups and counts by ones to determine the total
MuS2	**Perceptual multiples** • uses groups or multiples in counting and sharing concrete objects (e.g. skip counting by twos, fives or tens with all objects visible) • represents authentic situations involving equal sharing and equal grouping with drawings and objects (e.g. draws a picture to represent 4 tables that seat 6 people to determine how many chairs they will need; uses eight counters to represent sharing $8 between four friends)
MuS3	**Figurative (imagined units)** • uses perceptual markers to represent concealed quantities of equal amounts to determine the total number of items (e.g. to count how many whiteboard markers in four packs, knowing they come in packs of 5, the student counts the number of markers as 5, 10, 15, 20)
MuS4	**Repeated abstract composite units** • uses composite units in repeated addition using the unit a specified number of times (e.g. interprets 'four lots of three' additively and calculates 3 + 3 + 3 + 3 answering '12') • uses composite units in repeated subtraction using the unit a specified number of times (e.g. when asked 'how many groups of four can be formed from our class of 24?', the student repeatedly takes away four from 24 and counts the number of times this can be done. Says '20, 16, 12, 8, 4 and 0 so we can form six groups of four')
MuS5	**Coordinating composite units** • identifies and represents multiplication in various ways and solves simple multiplicative problems using these representations (e.g. modelling as equal groups, arrays or regions) • identifies and represents division in various ways such as sharing division or grouping division (e.g. sharing a carton of 12 eggs equally between four people, draws 12 dots and circles three groups of four with 3 in each share) • identifies and represents multiplication and division abstractly using the symbols × and ÷ (e.g. represents 3 groups of 4 as 3 × 4; uses 9 ÷ 3 to represent 9 pieces of fruit being equally shared by 3 people)
MuS6	**Flexible strategies for single digit multiplication and division** • draws on the structure of multiplication to use known multiples in calculating related multiples (e.g. uses multiples of 4 to calculate multiples of 8) • interprets a range of multiplicative situations using the context of the problem to form a number sentence (e.g. to calculate the total number of buttons in 2 containers, each with 5 buttons, uses the number sentence 2 ×

Multiplicative strategies	
	5 = ?; if a packet of 20 pens is to be shared equally between 4, writes 20 ÷ 4 = ?) • demonstrates flexibility in the use of single-digit multiplication facts (e.g. 7 boxes of 6 donuts is 42 donuts altogether because 7 × 6 = 42; multiplying any factor by one will always give a product of that factor i.e.: 1 × 6 = 6; if you multiply any number by zero the result will always be zero) • uses the commutative and distributive properties of multiplication to aid computation when solving problems (e.g. 5 × 6 is the same as 6 × 5; calculates 7 × 4 by adding 5 × 4 and 2 × 4) • applies mental strategies for multiplication to division and can justify their use (e.g. to divide 64 by 4, halves 64 then halves 32 to get an answer of 16) • explains the idea of a remainder as what is 'left over' from the division (e.g. an incomplete group, lot of, next row or multiple)
MuS7	**Flexible strategies for multiplication and division** • uses multiplication and division as inverse operations to solve problems or to justify a solution • uses known mental and written strategies such as using the distributive property, decomposition into place value or factors to solve multiplicative problems involving numbers with up to three digits and can justify their use (e.g. 7 × 83 equals 7 × 80 plus 7 × 3; to multiply a number by 72, first multiply by 12 and then multiply the result by 6; 327 × 14 is equal to 4 × 327 plus 10 × 327) • uses estimation and rounding to check the reasonableness of products and quotients (e.g. multiplies 200 by 30 to determine if 6138 is a reasonable answer to 198 × 31)
MuS8	**Flexible strategies for multi-digit multiplication and division** • solves multi-step problems involving multiplicative situations using appropriate mental strategies, technology and algorithms • interprets, represents and solves multifaceted problems involving all four operations with whole numbers
MuS9	**Flexible strategies for multiplication and division of rational numbers** • expresses a number as a product of its prime factors for a purpose • expresses repeated factors of the same number in index form (e.g. 2 × 2 × 2 × 3 × 3 = $2^3 \times 3^2$) • identifies and describes products of the same number as square or cube numbers (e.g. 3 × 3 is the same as 3^2 which is read as three squared) • describes the effect of multiplication by a decimal or fraction less than one (e.g. when multiplying whole numbers by a fraction or decimal less than 1 such as 15 × $\frac{1}{2}$ = 7.5)

Multiplicative strategies	
	• connects and converts decimals to fractions to assist in mental computation involving multiplication or division (e.g. to calculate 16 × 0.25, recognises 0.25 as a quarter, and determines a quarter of 16 or determines 0.5 ÷ 0.25, by reading this as one half, how many quarters and giving the answer as 2) • calculates the percentage of a quantity flexibly using multiplication and division (e.g. to calculate 13% of 1600 uses 0.13 × 1600 or 1600 ÷ 100 × 13) • uses multiplicative strategies efficiently to solve problems involving rational numbers including integers (e.g. calculates the average temperature for Mt Wellington for July to be -1.6 ˚C)
MuS10	**Flexible strategies for working multiplicatively** • uses knowledge of place value and multiplicative partitioning to multiply and divide decimals efficiently (e.g. 0.461 × 200 = 0.461 × 100 × 2 = 46.1 × 2 = 92.2) • flexibly operates multiplicatively with extremely large or very small numbers expressed in scientific notation (e.g. calculates the area of a computer chip measuring 2.56×10^{-6} m in width by 1.4×10^{-7} m in length) • chooses and uses appropriate strategies to solve multiple-step problems involving rational numbers • represents and solves multifaceted problems in a wide range of multiplicative situations including those involving very small or very large numbers (e.g. chooses to calculate the percentage of a percentage to determine successive discounts; determining the time it takes for sunlight to reach the earth)

Interpreting fractions

This sub-element emphasises the development of fraction sense, which is a crucial stage in learning how to reason proportionally. In developing fraction sense, students become increasingly able to recognise the part-whole description of a fraction, but also recognise and use fractions as numbers, measures, operators, ratios and as a division.

This sub-element describes how a student becomes increasingly able to use fractions as numbers that describe a relationship between two abstract measures of quantity. Rather than representing two numbers, the fraction $\frac{a}{b}$ represents the result of dividing one by the other. That is, $\frac{2}{3}$ is the 'number' that results from dividing 2 by 3.

Students learn to represent a fraction as a mathematical relationship between two quantities such as discrete countable objects or continuous measurable quantities. Representing fractions using number lines, area diagrams, concrete objects and arrays, enhances a students' ability to visually grasp concepts associated with interpreting fractions. They learn that equivalent fractions are proportional, and fractions can be used to comparatively describe proportional relationships like the number of yes responses were $\frac{2}{3}$ the number of no responses.

Students also learn to distinguish between fractions of different wholes and apply this knowledge when working with fractions operationally.

This sub-element is used in the sub-elements *Multiplicative strategies, Proportional thinking* and *Understanding chance*.

