# Demystifying the birthday paradox

Students will use simulations and theoretical probability to make sense of the well-known birthday paradox problem.

Students have the option to use at least one digital device per pair to interact with a simulation during this lesson.

## Visible learning

This learning episode incorporates Path content.

### Learning intentions

* To use complementary probabilities to solve problems.
* To use simulations to model complicated situations.

### Success criteria

* I can use complementary events to find the probability of an event.
* I can draw conclusions based on a simulation.

### Syllabus outcomes

A student:

* develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
* solves problems involving probabilities in multistage chance experiments and simulations **MA5-PRO-C-01**
* solves problems involving Venn diagrams, 2-way tables and conditional probability **MA5-PRO-P-01**

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## Activity structure

### Launch

1. Facilitate a class discussion to figure out if any students in the class share a birthday.
2. Conduct a finger vote ([bit.ly/DLSquick](https://app.education.nsw.gov.au/digital-learning-selector/LearningActivity/Card/560?clearCache=c14be2f4-9c26-ed25-458e-93aace1797)) to ask students how likely it is that 2 students in a class share a birthday, on a scale of 1 finger (very unlikely) to 5 fingers (very likely).
3. Students discuss in a Think-Pair-Share ([bit.ly/thinkpairsharestrategy](https://bit.ly/thinkpairsharestrategy)):
* How many students do you think need to be in a room to guarantee that 2 people share a birthday?
* How many students would there need to be for it to be likely that 2 share a birthday?
* How could we figure this out, without interrupting every classroom and office in the school?

These prompts can be displayed on slide 3 of the *Demystifying the birthday paradox* PowerPoint.

### Explore

#### 6 days in a year

1. Explain to students that when mathematicians are solving complicated problems, sometimes they will first try to make them simpler to better understand what is happening. To better understand this problem, we will say that there are only 6 days in a year instead of 365.
2. Assign visibly random groups of 3 ([bit.ly/visiblegroups](https://bit.ly/visiblegroups)) to vertical non-permanent surfaces ([bit.ly/VNPSstrategy](https://bit.ly/VNPSstrategy)). Provide each group with a 6-sided die and a whiteboard marker.
3. Groups are to conduct a simulation to represent how many students they need in a class for 2 students to share a birthday. The numbers of the dice represent the 6 days of the year and each roll is a student added to the class. Students should repeat the trial several times.

For example, if a group rolls a 3, then a 5 and then a 3. They would stop the trial and record that it took 3 students to have a shared birthday. In this example, day 3 is the shared birthday.

1. Groups neatly summarise the data obtained from their trials on the vertical non-permanent surfaces, before conducting a gallery walk ([bit.ly/DLSgallerywalk](https://bit.ly/DLSgallerywalk)) to observe the results obtained by their peers.
2. Groups return to their vertical non-permanent surfaces. Use a questioning strategy such as Pose-Pause-Pounce-Bounce (PDF 557KB) ([bit.ly/posepausepouncebounce](https://bit.ly/posepausepouncebounce)) to facilitate a class discussion on the data obtained from the simulation.

### Summarise

1. Assuming there are still 6 days in a year, have students consider a class of 6 students. Ask groups to calculate the probabilities below, which can be displayed on slide 5 of the *Demystifying the birthday paradox* PowerPoint. Sample solutions are provided under each prompt.
* The probability of a student being born on day 1.

$$\frac{1}{6}$$

* The probability of a student not being born on day 1.

$$1-\frac{1}{6}=\frac{5}{6}$$

* The probability of all students being born on day 1.

$$\left(\frac{1}{6}\right)^{6}=0.00002$$

* The probability of no students being born on day 1.

$$1-(\frac{1}{6})^{6}=0.99998$$

* The probability of at least 2 students sharing a birthday.

Student 1 is born on day 1. The probability of being born on day 1 is 1 (we are stating that this is 100% true).

Student number 2’s probability of not sharing that birthday is $\frac{5}{6}$.

Student number 3’s probability of not sharing a birthday with **either student 1 or 2** is $\frac{4}{6}$.

Student number 4’s probability of not sharing a birthday with student 1,2 or 3 is $\frac{3}{6}$.

Student number 5’s probability of not sharing a birthday with students 1–4 is $\frac{2}{6}$.

Student number 6’s probability of not sharing a birthday with students 1–5 is $\frac{1}{6}$.

So, the probability that **no** students share a birthday is $\frac{6}{6}×\frac{5}{6}×\frac{4}{6}×\frac{3}{6}×\frac{2}{6}×\frac{1}{6}=\frac{5}{324}$.

