



Education
& Training

Talking about Patterns & Algebra

Early Stage 1 to Stage 3

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Acknowledgements

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Overview

The *Patterns and Algebra* strand emphasises two components, *Number patterns* and *Number relationships*. Each of these components includes three aspects which show a progression from Early Stage One to Stage Four. While these aspects are shown separately, it is not suggested that each one should be taught in isolation.

Learning about *number patterns* involves students in:

- creating and continuing number patterns
- describing, discussing and labelling patterns
- finding specific terms of a sequence.

Learning about *number relationships* involves students in:

- using symbols and operations to show equality and inequality
- applying collection-based methods to generalise
- working out unknown elements to complete number sentences.

This section provides an overview of the distinctive learning and key concepts within the *Patterns and Algebra* strand for each stage. It does not describe the full scope of learning for each stage.

Particular attention has been given to emphasising the links between the *Patterns and Algebra* and the *Number* strands.

Focus: Number patterns

	Creating number patterns	Describing number patterns	Finding terms in a number pattern
ES1	Creating patterns using shapes, objects or pictures. Continuing number patterns that increase or decrease.	Describing a pattern made of shapes, objects or pictures in terms of distinguishing features and of the number of repeating elements.	Finding the next term in a number pattern that increases or decreases.
S1	Creating patterns using whole numbers that increase or decrease.	Describing a pattern involving whole numbers.	Determining a missing element within an additive pattern.
S2	Creating additive patterns using whole numbers, fractions and decimals.	Describing a pattern involving whole numbers, fractions or decimals.	Continuing a pattern to determine a higher or a lower term when given some consecutive elements.
S3	Creating patterns of multiples using whole numbers and by multiplying or dividing decimals by single digit numbers, 10, 100 or 1000. Completing a table of values for patterns involving one operation.	Describing a pattern in more than one way.	Determining a rule to describe a pattern and using it to derive the value of a higher term.
S4	Completing a table of values for patterns involving more than one operation.	Creating an algebraic equation that describes a pattern.	Using an algebraic expression that describes a pattern to calculate an element.

Focus: Number relationships

	Describing number relationships	Generalising about number relationships	Finding unknown elements
ES1	Using the term 'is the same as' to express equality. Recognising unequal groups.	Comparing groups to generalise about numbers.	Forming groups that have the same number of elements.
S1	Using the equals sign to mean 'is the same as'. Building all addition facts to 20. Building multiplication facts for ones, twos, fives and tens. Using the terms 'more than' and 'less than' to describe inequality.	Applying collection-based methods, including the commutative property of addition and multiplication to generalise about number relationships within Stage One content.	Completing number sentences involving one operation within Stage One content.
S2	Building all multiplication facts to 10×10 . Using symbols to show inequality.	Applying collection-based methods, including the associative property of addition and multiplication to generalise about number relationships within Stage Two content.	Completing number sentences involving one operation within Stage Two content.
S3	Describing number relationships using number sentences involving more than one operation.	Applying collection-based methods to solve number sentences that do not have a whole number solution.	Completing number sentences that involve more than one operation, fractions and decimals.
S4	Simplifying, expanding, factorising and comparing algebraic expressions.	Translating between words and algebraic symbols. Interpreting linear relationships created from simple number patterns and equations.	Solving algebraic equations.

Talking_{about} Patterns
& Algebra

Talking_{about} Patterns & Algebra

Early Stage 1

3

2

4

8

5

1

Overview of Early Stage 1 content

PAES1.1 Recognises, describes, creates and continues repeating patterns and number patterns that increase or decrease.

To recognise a pattern, students have to understand what a pattern is, so this is an important early focus. When young children splash blobs of paint on paper and an adult asks what it is, they sometimes reply, *It's just a pattern*. But is this what we mean in mathematics?

There needs to be an explicit focus on the meaning of *pattern* in mathematics. Toss a few counters of different colours on the floor and ask the students to discuss if you have made a pattern. Then arrange some counters in a pattern (say, a row of three blue, and under them a row of three yellow, then under them a row of three blue, and another row of three yellow) and ask, *Is this a pattern?* Ask the students what the difference is between the two groups of counters.

In terms of Patterns and algebra, we are talking about repeating patterns, and students need to have a clear understanding of this. We can provide some materials that assist in creating patterns, such as a long pegboard with one row of holes, or a strip of card with a row of circles on which students can place counters. In this way, students can be involved in discussing patterns, describing them, identifying the repeating components of patterns and suggesting number labels for patterns. Frequently return to the question, *Is this a pattern?* to generate discussion.

It is important to model the words *repeat*, *repeated* and *repeating*. For example, have a child use counters to make a three pattern and ask, What parts would you repeat to continue the pattern?

Also at this stage, students will develop an understanding of the concept of equality, and learn to form and compare groups to generalise about number.

Patterns and Algebra - Early Stage 1

Early Stage 1

Simple patterns

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Naming patterns

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Equality

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Early Stage 1

Simple patterns

What is a pattern?

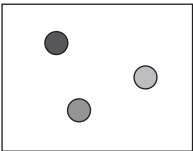
Key idea

Recognise, describe, create and continue repeating patterns

Materials

Three photocopies of BLM *What is a pattern?* cut to make six cards (p. 12)

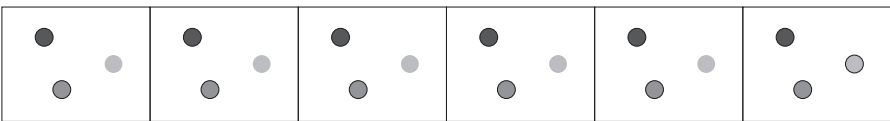
Show the students one of the cards.



Ask, *Is this a pattern?*

Allow the students to discuss their ideas, and lead them to understand that this, by itself, is not a pattern.