Some students will demonstrate the skills of *Interpreting fractions* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	Indicators
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is InF. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
InF1	**Creating halves** • demonstrates that dividing a whole into two parts can create equal or unequal parts • identifies the part and the whole in representations of one-half (e.g. joins two equal pieces back together to form the whole shape and can identify the pieces as equal parts of the whole shape)

Interpreting fractions	
	• creates equal halves using all of the whole (e.g. folds a paper strip in half to make equal pieces by aligning the edges; cuts a sandwich in half diagonally; partitions a collection into two equal groups to represent halving)
InF2	**Repeated halving** • makes quarters and eighths by repeated halving (e.g. locates halfway then halves each half; eight counters halved and then halved again into four groups of two) • identifies the part and the whole in representations of halves, quarters and eighths (e.g. identifies the fractional parts that make up the whole using fraction puzzles) • represents known fractions using various models (e.g. discrete collections, continuous linear and continuous area)
InF3	**Repeating fractional parts** • accumulates fractional parts (e.g. knows that two-quarters is inclusive of one-quarter and twice one-quarter, not just the second quarter) • checks the equality of parts by iterating one part to form the whole (e.g. when given a representation of one-quarter of a length and asked, 'what fraction is this of the whole length?', uses the length as a counting unit to make the whole) • identifies fractions in measurement situations and solves problems using halves, quarters and eighths (e.g. quarters in an AFL match; uses two $\frac{1}{2}$-cup measures in place of a 1-cup measure) • demonstrates that fractions can be written symbolically and interprets using part-whole knowledge (e.g. interprets $\frac{3}{4}$ to mean three one-quarters or three lots of $\frac{1}{4}$)
InF4	**Re-imagining the whole** • creates thirds by visualising or approximating and adjusting (e.g. imagines a strip of paper in 3 parts, then adjusts and folds) • identifies examples and non-examples of partitioned representations of fractions • divides a whole into different fractional parts for different purposes (e.g. exploring the problem of sharing a cake equally between different numbers of guests) • demonstrates that the more parts into which a whole is divided, the smaller the parts become
InF5	**Equivalence of fractions** • identifies the need to have equal wholes to compare fractional parts (e.g. compares the pieces of pizza when two identical pizzas are cut into six and eight and describes how one-sixth is larger than one-eighth)

Interpreting fractions	
	• creates fractions larger than 1 by recreating the whole (e.g. when creating four-thirds, demonstrates that three-thirds corresponds to the whole and the fourth third is part of an additional whole) • creates equivalent fractions by dividing the same-sized whole into different parts (e.g. shows two-sixths is the same as one-third of the same whole; creates a fraction wall) • uses partitioning to establish relationships between fractions (e.g. creates one-sixth as one-third of one-half)
InF6	**Fractions as numbers** • connects the concepts of fractions and division: a fraction is a quotient, or a division statement (e.g. two-sixths is the same as 2 ÷ 6 or 2 partitioned into 6 equal parts or to solve ‘Two chocolate bars shared among three people’ understands that it is 2 divided by 3, therefore they each get two-thirds of a chocolate bar) • justifies where to place fractions on a number line (e.g. to show two-thirds on a number line divides the space between 0 and 1 into three equal parts and indicates the correct location) • explains the equivalence of decimals to benchmark fractions (e.g. $\frac{1}{4}$ = 0.25, $\frac{1}{2}$ = 0.5, $\frac{3}{4}$ = 0.75, $\frac{1}{10}$ = 0.1, $\frac{1}{100}$ = 0.01)
InF7	**Comparing fractions** • understands the equivalence relationship between a fraction, decimal and percentage as different representations of the same quantity (e.g. $\frac{1}{2}$ = 0.5 = 50% because five is half of ten and fifty is half of 100) • identifies a fraction as a rational number that has relative size • reasons and uses knowledge of equivalence to compare and order fractions of the same whole (e.g. compares two-thirds and three-quarters of the same collection or whole, by converting them into equivalent fractions of eight-twelfths and nine-twelfths; explains that three-fifths must be greater than four-ninths because three-fifths is greater than a half and four-ninths is less than a half)
InF8	**Operating with fractions** • adds or subtracts fractions with the same denominators and justifies the need for a common denominator • uses strategies to calculate a fraction of a quantity (e.g. to calculate two-thirds of 27, determines one-third then doubles; to find three-eighths of 60, knows a quarter is equivalent to two-eighths and so finds a quarter by halving and halving again to get 15, halves to give 7.5 to find an eighth then adds 15 and 7.5 to accumulate three-eighths of 60 as 22.5)

Interpreting fractions	
	• explains the difference between multiplying and dividing fractions (e.g. recognises $\frac{1}{2} \times \frac{1}{4}$ as one-half of a quarter and $\frac{1}{2} \div \frac{1}{4}$ as how many quarters are in one half) • expresses one quantity as a fraction of another (e.g. 140 defective items from the 1120 that were produced represents one-eighth of all items produced) • demonstrates why dividing by a fraction can result in a larger number
InF9	**Operating with fractions proportionally** • demonstrates that a fraction can also be used as a ratio to compare the size of two sets (e.g. if the colour ratio of a black and white pattern is 2:3, $\frac{2}{5}$ is black and $\frac{3}{5}$ is white and the representation of black is $\frac{2}{3}$ of the white)

Proportional thinking

This sub-element addresses the proportional relationships between quantities, building from the sub-elements of Number and place value, Multiplicative strategies and Interpreting fractions. The ability to reason proportionally requires students to think multiplicatively. This sub-element includes understanding and working with percentages as well as comparing units in ratios, rates and proportions.

It begins with understanding the concept of a percentage. Percentages, as with fractions, represent a proportional relationship between quantities. They can be used as a number, an operator, a measurement and a ratio.

A ratio describes a situation in comparative terms, and a proportion is taken to mean when this comparison is used to describe a related situation in the same comparative terms. For example, if the ratio of trees to shrubs in a garden is 2 to 3, the comparison is the number of trees to the number of shrubs. Knowing that there are 30 plants in the garden, proportionally, the number of trees is 12 and the number of shrubs is 18. Applying the base comparison to the whole situation uses proportional reasoning. Proportional reasoning is knowing the multiplicative relationship between the base ratio and the proportional situation to which it is applied.

Learning to reason using proportion is a complex process that develops over an extended period. Proportional reasoning also includes numerical comparison tasks involving a comparison of different rates, for example, if one dog grows from 5 kilograms to 8 kilograms and another dog grows from 3 kilograms to 6 kilograms, which dog grew more compared to its original weight?

This sub-element applies to the sub-elements of *Number and place value*, *Understanding money, Understanding units of measurement, Understanding geometric properties, Interpreting fractions, Understanding chance* and *Interpreting and representing data.*

Some students will demonstrate the skills of *Proportional thinking* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	Indicators

Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is PrT.

The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.