A simpler way to express this is $P(unique birthdays)=\frac{5}{324}$.

And finally, the complement of no students sharing a birthday would be that **at least 2 students do share a birthday** which can be expressed as $P\left(at least 2 students share a birthday\right)=1-\frac{5}{324}=\frac{319}{324}$

1. Select non-volunteer students to discuss how the calculated probabilities related to the data obtained from their simulation.

### Apply

#### 365 days in a year

1. Have students return to desks. With one digital device between pairs of students, direct students to The Birthday Paradox Simulation ([bit.ly/bdaysimulation](https://bit.ly/bdaysimulation)).
2. Upon opening the interactive activity, there will be data displayed for a group size of 23. Students should discuss in their pairs what the dots represent.
3. Students are to select the **Clear data** button, drag the **Group size** slider to 6 then select the play button in the bottom left corner of the screen. The activity will begin running simulations where each dot is a simulation. The number along the horizontal axis is the number of birthday pairs in that size group.
4. Students discuss in a Think-Pair-Share:
* Why is the percentage of matches so different to the probability we calculated based on the dice?

This is because the GeoGebra simulation is now considering all 365 days of the year, 6 in this instance is only referring to the group size.

1. Allow students time to play with the simulation.
2. Either by explaining the results of the simulation or by continuing the process in the Summarise section, pairs are to develop solutions for at least one of the statements from the Launch section:
* How many students do you think need to be in a room to guarantee that 2 people share a birthday?
* How many students would there need to be for it to be likely that 2 share a birthday?
* How could we figure this out, without interrupting every classroom and office in the school?

‘The birthday paradox (8:02)’ ([bit.ly/vsaucebday](https://bit.ly/vsaucebday)) can be shown to students to assist in explaining the process.

## Assessment and differentiation

### Suggested opportunities for differentiation

**Launch**

* There are no correct answers during the launch and all students should be encouraged to participate and share their thoughts and reasoning.

**Explore**

* Students could be challenged to find the probability that 2 students are born on the same day without reducing the number of days in a year.
* Students may need to be reminded what ‘at least’ means in terms of probability.
* Less or more scaffolding can be provided to help students develop the pattern.

**Summarise**

* If students have access to a digital device, allow time to replicate the simulation using spreadsheet software.
* Students who are not ready to express the scenarios as formal probabilities could calculate probabilities from their simulations.

### Suggested opportunities for assessment

**Explore**

* Monitor student responses in the group work section to check for understanding.
* Monitor responses in class discussion to check for student understanding of complementary events and probability.
* Monitors students’ language for use of the terms ‘at least’ ‘and’ and ‘or’ to determine the operations used.

**Summarise**

* Monitor students’ justification of why the number of people needed is less than students think it will be.

## Sample solutions

### At least 2 people share the same birthday

**Probability with 2 people**

P(same) $=1×\frac{1}{365}$

**Probability with 3 people**

P(A’s birthday is unique) = $\frac{365}{365}$

P(B’s birthday is unique) = $\frac{364}{365}$

P(C’s birthday is unique) = $\frac{363}{365}$

P(A, B and C have unique birthdays) = $\frac{365}{365}×\frac{364}{365}×\frac{363}{365}$

 = 0.9918

P(at least A, B or C share birthday) = $1-(\frac{365}{365}×\frac{364}{365}×\frac{363}{365})$

 = 0.008204

**Probability with 4 people**

P(unique birthdays) = $\frac{365}{365}×\frac{364}{365}×\frac{363}{365}×\frac{362}{365}$

 = 0.9836

P(at least 2 share a birthday) = $1-(\frac{365}{365}×\frac{364}{365}×\frac{363}{365}×\frac{362}{365})$

 = 0.01636

**Probability with 5 people**

P(unique birthdays) = $\frac{365}{365}×\frac{364}{365}×\frac{363}{365}×\frac{362}{365}$ $×\frac{361}{365}$

 = 0.9729

P(at least 2 share a birthday) = $1-(\frac{365}{365}×\frac{364}{365}×\frac{363}{365}×\frac{362}{365}×\frac{362}{365})$

 = 0.02714

**Probability with n people**

P(unique birthdays) = $\frac{365}{365}×\frac{364}{365}×\frac{363}{365}×\frac{362}{365}×\frac{362}{365}×…×\frac{366-n}{365}$

P(at least 2 share a birthday) = $1-(\frac{365}{365}×\frac{364}{365}×\frac{363}{365}×\frac{362}{365}×\frac{362}{365}×…×\frac{366-n}{365})$

## References

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