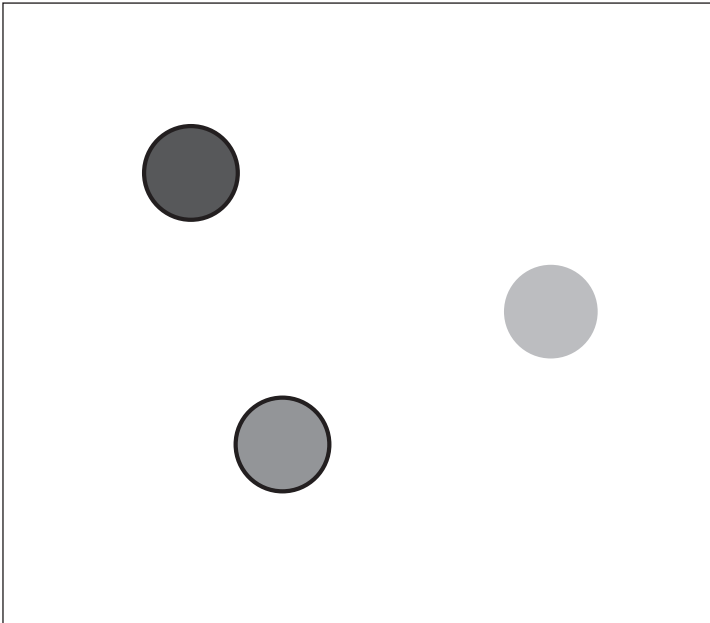
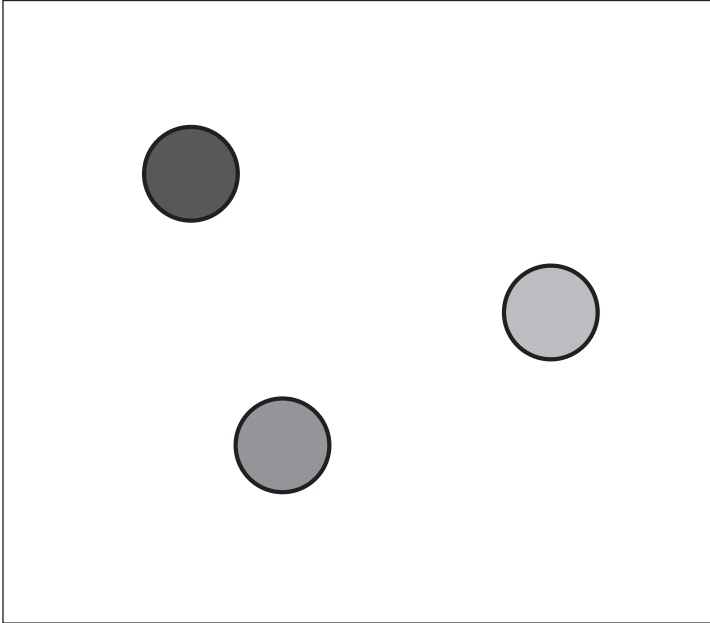
Next, line up the six cards.



Ask, *Is this a pattern?*

Lead the students in a discussion to understand that this is a pattern because there is a repeating element.

What is a pattern?



BLM - Early Stage 1

Sound and action patterns

Key idea

Recognise, describe, create and continue repeating patterns

Materials

None

Create patterns using a sequence of different sounds or actions, such as:

clap, clap, stamp, stamp, clap, clap, stamp, stamp ...

Ask the students to continue the patterns and describe them in words.

Create patterns using a single sound or action, such as:

clap, clap, clap [pause] clap, clap [pause] clap, clap, clap [pause] clap, clap ...

Ask the students to continue the patterns and describe them in words.

Have the students create their own sound or action patterns. Create a sound or action pattern and insert a deliberate error. Have the students raise their hands when they think that they hear or see the error.

For example, *clap, clap, finger-tap, clap, clap, finger-tap, clap, clap, finger-tap, clap, finger-tap ...*

Have the students explain why they think there is an error. Encourage them to use words to describe the pattern, rather than perform the actions.

Other outcomes addressed

NES1.1 Counts to 30, and orders, reads and represents numbers in the range 0 to 20.

Sound and action pattern representations

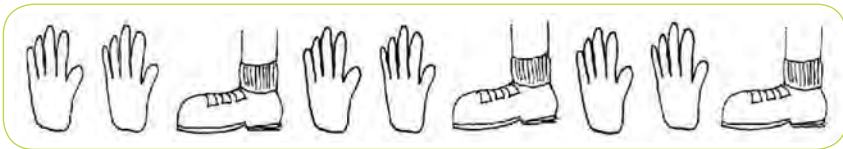
Key idea

Recognise, describe, create and continue repeating patterns

Materials

Multiple copies of *action word cards*

Ask the students to suggest how you could draw a clap and a stamp. Draw on the board sound and action patterns using claps and stamps.



Ask the students to describe the pattern. Also ask them, *How do you know that this is a pattern?* Reinforce the words *repeat*, *repeated* and *repeating*.

Patterns can also be represented by written words. For example, provide several cards with the word *clap* written on them and several with the word *tap* written on them. Have the students use the cards to create a pattern. For example:



Have the students demonstrate their sound and action patterns.

Patterns with objects, shapes and pictures

Key idea

Recognise, describe, create and continue repeating patterns

Materials

A collection of drawn shapes, objects and pictures

Create patterns with objects, drawn shapes or pictures.

For example:



Have the students describe the pattern.

Ask, *How could you continue this pattern?*

As a variation show a simple pattern, cover it and ask the students to make the same pattern. Ask them to explain how they know that their pattern is the same as the covered one.

Other outcome addressed

SGES1.3 Uses everyday language to describe position and give and follow simple directions.

Drawing patterns

Key idea

Recognise, describe, create and continue repeating patterns

Materials

Strips of card
Paper
Paper
Computer

Have the students draw patterns on strips of card. Ask them to explain why each one is a pattern.



Have the students cover one or more elements in their pattern and ask other students to work out what is under the cover.



Have the students create patterns using computer graphics and describe them. Print their patterns and ask the class to identify patterns that have the same number of repeating elements.