Proportional thinking	
PrT1	**Understanding percentages and relative size** • explains that a percentage is a proportional relationship between a quantity and 100 (e.g. 25% means 25 for every one hundred) • demonstrates that 100% is a complete whole (e.g. student explains that in order to get 100% on a quiz, you must answer every question correctly) • uses percentage to describe, represent and compare relative size (e.g. selects which beaker is 75% full, describes an object as 50% of another object) • uses the complement of a percentage to 100% to determine an amount (e.g. if 10% of the jellybeans in a jar are black then 90% are not black)
PrT2	**Determines a percentage as a part of a whole** • explains and fluently uses interchangeably the equivalence relationship between a fraction, decimal and percentage (e.g. $\frac{1}{2}$ = 0.5 = 50%; explains that at quarter time, 75% of the game is left to play) • uses key percentages and their equivalences to determine the percentage of a quantity (e.g. to solve 75% of 160, I know that 50% [half] of 160 is 80, and 25% [quarter] is 40 so 75% is 120) • calculates a percentage of an amount (e.g. interprets that a 25% discount on an $80 purchase means 25% × $80 and determines $20 is a quarter of $80) • expresses one quantity as a percentage of another (e.g. determines what percentage 7 is of 35) • uses the complement of the percentage to calculate the amount after a percentage discount (e.g. to calculate how much to pay after a 20% discount, calculates 80% of the original cost)
PrT3	**Identifies ratios as a part-to-part comparison** • represents and models ratios using diagrams or objects (e.g. in a ratio 1:4 of red to blue counters, for each red counter there are four blue counters) • interprets ratios as a comparison between two like quantities (e.g. ratio of students to teachers in a school is 20:1) • interprets a rate as a comparison between two different types of quantities (e.g. water flow can be measured at a rate of five litres per second) • expresses a ratio as equivalent fractions or percentages (e.g. the ratio of rainy days to fine days in Albany is 1:2 and so $\frac{1}{3}$ of the days are rainy; in a ratio of 1:1 each part represents one $\frac{1}{2}$ or 50% of the whole)
PrT4	**Using ratios and rates** • uses a ratio to increase or decrease quantities to maintain a given proportion (e.g. uses a scale ratio to determine distance on a map)

Proportional thinking	
	• uses rates to determine how quantities change (e.g. when travelling at a constant speed of 60 km/h how far would you have travelled in 30 minutes?)
PrT5	**Proportionality and the whole** • determines the whole given a percentage (e.g. given 20% is 13 mL, determines the whole is 65 mL) • identifies the common unit rate to compare rates expressed in different units (e.g. calculating the best buys; comparing the relative speed of two vehicles) • identifies, compares, represents and solves problems involving different rates in real world contexts • determines the equivalence between two rates or ratios by expressing them in their simplest form • describes how the proportion is preserved when using a ratio (e.g. uses the ratio 1:4:15 for the composition of silver, copper and gold to determine the mass of copper in a rose gold ring that weighs 8 grams)
PrT6	**Applying proportion** • increases and decreases quantities by a percentage (e.g. to determine percentage increases and percentage discounts) • uses common fractions and decimals for proportional increase or decrease of a given amount • expresses a percentage increase using a multiplier (e.g. adding 3% is the same as multiplying by 1.03) • uses percentages to calculate interest payable on loans • identifies and interprets situations where direct proportion is used (e.g. hours worked and payment received) • identifies and interprets situations where indirect proportion is used (e.g. speed and distance travelled; recognises π as the proportional relationship between the circumference of a circle and its diameter) • uses ratio and scale factors to enlarge or reduce the size of objects (e.g. interprets the scale used on a map and determines the real distance between two locations)
PrT7	**Flexible proportional thinking** • identifies proportional relationships in formulas and uses proportional thinking flexibly to explore this relationship (e.g. recognises the proportional relationship between concentration and volume of a solution in the formula c = $\frac{n}{v}$ and uses this relationship to make decisions when diluting solutions) • identifies, represents and chooses appropriate strategies to solve percentage problems involving proportional thinking (e.g. percentage of a percentage for calculating successive discounts; uses percentages to

Proportional thinking	
	calculate compound interest on loans and investments; uses percentage increases or decreases as an operator, such as a 3% increase is achieved by multiplying 1.03, and 4 successive increases is multiplying by 1.03^4 to make meaning of the formula)

Number patterns and algebraic thinking

Figuring out how a pattern works brings predictability and allows the making of generalisations. This sub-element describes how a student becomes increasingly able to identify a pattern in the environment, to being able to recognise, represent, describe and use generalisable patterns in everyday contexts.

Our innate ability to perceptually subitise small quantities can be increased conceptually to partition and quantify larger quantities through using pattern recognition. As students become increasingly able to connect patterns with the structure of numbers, they create a foundation for algebraic thinking, that is, thinking about generalised quantities. For example, number patterns are evident in house numbers on opposite sides of streets or the seating plan in a plane. Algebra enables the 'generalisation' of patterns so that students can apply them from one situation to another.

Algebraic thinking is also used to capture the relationship between quantities. These relationships can be categorised into general groupings based on their behaviours such as linear and non-linear relationships. Recognising whether a relationship is growing linearly or exponentially can provide valuable information as to its future behaviour and has multiple applications, such as monitoring the spread of disease, financial decision making, project management and data analytics.

Some students will demonstrate the skills of *Number patterns and algebraic thinking* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	Indicators
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is NPA. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
NPA1	**Recognises patterns** • identifies and describes patterns in everyday contexts (e.g. brick pattern in a wall or the colour sequence of a traffic light) • identifies 'same' and 'different' in comparisons • copies simple patterns using shapes and objects • identifies numbers in standard pattern configurations without needing to count individual items (e.g. numbers represented on dominos or a standard dice)

Number patterns and algebraic thinking	
NPA2	**Identifying and creating patterns** • identifies the pattern unit with a simple repeating pattern (e.g. continues the repeating pattern red, blue, red, blue with red then blue) • creates repeating patterns involving the repetition of a pattern unit with shapes, movements, objects and numbers (e.g. circle, square, circle, square; stamp, clap, stamp, clap; 1,2,3 1,2,3 1,2,3) • continues a pattern involving shapes or objects • determines a missing element within a pattern involving shapes or objects • conceptually subitises by identifying patterns in standard representations (e.g. patterns within ten frames, using finger patterns to represent a quantity)
NPA3	**Continuing and generalising patterns** • represents growing patterns where the difference between each successive term is constant using concrete materials, then summarising the pattern numerically (e.g. constructs a pattern using concrete materials such as toothpicks, then summarises the number of toothpicks used as 4, 7, 10, 13 ...) • describes rules for continuing growing patterns where the difference between each successive term is the same (e.g. to determine the next number in the pattern 3, 6, 9, 12 … you add 3; for 20, 15, 10 … the rule is described as each term is generated by subtracting five from the previous term) **Relational thinking** • uses the equals sign to represent 'is equivalent to' or 'is the same as' in numerical sentences (e.g. when asked to write an expression that is equivalent to 5 + 3 the student responds 6 + 2 and then writes 5 + 3 = 6 + 2) • solves number sentences involving unknowns using the inverse relationship between addition and subtraction (e.g. 3 + ? = 5 and knowing 5 – 3 = 2 then ? must be 2)
NPA4	**Generalising patterns** • represents growing patterns where each successive term is determined by multiplying the previous term by a constant, using concrete materials, then summarises the pattern numerically (e.g. constructs a pattern using concrete materials such as tiles then summarises the pattern as 2, 6, 18, 54 ...) • describes rules for continuing patterns where each successive term is found by multiplying or dividing the previous term by the same factor (e.g. to determine the next term in the pattern 1, 3, 9, 27 … multiply by 3) **Relational thinking** • uses relational thinking to determine the missing values in a number sentence (e.g. 6 + ? = 7 + 4)