Identifying a repeating component of a pattern

Key idea

Recognise, describe, create and continue repeating patterns

Materials

Students' drawn patterns from *Drawing patterns* (p. 17)

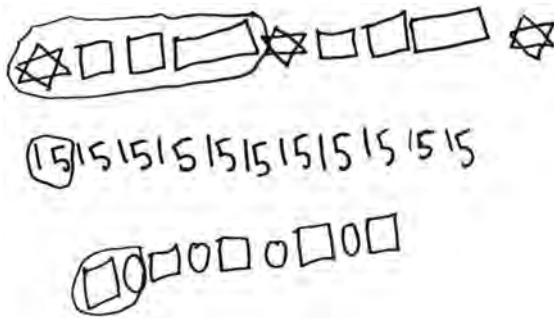
Scissors

Computer

Select a pattern that a student has drawn and ask the class, *What part of the pattern is repeated?*

Cut from the pattern a repeating component. Ask the students to use this component to check the rest of the pattern. Have the students repeat the activity with other patterns that they have drawn.

Then have the students create a repeating component on a computer and use the copy and paste functions to make a pattern.



People patterns

Key idea

Recognise, describe, create and continue repeating patterns

Materials

None

Ask a girl to stand up, then ask a boy to stand beside her. Continue to alternate boys and girls until there are eight students standing in a row. Ask, *Can you see a pattern in this row of children? Why is it a pattern?*

Create other patterns such as,

black shoes, runners, black shoes, runners ...

jumper, jumper, no jumper, jumper, jumper, no jumper ...

Ask, *What is a pattern?*

Select five students to stand in a row as follows:

boy, girl, boy, boy, girl

and ask, *Is this a pattern?*

Select another five students to stand in the row and create a pattern. Ask, *Is this a pattern now?*

Naming patterns

Labelling patterns

Key idea

Recognise, describe, create and continue repeating patterns

Materials

Collections of objects or *Shape cards*
Numeral cards
blank cards

Make a pattern using six objects or shapes. For example:



Have the students describe the pattern and give it a number name. Ask them to explain their choice of name.

If they suggest *six pattern*, extend the pattern:



Ask:

How many elements are there in the part of the pattern that repeats?

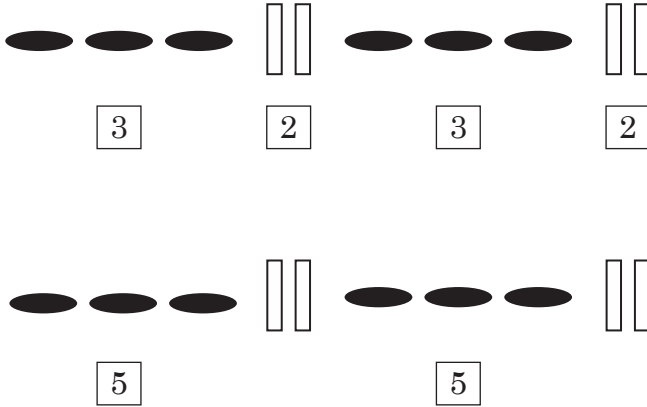
Would you still call this a six pattern?

Display the following pattern and ask a student to name it.



If a student calls it a *one-two pattern*, ask if there could be any other name for it. Calling it a *three pattern* develops concepts of number combinations.

Provide numeral cards, collections of objects and shape cards. Have the students use them to create and label patterns. For example:



Other outcomes addressed

NES1.2 combines, separates and compares collections of objects, describes using everyday language and records using informal methods.

Number patterns that increase or decrease

Sock numbers

Key idea

Continue simple number patterns that increase or decrease

Materials

Copies of the BLM, *Pair of socks* (p. 24), each cut to make two cards

Display a *Pair of socks* card, ask the students, *How many socks are there?* and record the number two. Place a second *Pair of socks* card above the first and ask, *How many socks are there now?* and record the number four next to the second card. Continue adding *Pair of socks* cards and recording the totals to build the count-by-twos number sequence.

Labelling number sequences is a form of generalising, which is important for algebraic thinking.



Other outcomes addressed

NES1.1 Counts to 30, and orders, reads and represents numbers in the range 0 to 20.

Counting backwards by sock numbers

Key idea

Continue simple number patterns that increase or decrease

Materials

Eleven copies of the BLM, *Pair of socks* (p. 24), each cut to make two cards. Then, cut one *Pair of socks* card into single socks.

Display ten *Pair of socks* cards and record how many socks there are. Remove one card and record how many socks are left. Then keep repeating the process to build the count-by-twos backward number sequence.

Use a card displaying a single sock and the *Pair of socks* cards to repeat the activity, beginning with a single sock. Once all of the cards are placed down, remove one *Pair of socks* card at a time.



Other outcomes addressed

NES1.1 Counts to 30, and orders, reads and represents numbers in the range 0 to 20.

Pair of socks



BLM - Early Stage 1

Numeral card counting

Key idea

Continue simple number patterns that increase or decrease

Materials

Numeral cards from 1 to 20

Place the *numeral cards* face-down in order from 1 to 20. Have a student turn over every second card to build the count-by-twos number sequence. Have the class read the sequence.

Turn all of the *numeral cards* face-down in order from 1 to 20. Have a student turn over the first and then every following second card to build the sequence of odd numbers. Have the class read the sequence.

Turn all of the *numeral cards* face-up in order from 1 to 20. Have a student turn over the second-last card (19) and then continue turning over every second card, thus building the count-by-twos backward number sequence. Repeat the activity using the *numeral cards* 1-19 to build the backwards sequence of odd numbers.

Similar activities can be done with the numeral cards 1-30.

Other outcomes addressed

NES1.1 Counts to 30, and orders, reads and represents numbers in the range 0 to 20.

Whisper-talk counting

Key idea

Continue simple number patterns that increase or decrease

Materials

None

Build the sequence of multiples of three by whispering *one*, *two* and then saying aloud *three*, whispering *four*, *five* and saying *six*, and so on.

Other outcomes addressed

NES1.1 Counts to 30, and orders, reads and represents numbers in the range 0 to 20.

Number patterns that increase

Key idea

Continue simple number patterns that increase or decrease

Materials

Number line from 1 to 30
Pegs

Display the number line. Begin clipping a peg on every second number. Ask the students, *Who can place the next peg?* Continue until all even numbers have been identified. Point to each peg and have the students say the corresponding number.

Repeat for other number sequences.

Calculator sequences

Key idea

Continue simple number patterns that increase or decrease

Materials

Calculators with a constant addition function

Provide each student with a calculator and demonstrate how to use the constant addition function. Key in $\boxed{1}$ then $\boxed{+}$, then repeatedly press $\boxed{=}$ to successively display the terms of the count-by-ones sequence.

Have the students record each number as it is displayed and describe the number sequence.

Note that some calculators need the $\boxed{+}$ key to be pressed twice to perform constant addition.

Number patterns that decrease

Key idea

Continue simple number patterns that increase or decrease

Materials

Number line from 1 to 30
Pegs

Display the number line. Begin clipping a peg on every second number, starting from 30. Ask the students, *Who can place the next peg?* Continue until all even numbers in descending order have been identified. Point to each peg in descending order and have the students say the corresponding number.

Repeat for other number sequences.

Equality

Expressing equality

Key idea

Use the term ‘is the same as’ to describe equality of groups

Materials

Counters

Use counters of two different colours to create and display the following pattern:



Ask the students to describe and name the pattern.

Provide the students with a collection of counters and ask them to create different *four patterns*. Select students to share their patterns and compare them to the pattern above.

Discuss the combinations of elements and the equality of the groups. For example, *Two and two is the same as three and one*.

Repeat for patterns with other numbers of elements.

Other outcomes addressed

NES1.2 Combines, separates and compares collections of objects, describes using everyday language and records using informal methods.

Problem posing

Key idea

Use the term ‘is the same as’ to describe equality of groups

Materials

Counters

Pose problems that focus on equality of groups, such as the following:

I have seven counters altogether. If there are five counters in my left hand, how many are in my right hand?

Extend the problem to: *I have seven counters altogether. How many might I have in each hand?*

Other outcomes addressed

NES1.2 Combines, separates and compares collections of objects, describes using everyday language and records using informal methods.

Equal groups

Key idea

Use the term 'is the same as' to describe equality of groups

Materials

Large sheets of card or paper

Have the students make posters showing different ways of expressing or representing different numbers. The posters can be displayed on a wall and added to over time. For example:

2 and 2

1 and 1 and 1 and 1

4

Four ones




3 and 1

1 and 2 and 1

Two 2s

4

○ ○ ○ ○

Other outcomes addressed

NES1.2 Combines, separates and compares collections of objects, describes using everyday language and records using informal methods.

Talking about Patterns
& Algebra

Talking about Patterns & Algebra

Stage 1

3

2

4

8

5

1

Overview of Stage 1 content

PAS1.1 Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships.

At this stage, the students will investigate, describe and discuss increasingly complex number patterns and number relationships. They will generalise about number patterns and be able to name and describe patterns of odd and even numbers. The students learn to determine a missing element within an additive pattern. They will explore, describe and discuss the relationships between addition and subtraction facts and how the order in which you add or multiply affects the result.

While building concepts, the students will use informal language as well as more formal terms in their spoken and written language.

The use of materials, such as number lines and hundred charts, will support the students to identify and create number sequences.



Patterns and Algebra - Stage 1

Generating number sequences

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Generating number sequences

Investigating numbers

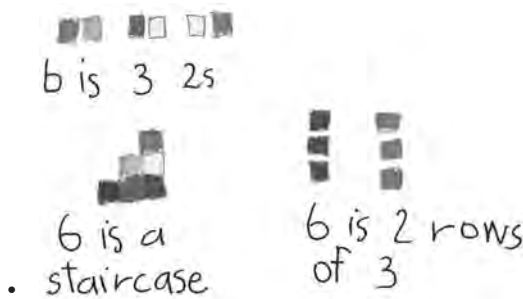
Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

Interlocking cubes, counters, blocks

Have the students represent numbers using objects such as blocks, counters or interlocking cubes and record their findings in drawings, words and symbols. For example, a student could represent six as three pairs of interlocking cubes, or as two rows of three cubes, or as a staircase, and record this as follows:



Ask questions such as, *What other numbers are made up of rows of three?* to provide opportunities for the students to make generalisations and build number sequences.

Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Generating number sequences

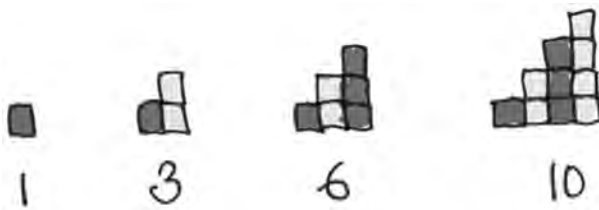
Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

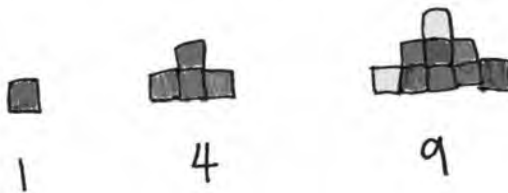
Interlocking cubes

Have the students use interlocking cubes to build the sequence of *staircase numbers*. Then have them draw each term of the sequence and record the number of cubes needed to represent it.



Have the students use their drawings to discuss the pattern of numbers created.

Students could use the cubes to create other number sequences. For example, square numbers, multiples of five and multiples of three.



Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Counting by twos

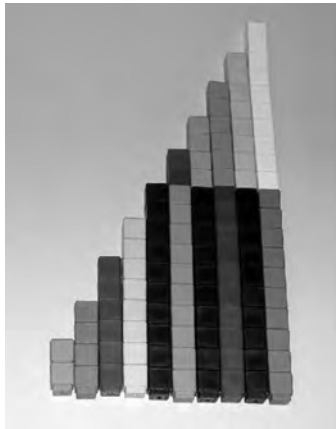
Key idea

Continue simple number patterns that increase or decrease

Materials

Interlocking cubes

Have the students use interlocking cubes to make a staircase pattern for the multiples of two. Have the students record and read the sequence.



Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Generating counting sequences

Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

Popsticks

Have the students use popsticks to make a row of triangles and record the total number of popsticks used after each triangle has been added to the row. Have the students discuss the counting sequence created.



Have the students make other shapes to create and discuss other counting sequences.

Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Sausage counting

Key idea

Continue simple number patterns that increase or decrease

Materials

Ten copies of the BLM, *Sausages* (p. 42)

Explain to the students that butchers hang sausages in threes. Display ten *Sausages* cards on the floor. Have the students count the total number of sausages and record the total on the board. Remove one *Sausages* card at a time and have the students count and record the total number of the remaining sausages.



Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Sausages



BLM - Stage 1

Skip counting to 24

Key idea

Generate, describe and record number patterns using a variety of strategies. Build number relationships by relating multiplication and division facts to at least 10×10

Materials

None

Ask, *In how many different ways can you skip count to 24?*
Have the students record the counting sequences that they find.

Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Counting toes

Key idea

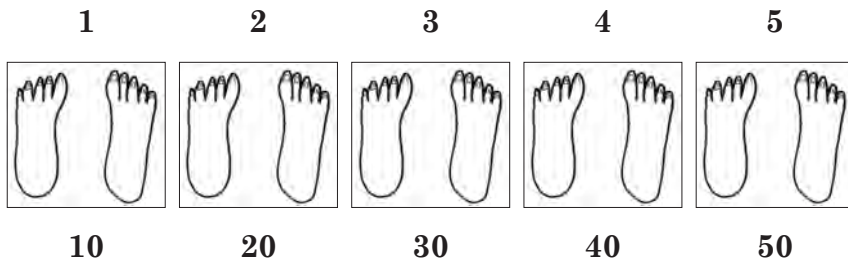
Continue simple number patterns that increase or decrease

Materials

Ten copies of the BLM *Pair of feet* (p. 45)

Display one *Pair of feet* card and ask the students to count how many toes there are altogether and record the number ten. Add another *Pair of feet* card and have the students count and record the total number of toes. Continue adding *Pair of feet* cards and recording the number of toes to build the count-by-tens number sequence.

Display on a wall the ten *Pair of feet* cards with the number of pairs of feet recorded above the cards and the number of toes below as follows:

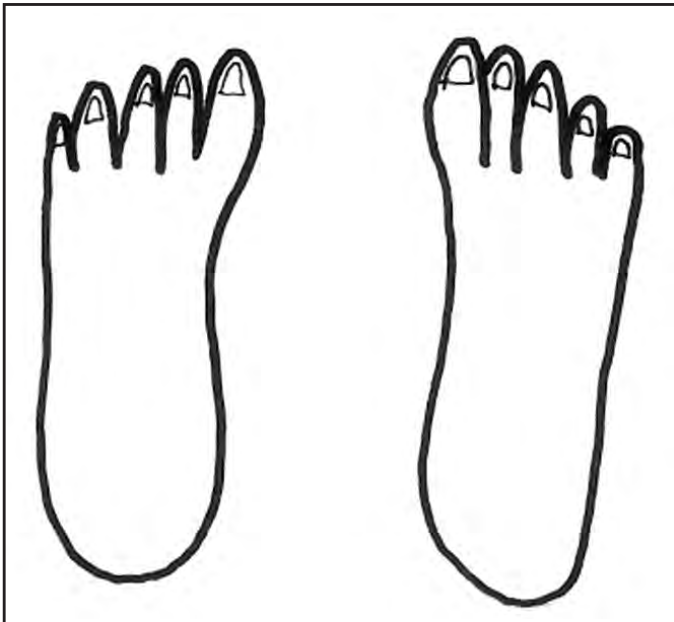
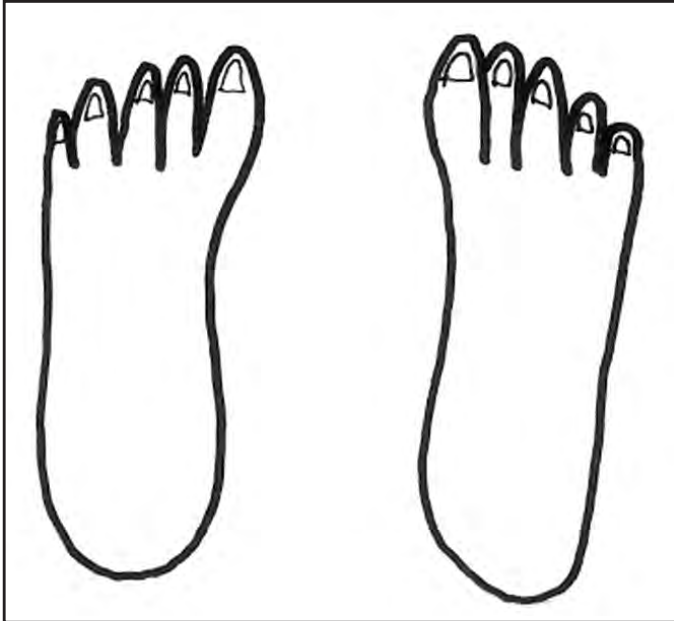


Discuss the relationship between the two sequences of numbers.

Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Pair of feet



Silent counting

Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

None

Have the students silently count three claps then on the fourth clap say, *four*. Continue silently counting three claps and counting out loud every fourth clap.

Repeat the activity and have a student record on the board the numbers counted out loud. Ask the students to describe the sequence.

Modify the activity to create other sequences. For example, have the students:

- repeatedly count the fingers of one hand, tapping the first four fingers while counting them silently, then counting the thumb out loud
- turn their hands palm-up and silently count the first two parts of each finger then count out loud the third part.

Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Area number patterns

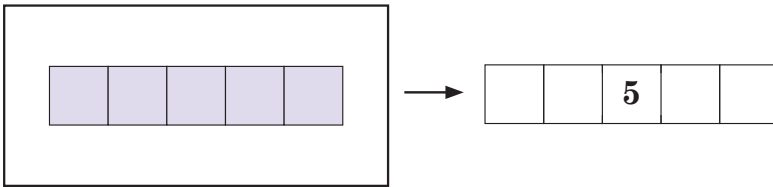
Key idea

Create, represent and continue a variety of number patterns and supply missing elements

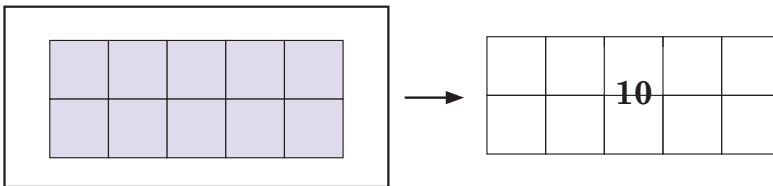
Materials

Paper
 Wooden or cardboard square tiles
 Scissors
 Pencils

Place five tiles in a row on a sheet of paper and draw the outline. Cut out the rectangle, record the total number of tiles in the rectangle and mark the position of each tile.



Next, draw and cut out a rectangle made from two rows of five tiles. Record the number of tiles on it and mark the position of each tile.



Continue making rectangles, each one increasing in area by a row of five tiles until there are five or six rectangles.

Place the rectangles on the floor in random order with the numbers face down and then have the students rearrange them in a row from smallest to largest. Turn the rectangles over and ask, *What can you tell me about this sequence of numbers? How did we create this sequence? What would the next number in the sequence be?*

Repeat the activity with the students working in pairs to create other number sequences.

Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

MS1.2 Estimates, measures, compares and records areas using informal units.

Continuing number sequences

Continuing number sequences

Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

None

Record the sequence 3, 6, 9, 12, 15, on the board and ask the students to work in pairs to continue the sequence and describe it. Repeat the activity for the following sequences:

30, 27, 24, 21, 18, ...

1, 3, 6, 10, 15, ...

1, 4, 7, 10, 13, ...

1, 2, 4, 8, 16, ...

1, 3, 7, 13, 21, ...

Have the students set similar tasks for their peers.

Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

What other numbers belong?

Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

Cards, each with some terms of a sequence displayed in random order

Blank cards

Display *Card 1* and say, *The numbers of a sequence have been jumbled on this card. What other numbers could be on this card? How do you know?*

Repeat the activity for *Card 2*.

Card 1

31	19	7
37	1	
13		25

Card 2

25		5	
30	15		35
20		10	

If the students are unable to suggest additional numbers, have them order the displayed numbers.

Display *Card 3* and ask, *Which number is missing from the sequence? How do you know?*

Card 3

66	36	
56		16
26		6

Provide the students with blank cards and have them create similar problems for others to solve.

Find the missing numbers

Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

None

Write a number sequence on the board, leaving out one or more of the terms.

Have the students identify each missing number and explain their reasoning.

The following are examples of sequences that could be used for this activity:

2, 5, 8, 11, ?, 17, 20

3, 6, 12, ?, 48, 96

20, 19, 17, 14, ?, 5

$\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, ?, $5\frac{1}{2}$

?, 10, 20, 30, 40, 50

$\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, ?, $3\frac{1}{2}$, 4, ?, 5

100, 92, 84, ?, 68, 60, 52

2, 5, 10, 17, ?, 37

Students may find different ways of working out the same missing number.

$$2, 5, 10, 17, ?, 37$$

$$+3 \quad +5 \quad +7 \quad +9 \quad +11$$

$$17 + 9 = 26$$

I count with the next odd number all the time

Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Numeral card patterns

Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

Numeral cards

Select ten numeral cards that are part of a sequence and display them in a line, with one or more of the cards turned face down. For example:

5 10 15 20 30 35 40

Have the students explain how they determined the missing numbers.

Organise the students into groups. Have the students take turns to create similar problems for others in the group to solve.

Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Number line and number chart patterns

Number line patterns

Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

Ten cards of alternating colours each one showing a different decade in the range 1-100

Pegs

Place the ten cards end-to-end in order to form a number line. Clip a peg onto the numeral 4 on the number line and ask the students to place the next peg on the number that is ten more. Ask the students to explain how they determined their answer. Have the students continue adding ten and clipping a peg on each number in the sequence. Discuss the pattern created.

Create another sequence, starting from 4 and adding 20 each time. Compare the pattern created with the previous pattern.



Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

Hundred chart patterns

Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

Two sets of cards of alternating colours each card showing a different decade in the range 1-100

Transparent counters

Place one set of cards in rows under each other to construct a hundred chart and the second set horizontally to form a number line. Discuss the relationship between the number line and the hundred chart.

Have the students identify the sequence of counting by tens and place counters on the corresponding numerals on both the hundred chart and the number line. Compare and discuss the different visual representations of this sequence. Repeat the activity for different sequences.

Other outcomes addressed

NS1.1 Counts, orders, reads and represents two- and three-digit numbers.

Non-standard number chart patterns

Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

Ten cards of alternating colours each one showing a different decade in the range 1-100

Arrange the cards in rows of 20. Have the students discuss any patterns that they are able to identify in this chart.

Focus the students' attention to the column showing the following sequence, 10, 30, 50, 70, 90. Ask the students the following questions.

What are the next three numbers in the sequence?

How are you working them out?

How would you describe this sequence?

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

Repeat the above activity with sequences from other columns in the chart.

Cut each card in half to form strips of five numbers and arrange them in rows one under another. Alternatively, place the cards in four rows, with 1 to 25 in the top row, 26

to 50 in the next, then 51 to 75 in the next and 76 to 100 in the last row. Identify and discuss the visual arrangement of various sequences.

Other outcomes addressed

NS1.1 Counts, orders, reads and represents two- and three-digit numbers.

Square number charts

Key idea

Create, represent and continue a variety of number patterns and supply missing elements

Materials

2 cm grid paper

Have the students use the grid paper to create square number grids of different sizes. For example, a *3 by 3* number grid showing the numbers from 1 to 9, or a *4 by 4* grid showing the numbers from 1 to 16.

Ask the students to colour every even number one colour and every odd number a second colour.