Number patterns and algebraic thinking	
	• uses equivalent number sentences involving addition or subtraction to calculate efficiently or to find an unknown (e.g. 527 + 96 = ? is the same as 527 + 100 – 4 = ? ; If 6 + ? = 8 + 3, then as I know 8 = 6 + 2, I can write 8 + 3 as 6 + 2 + 3, which is the same as 6 + 5 therefore '?' is 5) • solves number sentences involving unknowns using the inverse relationship between multiplication and division (e.g. to determine the missing number in 2 × ? = 10 knowing 10 ÷ 2 is equal to five then ? must be five)
NPA5	**Generalising patterns** • creates and interprets tables used to summarise patterns (e.g. the cost of hiring a bike based on the cost per hour) • identifies a single operation rule in numerical patterns and records it in words (e.g. European dress size = Australian dress size + 30) • relates the position number of shapes within a pattern to the rule for the sequence (e.g. number of counters = shape number + 2) • predicts a higher term of a pattern using the pattern's rule • extends number patterns to include rational (e.g. 2, $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, 3 …; 2, -4, 8, -16 …; 10, 9.8, 9.6, 9.4 …) **Relational thinking** • balances number sentences involving one or more operations following conventions of order of operations (e.g. 5 x 2 + 4 = 4 x 2 + ?; 6 +? x 4 = 9 x 2) • identifies and uses equivalence in number sentences to solve multiplicative problems (e.g. uses a number balance or other materials to model the number sentence 6 x 4 = 12 x ? in order to solve a problem)
NPA6	**Representing unknowns** • creates algebraic expressions from word problems involving one or more operations (e.g. when n = number of egg cartons, then the number of eggs can be represented by the expression 12n) • uses words or symbols to express relationships involving unknown values (e.g. number of apples packed = 48 x number of boxes packed; C = 20 + 10h) • evaluates an algebraic expression or equation by substitution (e.g. uses the formula for force '*F*', *F* = *ma* to calculate the force given the mass 'm' and the acceleration 'a')
NPA7	**Algebraic expressions** • creates and identifies algebraic equations from word problems involving one or more operations (e.g. if a taxi charges $5 call out fee then a flat rate of $2.30 per km travelled, represents this algebraically as C = 5 + 2.3*d* where *d* is the distance travelled in km and C is the total cost of the trip) • identifies and justifies equivalent algebraic expressions

Number patterns and algebraic thinking	
	• interprets a table of values in order to plot points on a graph
NPA8	**Algebraic relationships** • interprets and uses formulas and algebraic equations that describe relationships in various contexts (e.g. uses $A = \pi r^2$ to calculate the area of a circular space; uses $A = P(1 + \frac{r}{n})^{nt}$ when working with compound interest; uses $v = u + at$ to calculate the velocity of an object) • plots relationships on a graph using a table of values representing authentic data (e.g. uses data collected in a spread sheet to plot results of a science experiment)
NPA9	**Linear and non-linear relationships** • identifies the difference between linear and non-linear relationships in everyday contexts (e.g. explains that in a linear relationship, the rate of change is constant such as the cost of babysitting by the hour, whereas in a non-linear relationship the rate of change will vary and it could grow multiplicatively or exponentially such as a social media post going viral) • describes and interprets the graphical features of linear and non-linear growth in authentic problems (e.g. in comparing simple and compound interest graphs; uses a line of best fit to describes the relationship between scientific data plotted on a graph)

Understanding money

Financial decisions require the capacity to carry out calculations with money that draws on the numerical skills captured in the element of Number sense and algebra and the measurement skills of understanding the base unit. Students develop a better understanding of money through identifying situations where money is used and applying their knowledge of the value of money, along with their number skills, to situations involving the purchasing, budgeting and justification for the use of money.

Decimal currency uses a metric system with 100 cents equivalent to one dollar. The dollar is the base unit of measurement and a cent is one hundredth of the equivalent value. Due to the withdrawal from circulation of 1-cent and 2-cent coins, monetary amounts such as $3.42 can no longer be represented with coins, however it is still used in digital currency.

Understanding how to use currency draws on both additive and multiplicative strategies. Giving change requires being able to round values and work with multiples of 5, 10, 20 or 50. Applications of *Understanding money* can be found in the calculation of interest (both simple and compound) and in determining 'best buys'. These are described in the *Proportional thinking* sub-element.

This sub-element addresses the financial numeracy skills necessary to access the Australian Curriculum and to support the development of numeracy skills required to become a financially literate member of society.

Some students will demonstrate the skills of *Understanding money* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	Indicators
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is UnM. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
UnM1	**Face value** • identifies situations that involve the use of money • identifies and describes Australian coins based on their face value
UnM2	**Sorting money** • sorts and orders Australian coins based on their face value • sorts and then counts the number of Australian coins with the same face value

Understanding money	
UnM3	**Counting money** • determines the equivalent value of coins sorted into one denomination • counts small collections of coins according to their value • writes the value of a small collection of coins in whole dollars, or whole cents using numbers and the correct dollar sign or cent symbol
UnM4	**Equivalent money** • understands that the Australian monetary system includes both coins and notes and how they are related (e.g. orders money based on its monetary value) • determines the equivalent value of coins to $5 using any combination of 5c, 10c, 20c or 50c coins • represents different values of money in multiple ways
UnM5	**Counting money** • counts a larger collection of coins by making groups (e.g. counts the coins in a money box by sorting the 5c, 10c and 20c pieces into $1 groups) • determines the amount of money in a collection, including both notes and coins, using basic counting principles and the standard form of writing dollars and cents in decimal format, to two decimal places
UnM6	**Working with money additively** • calculates the total cost of several different items in dollars and cents • counts the change required for simple transactions to the nearest five cents • calculates the change, to the nearest five cents, after a purchase using additive strategies • determines the conditions for a profit or a loss on a transaction
UnM7	**Working with money multiplicatively** • calculates the total cost of several identical items in dollars and cents • connects the multiplicative relationship between dollars and cents to decimal notation (e.g. explains that a quarter of dollar is equal to $0.25 or 25 cents; calculates what 150 copies will cost if they are advertised at 15c a print and expresses this in dollars and cents as $22.50) • solves problems, such as repeated purchases, splitting a bill or calculating monthly subscription fees, using multiplicative strategies • makes and uses simple financial plans (e.g. creates a classroom budget for an excursion; planning for a school fete)
UnM8	**Working with money proportionally** • calculates the percentage change [10, 20, 25 and 50%] with and without the use of technology (e.g. using GST as 10% multiplies an amount by 0.1 to

Understanding money	
	calculate the GST payable or divides the total paid by 11 to calculate the amount of GST charged; calculates the cost after a 25% discount on items) • calculates income tax payable using taxation tables • interprets an interest rate from a given percentage and calculates simple interest payable on a short-term loan (e.g. calculates the total interest payable on a car loan)
UnM9	**Working with money proportionally** • determines the 'best buy' using a variety of strategies (e.g. comparing cost per 100 g or comparing the cost of a single item on sale versus a multi-pack at the regular price) • determines the best payment method or payment plan for a variety of contexts using rates, percentages and discounts (e.g. decides which phone plan would be better based on call rates, monthly data usage, insurance and other upfront costs) • calculates the percentage change including the profit or loss made on a transaction (e.g. profit made from on selling second-hand goods through an online retail site)
UnM10	**Working with money proportionally** • calculates compound interest and connects it to repeated applications of simple interest • identifies and evaluates the costs associated with a major purchase (e.g. in purchasing a car calculates the depreciation, ongoing maintenance, insurance and the effect of loan repayments on disposable income)

MEASUREMENT AND GEOMETRY

Understanding units of measurement

This sub-element describes how a student becomes increasingly able to identify attributes that can be measured and the units by which they are measured. They initially use direct comparison to recognise and understand what it means to have more or less of a particular attribute.