1	2	3
4	5	6
7	8	9

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Ask the students the following questions:

Which grids result in a checked pattern?

Which grids result in a striped pattern?

Why?

Equality

Number posters

Key idea

Use the equals sign to record equivalent number relationships

Materials

Large sheets of card or paper

Have the students make posters to display different ways of expressing a specific number. These could be added to over time. This is the same task as described in Early Stage 1, *Equal groups* p. 32. However, this example includes the use of formal symbols to record number expressions.

$$\begin{array}{c} 5 + 5 + 1 \\ 12 - 1 \quad 6 + 5 \\ \boxed{11} \\ 20 - 9 \quad 10 + 1 \\ 2 + 2 + 2 + 2 + 2 + 1 \end{array}$$

Have the students use the posters to record equivalent number relationships.

Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Commutativity

Turn around number facts

Key ideas

Use the equals sign to record equivalent number relationships

Build number relationships by relating addition facts to at least 20

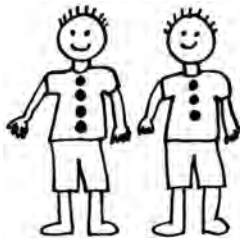
Materials

None

Have the students build addition and subtraction facts to at least 20 by answering questions such as, *What is $17 + 2$? ... If $17 + 2$ is 19, then what is $2 + 17$? How do you know?*

On the number posters from the previous activity, record pairs of addition facts which show the commutative property. For example, $6 + 5 = 11$ and $5 + 6 = 11$.

Set the students the task of drawing a picture that illustrates a pair of addition facts. The example below shows 4 + 3 buttons and it also shows 3 + 4 buttons, depending on which person's buttons you count first.



Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

Rearranging rectangles

Key idea

Use the equals sign to record equivalent number relationships

Materials

Grid paper
Square tiles

Place 16 tiles in a group on the floor and ask a student to use the tiles to create a rectangle. Ask:

How many rows are there?

How many tiles are there in each row?

How many tiles are there altogether?

How could we write this using words?

How could we write this using symbols?

What other rectangles could we make using the same number of tiles?

Repeat the questioning for each new rectangle created, recording the answers.

What do you notice about the number of tiles in all of these rectangles?

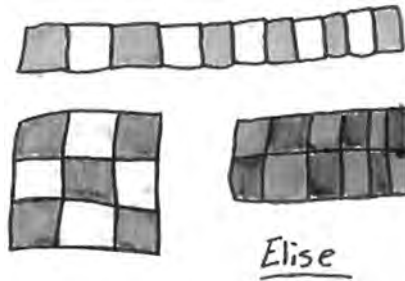
Select two of the rectangles created.

Ask,

What could we say about the total number of tiles in each of these two rectangles?

How could we write this using symbols?

Provide each student with 12 square tiles to investigate the different rectangles that can be created. Have the students use grid paper to represent the rectangles that they have created and record accompanying number sentences. Have the students share their investigations. Allow the students to repeat the activity using a different number of tiles.



Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Teddy groups

Key idea

Use the equals sign to record equivalent number relationships

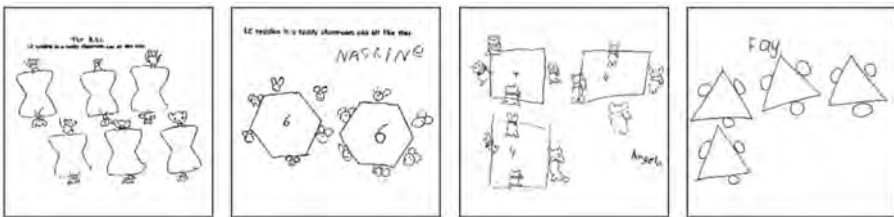
Materials

Triangle, square, hexagon, trapezium pattern blocks
Teddy counters

Provide groups of students with 24 teddies. Have the students investigate in how many different ways the teddies could be grouped so that there is an equal number of teddies in each group. Have the students record and share their findings.

As a variation, provide the students with pattern blocks and tell them that the blocks represent tables for the teddies to sit at.

These are the drawings recorded by a group of students who worked with 12 teddies.



Discuss the students' findings. Compare the total number of teddies in each arrangement and discuss how the equivalence could be recorded using symbols.

Other outcomes addressed

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division.

Is it true?

Key ideas

Use the equals sign to record equivalent number relationships

Build number relationships by relating addition and subtraction facts to at least 20

Materials

None

Present a list of equations such as the following to the students and ask them to identify which are correct.

$$14 + 2 = 2 + 14$$

$$18 + 3 = 3 + 18$$

$$56 + 79 = 79 + 56$$

Have the students explain the strategies that they used to determine if the equations were correct.

Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

Rainbow relationships

Key ideas

Use the equals sign to record equivalent number relationships

Build number relationships by relating addition and subtraction facts to at least 20

Make generalisations about number relationships

Materials

Coloured felt-tip pens

Ask the students to recall addition facts that equal seven and record these on the board. Discuss how the facts could be recorded to identify all possible solutions.

Record the addition facts in the following order:

$$\begin{array}{l}
 0 + 7 = 7 \\
 1 + 6 = 7 \\
 2 + 5 = 7 \\
 3 + 4 = 7 \\
 4 + 3 = 7 \\
 5 + 2 = 7 \\
 6 + 1 = 7 \\
 7 + 0 = 7
 \end{array}$$

Have the students identify pairs of equations that contain the same numbers and join them with arcs. Discuss the equality between each pair of equations.

Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

Addition and subtraction relationships

Completing number sentences

Key ideas

Use the equals sign to record equivalent number relationships

Build number relationships by relating addition and subtraction facts to at least 20

Materials

Incomplete number sentences on cards or on the board

Write a number sentence involving number bonds on the board such as the following:

$$3 + 5 = 4 + 4$$

Ask the students to write another number sentence where each side equals eight.