In making the transition from informal to formal units, a student attends to the structure of units and the process used to measure certain attributes. While attributes can initially be measured informally, students progress to the more abstract use of formulas to calculate attributes such as perimeter, area, volume and surface area, after they have established a sound understanding of the attributes. The formal units used in measurement are the base and derived units within the metric system. The relationship between units are described using prefixes and the base ten system.

Students are introduced to angles as a measure of turn, through visually connecting concrete examples of angles represented in the real world, to classifications of angles based upon the measurement of the size of the angle such as a right angle. As students progress from initially comparing angles visually, they move to formally measuring the size of an angle and learn to use equipment such as a protractor and units such as degrees.

Estimation is an important skill associated with measurement. As students develop more sophisticated knowledge and skills for measuring attributes, they also develop more sophisticated strategies for estimating measurements. Practising estimation skills assists students in developing a mental frame of reference for size and determining the more appropriate and efficient unit to use.

Experience helps students develop estimates associated with commonly available reference objects for many attributes (e.g. a litre of milk, a cupful in cooking or a kilogram of flour). Developing formal units of measurement is vital in areas as diverse as medicine and trades. The relationship between units of measurement is applied when working with ratios, rates and proportions as well as decimals and percentages.

While time is also an attribute that is measured in units, it is dealt with in the sub-element *Measuring time*.

Some students will demonstrate the skills of *Understanding measurement* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Understanding units of measurement

Level	Indicators
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is UuM. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
UuM1	**Describing the size of objects** • uses gestures and informal language to identify the size of objects (e.g. holds hands apart and says 'it's this big') • uses everyday language to describe attributes in absolute terms that can be measured (e.g. my tower is tall, this box is heavy, it is warm today)
UuM2	**Comparing and ordering objects** • uses direct comparison to compare two objects and indicates whether they are the same or different based on attributes such as length, height, mass or capacity (e.g. compares the length of two objects by aligning the ends; pours sand or water from one container to another to decide which holds more) • uses comparative language to compare two objects (e.g. states which is shorter or longer, lighter or heavier) • orders three or more objects by comparing pairs of objects (e.g. decides where to stand in a line ordered by height by comparing their height to others directly)
UuM3	**Using informal units of measurement** • measures an attribute by choosing and using multiple identical, informal units • selects the appropriate size and dimensions of an informal unit to measure and compare attributes (e.g. chooses a linear unit such as a pencil to measure length, or a square unit such as a tile to measure area) • chooses and uses appropriate uniform informal units to measure length and area without gaps or overlaps (e.g. uses the same sized paper clips to measure the length of a line; uses tiles, rather than counters to measure the area of a sheet of paper because the tiles fit together without gaps) • uses multiple uniform informal units to measure and make direct comparisons between the mass or capacity of objects (e.g. uses a balance scale and a number of same-sized marbles to compare mass; uses a number of cups of water or buckets of sand to measure capacity) • counts the individual uniform units used by ones to compare measurements (e.g. I counted 4 matchsticks across my book and the shelf is 5 matchsticks wide, so I know my book will fit)

Understanding units of measurement	
	Estimating measurements • estimates the total number of uniform informal units needed to measure or compare attributes (e.g. uses a handspan or a finger width; stands an arm length apart) • checks an estimate using informal units to compare to predicted measurement
UuM4	**Repeating a single informal unit to measure** • measures length using a single informal unit repeatedly (e.g. uses one paper clip to measure the length of a line, making the first unit, marking its place, then moving the paper clip along the line and repeating this process) • compares the area of two or more shapes using an informal single unit of measure repeatedly (e.g. using a sheet of paper to measure the area of a desktop) • measures an attribute by counting the number of units used **Estimating measurements** • uses familiar household items as benchmarks when estimating mass and capacity (e.g. compares capacities based on knowing the capacity of a bottle of water) **Describing turns** • describes a turn in both direction and the amount of turn (e.g. a quarter turn to the right, a full turn on the spot)
UuM5	**Using abstract units** • uses the array structure to calculate area measured in square units (e.g. draws and describes the column and row structure to represent area as an array, moving beyond counting of squares by ones) • uses rows, columns and layers to calculate the volume in cubes of a rectangular prism (e.g. My prism has four rows of two cubes in the first layer and I've made it three layers high so that's $4 \times 2 = 8$ and $3 \times 8 = 24$, so the volume is 24 cubes) • estimates the measurement of an attribute by visualising between known informal units (e.g. uses a cup to measure a half cup of rice; determines that about three sheets of paper would fit across a desk, and close to six might fit along it, so the area of the desk is about eighteen sheets of paper) • explains the difference between different attributes of the same shape or object (e.g. area and perimeter, mass and capacity) **Angles as measures of turn** • describes the size of an angle as an amount of turn (e.g. the angle between the blades gets bigger as you open the scissors)

Understanding units of measurement	
UuM6	**Using formal units** • measures, compares and estimates length, perimeter and area using standard metric units (e.g. I drew around my hand on centimetre grid paper and counted to find the area is about 68 square centimetres) • uses scaled instruments to measure length, mass, capacity and temperature • estimates measurements of an attribute using formal units (e.g. estimates the width of their thumb is close to a centimetre; compares capacities based on the capacity of a 600 ml bottle of water) **Angles as measures of turn** • compares angles to a right angle and classifies them as equal to, less than or greater than a right angle
UuM7	**Using formal units and formulas** • calculates perimeter using properties of two-dimensional shapes to determine unknown lengths • measures and calculates the area of different shapes using formal units and a range of strategies **Angles as measures of turn** • estimates and measures angles in degrees up to one revolution (e.g. uses a protractor to measure the size of an angle)
UuM8	**Converting units** • converts between metric units of measurement • describes the relationship between metric units of measurement and the base-ten place value system **Using formal units and formulas** • establishes and uses formulas for calculating the area of rectangles and triangles **Angles as measures of turn** • measures and uses key angles [45˚, 90˚, 180˚, 360˚] to define other angles according to their size (e.g. measures a right angle to be 90˚ and uses this to determine if two lengths are perpendicular)
UuM9	**Using formal units and formulas** • establishes and uses formulas for calculating the area of parallelograms, trapeziums, rhombuses and kites • establishes and uses formulas for calculating the volume of a range of prisms

Understanding units of measurement	
	Circle measurements • informally estimates the circumference of a circle using the radius or diameter • establishes the relationship between the circumference and the diameter of a circle as the constant (pi) • calculates the circumference and the area of a circle using pi and a known diameter or radius
UuM10	**Using formal units and formulas** • uses dissection and rearrangement to calculate area and volume • uses formal units and formulas to calculate the surface area of prisms, cylinders, cones and pyramids • uses the conversion between units of volume and capacity to calculate the capacity of objects based on the internal volume and vice versa • identifies appropriate units to use according to the level of precision required (e.g. building plans show measurements in mm, but to purchase enough carpet you need to measure the length and width of the room and round up to the nearest whole metre) • uses and applies Pythagoras theorem to authentic contexts (e.g. determines the height of a television screen, given the diagonal length of the screen is 110cm and having measured its length as 88.6cm) • uses and applies properties of congruent and similar triangles to authentic contexts to determine unknown angles and sides • uses trigonometry to calculate the unknown lengths or angles in authentic problems • chooses an appropriate method to solve problems involving right triangles in authentic contexts

Understanding geometric properties

This sub-element describes how a student becomes increasingly able to identify the properties of shapes and objects and how they can be combined or transformed. Later this requires students to use their understanding of the sub-element *Understanding units of measurement.*

Being able to use spatial reasoning and geometric properties to solve problems is important for a range of authentic applications and can be applied across several learning areas. Builders, artists and engineers use the properties of geometric shapes and objects within their designs to provide strength and stability, and for aesthetic appeal.

Knowledge of how certain shapes will tessellate allows tilers, paving companies and architects to design spaces efficiently and creatively. Symmetry is a fundamental aspect of geometry both in the environment and in design.

Some students will demonstrate the skills of *Understanding geometric properties* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	Indicators
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is UGP. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
UGP1	**Familiar shapes and objects** • uses everyday language to describe and compare shapes and objects (e.g. round, small, flat, pointy) • locates and describes similar shapes and objects in the environment • names familiar shapes in the environment (e.g. circle, triangle, square) **Angles** • identifies and describes a turn in either direction (e.g. turn the doorknob clockwise; turn to your left))
UGP2	**Features of shapes and objects** • identifies and describes features of shapes and objects (e.g. sides, corners, faces, edges and vertices) • sorts and classifies familiar shapes and objects based on obvious features (e.g. triangles have three sides; a sphere is round like a ball)

Understanding geometric properties	
	Transformations • identifies features of shapes of different sizes and in different orientations in the environment following basic one-step translations, reflections or rotations (e.g. using a half turn, flipping the shape over) • explains that the shape or object does not change when presented in different orientations (e.g. a square remains a square when rotated) **Angles** • identifies angles in the environment (e.g. an angle formed when a door is opened; identifies there are four angles in a square)
UGP3	**Properties of shapes and objects** • identifies the relationship between the number of sides of a two-dimensional shape and the number of corners (e.g. if the shape has four sides, it has four corners) • describes and identifies the two-dimensional shapes represented by the faces of three-dimensional objects (e.g. recognises the faces of a triangular prism as triangles and rectangles) • represents shapes and objects (e.g. drawing and sketching; model building such as skeletal models and centi-cubes; using digital drawing packages) **Transformations** • determines whether a shape has line symmetry (e.g. folds paper cut-outs of basic shapes to demonstrate which has line symmetry and which does not) • identifies symmetry in the environment • identifies and creates patterns involving one- and two-step transformations of shapes (e.g. uses pattern blocks to create a pattern and describes how the pattern was created) **Angles** • compares angles to a right angle, classifying them as greater than, less than or equal to a right angle
UGP4	**Properties of shapes and object** • classifies two-dimensional shapes according to their side and angle properties (e.g. describes a square as a regular rectangle) • identifies key features of shapes (e.g. explains that quadrilaterals have two diagonals however they are not always equal in length) • aligns three-dimensional objects to their two-dimensional nets • identifies the relationship between the number of faces, edges and the number of vertices of a three-dimensional object (e.g. uses a table to list the number of faces, edges and vertices of common three-dimensional objects and identifies the relationships in the data)

Understanding geometric properties	
	Transformations • identifies that shapes can have rotational symmetry (e.g. 'this drawing of a flower is symmetrical; I can spin it around both ways and it always looks exactly the same') • creates symmetrical designs using a range of shapes and identifies the type of symmetry as appropriate • creates tessellating patterns with common shapes, deciding which will tessellate and which will not by referring to their sides and angles **Angles** • estimates, compares and constructs angles (e.g. uses a ruler and protractor to construct a 45˚ angle; compares the size of angles in the environment and estimates their size) • describes angles in the environment according to their size as acute, obtuse, right, straight, reflex or a revolution and identifies them in shapes and objects (e.g. identifies slope as angles in the environment such as the ramp outside of the school block)
UGP5	**Properties of shapes and objects** • classifies three-dimensional objects according to their properties (e.g. describes the difference between a triangular prism and a triangular pyramid) • relates pyramids and prisms to their two-dimensional nets **Transformations** • uses combinations of reflecting, translating and rotating shapes to describe and create patterns and solve problems • identifies tessellations used in the environment and explains why some combinations of shapes will tesselate whilst others will not (e.g. tiling a wall using a combination of different shaped tiles; exploring regular and semi-regular tessellations in architectural design) • explains the result of changing critical and non-critical properties of shapes (e.g. if I enlarge a square, it's still a square, or if I rotate a square it remains a square but if I change the length of one of its sides, it's no longer a square) **Angles** • identifies supplementary and complementary angles and uses them to solve problems • identifies that angles at a point add to 360° and that vertically opposite angles are equal and reasons to solve problems
UGP6	**Properties of shapes and objects** • investigates properties of a triangle (e.g. explains why the longest side is always opposite the largest angle in a triangle; recognises that the combined

Understanding geometric properties	
	length of two sides of a triangle must always be greater than the length of the third side) • uses relevant properties of common geometrical shapes to determine unknown lengths and angles **Transformations** • enlarges and reduces shapes according to a given scale factor and explains what features change and what stay the same (e.g. says 'when I double the dimensions of the rectangle, all of the lengths are twice as long as they were, but the size of the angles stay the same) • applies angle properties to solve problems that involve the transformation of shapes and objects and how they are used in practice (e.g. when determining which shapes tessellate) **Angles** • uses angle properties to identify perpendicular and parallel lines • demonstrates that the angle sum of a triangle is 180˚ and uses this to solve problems • identifies interior angles in shapes to calculate angle sum • uses angle properties to identify and calculate unknown angles in familiar two-dimensional shapes
UGP7	**Geometric properties** • uses the Pythagoras theorem to solve right-angled triangle problems • determines the conditions for triangles to be similar • determines the conditions for triangles to be congruent **Transformations** • uses the enlargement transformation to explain similarity and develop the conditions for triangles to be similar • solves problems using ratio and scale factors in similar figures **Angles** • uses angle properties to reason geometrically, in order to solve spatial problems (e.g. applies an understanding of the relationship between the base angles of an isosceles triangle to determine the size of a similar shape in order to solve a problem) • uses trigonometry to calculate the unknown angles and unknown distances in authentic problems (e.g. measures the height of a tree using a clinometer to measure the angle of inclination and trigonometry to approximate the vertical height; calculates the angle of inclination for a ramp)

Positioning and locating

This sub-element describes how students become increasingly able to recognise the attributes of position and location, and to use positional language to describe themselves and objects in the environment using maps, plans and coordinates.

A student learns to reason with representations of shapes and objects regarding position and location, and to visualise and orientate objects to solve problems in spatial contexts. The use of scales on maps is an application of proportional reasoning.

There are aspects of this progression that are explored in more depth in other learning areas such as the construction and interpretation of maps within HASS.

Some students will demonstrate the skills of *Positioning and locating* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	**Indicators**
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is PoL. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
PoL1	**Position to self** • locates positions in the classroom relevant to self (e.g. hangs their hat on their own hook, puts materials in their own tray; says ‘my bag is under my desk’) • orients self to other positions in the classroom (e.g. collects a box of scissors from the shelf at the back of the classroom) • follows simple instructions using positional language (e.g. please stand near the door, you can sit on your chair, put your pencil case in your bag, crawl through the tunnel)
PoL2	**Position to other** • uses positional terms with reference to themselves (e.g. sit next to me, you stood in front of me, this is my left hand) • interprets a simple diagram or picture to describe the position of an object in relation to other objects (e.g. the house is between the river and the school) • gives and follows simple directions to move from one place to another using familiar reference points (e.g. walk past the flagpole around the vegetable patch and you will find Mr Smith’s classroom)

Positioning and locating	
PoL3	**Using an informal map** • draws an informal map or sketch to provide directions • describes and locates relative positions on an informal map • orients an informal map using recognisable landmarks and current location • locates self on an informal map to select an appropriate path to a given location
PoL4	**Using formal maps and plans** • locates position on maps using grid references • describes routes using landmarks and directional language • interprets keys, simple scales and compass directions contained within a map to locate features
PoL5	**Using proportional thinking for scaling** • interprets the scale used to create plans, drawings or maps • interprets and uses plans and maps involving scale • describes and interprets maps to determine the geographical location and positioning of states and territories within Australia and of countries relative to Australia • uses more formal directional language such as compass directions and coordinates to locate position

Measuring time

This sub-element describes how a student becomes increasingly aware of reading and describing the passage of time and how elapsed time can be measured.

A student initially describes the passage of time associated to regularly occurring events, such as having breakfast in the morning, going to school on weekdays and being five years old. As their knowledge of time becomes more sophisticated, they learn to apply units and conventions associated with measuring and recording the succession and duration of time.

Historically we have moved from measuring time by the sun, moon and the stars using sundials and astrological measurements to using mechanical analogue and electric digital clocks, stop watches and other timing mechanisms. Time itself provides us with a measure of change by which we can record specific events. A formal precise set of units needs to be used to allow for comparison and consistency. Time is measured in two ways, as the precise moment that an event occurred and as a duration or time interval for an event.

The attribute of time and its measurement supports the learning of proportional thinking and reasoning to describe change over time.

Aspects of this progression are explored in more depth within other learning areas such as Science and HASS.

Some students will demonstrate the skills of *Measuring time* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	Indicators
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is MeT. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
MeT1	**Sequencing time** • uses the language of time to describe events in relation to past, present and future (e.g. yesterday I.., today I .., tomorrow I will .., next week I will ..) • applies an understanding of passage of time to sequence events using everyday language (e.g. I play sport on the weekend and have training this afternoon; the bell is going to go soon; we have cooking tomorrow) • uses direct comparison to compare time duration of two actions, knowing they must begin the actions at the same time (e.g. who can put their shoes on in the shortest time) • measures time duration by counting and using informal units (e.g. counting to 20 while children hide when playing hide and seek)

Measuring time	
MeT2	**Units of time** • uses and justifies the appropriate unit of time to describe the duration of events (e.g. uses minutes to describe time taken to clean teeth; uses hours to describe the duration of a long-distance car trip) • identifies the clockface is a circle subdivided into 12 parts and uses these to allocate hour markers • identifies that hour markers on a clock can also represent quarter-hour and half-hour marks and shows that there is a minute hand and an hour hand on a clock • identifies the direction of clockwise and anticlockwise relating it to the hands of the clock • reads time on analogue clocks to the hour, half-hour and quarter-hour • names and orders days of the week and months of the year • uses a calendar to identify the date and determine the number of days in each month
MeT3	**Measuring time** • uses standard instruments and units to describe and measure time to hours, minutes and seconds (e.g. measures time using a stopwatch; sets a timer on an appliance; estimates the time it would take to walk to the other side of the school oval and uses minutes as the unit of measurement) • reads and interprets different representations of time (e.g. on an analogue clock, watch or digital clock) • identifies the minute hand movement on an analogue clock and the 60-minute markings, interpreting the numbers as representing lots of five (e.g. interprets the time on an analogue clock to read seven forty, by reading the hour hand and the minute hand and explaining how they are related) • uses smaller units of time such as seconds to record duration of events • uses a calendar to calculate time intervals in days and weeks, bridging months
MeT4	**Relating units of time** • identifies the relationship between units of time (e.g. months and years; seconds, minutes and hours) • uses am and pm notation to distinguish between morning and afternoon using 12-hour time • determines elapsed time using different units (e.g. hours and minutes, days and weeks) • interprets and uses a timetable • constructs timelines using a time scale (e.g. chronologically sequences history of the school)

Measuring time	
MeT5	**Converting between units of time** • interprets and converts between 12-hour and 24-hour digital time, and analogue and digital representations of time to solve duration problems • converts between units of time, using appropriate conversion rates, to solve problems involving time (e.g. uses that there are 60 seconds in a minute to calculate the percentage improvement a 1500m runner made to their personal best time) • uses rates involving time to solve problems (e.g. travelling at 60 km/h, how far will I travel in 30 minutes?)
MeT6	**Measuring time with large and small timescales** • uses appropriate metric prefixes to measure both large and small durations of time (e.g. millennia, nanoseconds) • constructs timelines using an appropriate scale (e.g. chronologically sequences historical events)
MeT7	**Measuring how things change over time** • investigates, describes and interprets data collected over time (e.g. uses a travel graph to describe a journey; interprets data collected over a period of time using a graphical representation and makes a prediction for the future behaviour of the data)

STATISTICS AND PROBABILITY

Understanding chance

Our modern understanding of probability dates from the second half of the 17th century with the analysis of games of chance. Probability is a measure of the likelihood of an event. Weather forecasting, the generation of insurance premium costs, sports and events management all use the numeracy required in understanding chance as a foundation to their field.

Students begin with recognising that events may or may not happen and they begin to describe familiar events that involve chance. As their understanding of chance situations becomes more sophisticated, they are able to describe outcomes of chance experiments, develop an understanding of randomness, recognise bias, make predictions and explain why expected results may differ from the actual results of chance events.

This sub-element describes how a student becomes increasingly able to use the language of chance and the numerical values of probabilities when determining the likelihood of an event and comparing chance events in relation to variation and expectation. Understanding chance is often essential to interpret data.

Some students will demonstrate the skills of *Understanding chance* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	Indicators
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is UnC. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
UnC1	**Describing chance** • describes everyday occurrences that involve chance • makes predictions on the likelihood of simple, everyday occurrences as to it will or won't, might or might not happen (e.g. I might be able to come and play at your house today; next year I will be … years old; my tower might not fall down)
UnC2	**Comparing chance** • describes and orders the likelihood of events in non-quantitative terms such as certain, likely, highly likely, unlikely, impossible (e.g. if there are more blue than red marbles in a bag, blue is more likely to be selected; I am certain that I won't win the competition because I didn't enter)

Understanding chance	
	• records outcomes of chance experiments in tables and charts • demonstrates that outcomes of chance experiments may differ from expected results (e.g. we will not get the same results every time we roll a dice) • draws conclusions that recognise variation in results of chance experiments (e.g. you rolled a lot of sixes this game, I hope I get more sixes next time)
UnC3	**Fairness** • identifies all possible outcomes of one-step experiments and records outcomes in tables and charts • explains why outcomes of chance experiments may differ from expected results (e.g. just because there are six numbers on a dice doesn't mean you are going to roll a 6 every six rolls, you may not roll a 6 in the entire game) • explains that 'fairness' of outcomes is related to the notions of equal likelihood of all possible outcomes (e.g. uses phrases such as fifty-fifty when there are two outcomes and when two events occurring are equally likely) • identifies unfair elements in games that affect the chances of winning (e.g. having an unequal number of turns; weighted dice) • explains that the probabilities of all chance events are either 'impossible', 'certain to happen' or lie somewhere in between • identifies events where the chance of one event occurring will not affect the occurrence of the other (e.g. if a coin is tossed and heads have come up seven times in a row, it is still equally likely that the next toss will be either a head or a tail)
UnC4	**Probabilities** • expresses the theoretical probability of an event as the number of ways an event can happen out of the total number of possibilities • identifies a range of chance events that have a probability from 0 – 1 (e.g. you have zero probability of rolling a 7 with one roll of a standard 6-sided dice; the probability that tomorrow is Wednesday given today is Tuesday is 1) • describes probabilities as fractions of one (e.g. the probability of an even number when rolling a dice is $\frac{3}{6}$) • expresses probabilities as fractions, decimals, percentages and ratios recognising that all probabilities lie on a measurement scale of 0 to 1 (e.g. uses numerical representations such as 75% chance of rain or 4 out 5 people liked the story; explains why you can't have a probability less than zero)

Understanding chance	
UnC5	**Calculating probabilities** • determines the probability of compound events and explains why some results have a higher probability than others (e.g. tossing two coins) • represents diagrammatically all possible outcomes (e.g. tree diagrams, two-way tables, Venn diagrams) • measures and compares expected results to the actual results of a chance event over a number of trials and compares and explains the variation in results (e.g. uses probability to determine expected results of a spinner prior to trial) • recognises that the chance of something occurring or its complement has a total probability of 1 (e.g. the probability of rolling a 3 is $\frac{1}{6}$ and the probability of not rolling a 3 is $\frac{5}{6}$) • calculates and explains the difference between the probabilities of chance events with and without replacement (e.g. if we put all of the class names in a hat and draw them out one at a time without putting the name back in, the probability of your name getting called out increases each time because the total number of possible outcomes decreases) • calculates the probabilities of future events based on historical data (e.g. uses historical rainfall data to plan the date for an outdoor event)
UnC6	**Probabilistic reasoning** • recognises combinations of events and the impact they have on assigning probabilities (e.g. and, or, not, if not, at least) • solves conditional probability problems informally using data in two-way tables and authentic contexts • evaluates chance data reported in media for meaning and accuracy • applies probabilistic/chance reasoning to data collected in statistical investigations when making decisions acknowledging uncertainty

Interpreting and representing data

This sub-element describes how a student becomes increasingly able to recognise, use and interpret visual and numerical displays to describe data associated with statistical investigations, and to critically evaluate investigations by others. It describes how a student becomes increasingly able to employ the sequence of steps involved in a statistical investigation: posing questions, collecting and analysing data, and drawing conclusions.

Students progress from being able to work with one variable data in one-to-one data displays to working with bivariate data and more sophisticated one-to-many data displays. The development of students' ability to measure and interpret data relies on their development of number sense.

Making sense of data draws on knowing the concepts and tools that are being used to describe the global features of data. A student understands how these concepts and tools make meaning of data in context and develops the ability to think critically about any claims, either questioning or confirming them.

Some students will demonstrate the skills of *Interpreting and representing data* using augmentative and alternative communication strategies. This may include digital technologies, sign language, braille, real objects, photographs and pictorial representations.

Level	Indicators
Each sub-element level has been identified by upper-case initials and, in some cases, lower-case letters of the sub-element name followed by ascending numbers. The abbreviation for this sub-element is IRD. *The listing of indicators within each level is non-hierarchical. Subheadings have been included to group related indicators. Where appropriate, examples have been provided in brackets following an indicator.*	
IRD1	**Emergent data collection and representation** • poses and answers simple questions and collects responses (e.g. collects data from a simple yes/no question by getting respondents to form a line depending upon their answer) • displays information using real objects, drawings or photographs (e.g. collects leaves from outside the classroom and displays them in order of size) • sorts and classifies shapes and objects into groups based on their features or characteristics and describes how they have been sorted (e.g. sorts objects by colour) • identifies things that vary or stay the same in everyday life (e.g. it is always dark at night; although jellybeans are the same size, they can be different colours)

Interpreting and representing data	
IRD2	**Basic one-to-one data displays** • poses questions that could be investigated from a simple numerical or categorical data set (e.g. number of family members, types of pets, where people live) • displays and describes one variable data in lists or tables • communicates information through text, pictures-graphs and tables using numbers and symbols (e.g. creates picture graphs to display one-variable data) • responds to questions and interprets general observations made about data represented in simple one-to-one data displays (e.g. responds to questions about the information represented in a simple picture graph that uses a one-to-one representation)
IRD3	**Collecting, displaying and interpreting categorical data** • designs simple survey questions to collect categorical data • collects, records and displays one variable data in variety of way such as tables, charts, plots and graphs using the appropriate technology (e.g. uses a spread sheet to record data collected in a simple survey and generates a column graph to display the results) • displays and interprets categorical data in one-to-many data displays • interprets categorical data in simple graphical displays such as bar and column graphs, pie charts and makes simple inferences • makes comparisons from categorical data displays using relative heights from a common baseline (e.g. compares the heights of the columns in a simple column graph to determine the tallest and recognises this as the most frequent response)
IRD4	**Collecting, displaying and interpreting numerical data** • collects and records discrete numerical data using an appropriate method for recording (e.g. uses a frequency chart to record the experimental results for rolling a dice) • constructs graphical representations of numerical data and explains the difference between continuous and discrete data • explains how data displays can be misleading (e.g. whether a scale should start at zero; not using uniform intervals on the axes) • interprets data displayed using a multi-unit scale, reading values between the marked units
IRD5	**Collecting, displaying continuous data** • poses questions based on variations in continuous numerical data and chooses the appropriate method to record results (e.g. collects information on the heights of buildings or daily temperatures, tabulates the results and represents these graphically)

Interpreting and representing data	
	• uses numerical and graphical representations relevant to the purpose of the collection of the data and explains their reasoning (e.g. 'I can't use a frequency histogram for categorical data because there is no numerical connection between the categories'; converts their data to percentages in order to compare the girls results to that of the boys, as the total number of boys and girls who participated in the survey was different) • determines and calculates the most appropriate statistic to describe the spread of data • calculates simple descriptive statistics such as mode, mean or median as measures to represent typical values of a distribution • compares the usefulness of different representations of the same data • determines the location and calculates the spread of data using range
IRD6	**Interpreting graphical representations** • uses features of graphical representations to make predictions • summarises data using fractions, percentages and decimals (e.g. $\frac{2}{3}$ of a class live in the same suburb) • explains that continuous variables depicting growth or change often vary over time (e.g. growth charts, temperature charts) • interprets graphs depicting motion such as distance–time graphs • interprets and describes patterns in graphical representations in real-life situations (e.g. rollercoasters, flight trajectory) • investigates the association of two numerical variables through the representation and interpretation of bivariate data (e.g. uses scatter plots) • investigates, represents and interprets time series data (e.g. interrogates a time series graph showing the change in costs over time) • interprets the impact of changes to data (e.g. the impact of outliers on a data set)
IRD7	**Sampling** • determines whether to use data from a sample or a population • determines what type of sample to use from a population • makes reasonable statements about a population based on evidence from samples • plans, executes and reports on sampling-based investigations, taking into account validity of methodology and consistency of data, to answer questions formulated by the student

Interpreting and representing data	
IRD8	**Recognising bias** • applies an understanding of distributions to evaluate claims based on data (e.g. the predictive accuracy of a sample depends on both the size of a sample and how well it represents the population) • identifies and explains bias as a possible source of error in media reports of survey data • justifies criticisms of data sources that include biased statistical elements (e.g. inappropriate sampling from populations)