Have the students work in pairs to write every number sentence that has sums on both sides equal to eight. Repeat the process starting with a number sentence involving subtraction. For example:

$$5 - 3 = 4 - 2$$

Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

Adding and subtracting zero

Key idea

Make generalisations about number relationships

Materials

None

Record the following number sentences on the board and discuss the answers.

$$4 + 0 =$$

$$4 - 0 =$$

$$999 + 0 =$$

$$999 - 0 =$$

Ask:

Is it true that $7 + 0 = 7 - 0$? Why?

What happens when you add zero to a number?

What happens when you subtract zero from a number?

What happens when you add one to any number?

What happens when you subtract one from any number?

Have the students investigate adding *one* to any even number and adding *one* to any odd number and share their findings. Similarly, investigate adding or subtracting *two* to any number.

Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

Adding and subtracting odd and even numbers

Key idea

Make generalisations about number relationships

Materials

None

Have the students investigate the following questions and share their findings with other students:

What happens when you add two even numbers?

What happens when you add two odd numbers?

What happens when you add an even number and an odd number?

What happens when you add three even numbers?

What happens when you add three odd numbers?

What happens when you find the difference between two even numbers?

What happens when you find the difference between two odd numbers?

What happens when you find the difference between an even number and an odd number?

Other outcomes addressed

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers.

Talking_{about} Patterns
& Algebra

Talking_{about} Patterns & Algebra

Stage 2

3

2

4

8

5

1

Overview of Stage 2 content

PAS2.1 Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values.

In Stage 2, students identify, describe and discuss patterns in sequences of multiples as well as patterns using fractions and decimals. Opportunities should also be provided for students to investigate patterns in number sequences where the differences between consecutive numbers are not the same.

The students investigate relationships among multiplication and division facts as well as relationships between sequences. For example, relating the sequence of multiples of four to the sequence of multiples of eight.

Students build multiplication facts to 10×10 and explore, describe and discuss the associative property of addition and multiplication.



Patterns and Algebra - Stage 2

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The shape of number sequences

- | | |
|--------------------|-----|
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Investigating sequences of multiples

Generating sequences by counting

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

Copies of BLM *1–100 chart* (p. 150)

Have the students generate the sequence for multiples of four by silently counting three numbers and then counting out loud every fourth number to 40. Repeat the activity and have the students record the numbers that are said aloud. Tell the students that the numbers recorded are the multiples of four.

Organise the students into pairs and give each pair a *Hundred chart*. Ask the students to colour in each multiple of four.

Repeat this activity for other multiples.

Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

Generating sequences by making shapes

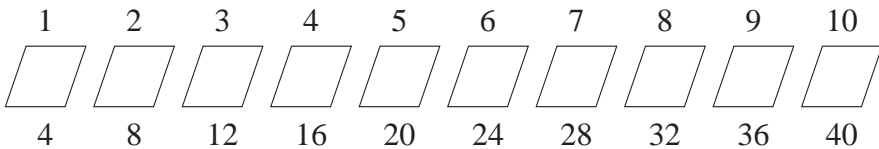
Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

Popsticks or matchsticks

Provide the students with popsticks or matchsticks and ask them to use four sticks to create a rhombus. Have the students record the number of rhombuses created and the total number of sticks used. Have the students repeat the process to create nine additional rhombuses.



After completing the task have the students discuss how the sequence of multiples could be continued without making the shapes.

Ask:

Can you work out how many popsticks you would need if you wanted to make 15 rhombuses?

How could you work this out?

If I used 80 popsticks, how many rhombuses could I make?

Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

Generating whole number sequences on a calculator

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

Calculators with a constant function

Have the students use the constant function on a calculator to generate the first nine multiples of 37. Record these multiples in the first row of a table on the board.

Repeat the process to generate the next nine multiples and record these in the second row of the table.

Organise the students into pairs and have them copy the table onto paper. Then have them continue the sequence to 999, using the constant function on a calculator. This is done by keying in 37, then $\boxed{+}$, then repeatedly pressing the $\boxed{=}$ key.

37	74	111	148	185	222	259	296	333
370	407	444	481	518	555	592	629	666
703	740	777	814	851	888	925	962	999

Ask:

What patterns can you see?

Look at the third column. What is the difference between the first two numbers in the column and between the second and third numbers in the column?

Why does this occur?

Does this occur elsewhere? Why?

What do you think will be the next number in the table?

Have the students determine and record the next nine multiples in the sequence.

Ask: *What do you notice about the pattern?*

Other outcomes addressed

NS2.2 Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers.

Investigating sequences of multiples

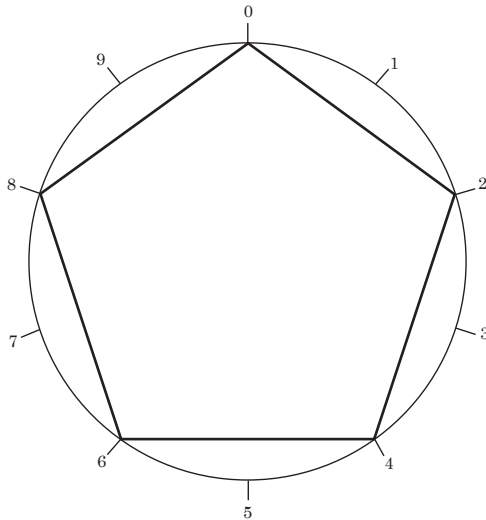
Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

A 0-9 wheel drawn on the board
Copies of BLM, 0-9 wheel (p. 81)

Have the students generate the sequence of multiples of two and record these on the board. Focus the students' attention to the last digit in each number in the sequence. On a 0-9 wheel, start from *two* and join in order with a straight line the last digit of each multiple in the sequence.

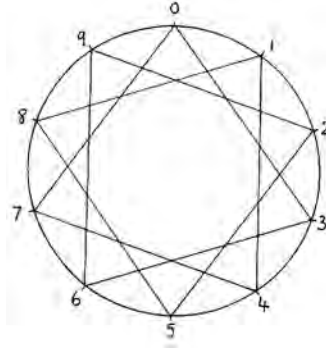


Ask:

What shape has been created?

What do you notice about the numbers in the sequence?

Have the students work in pairs to repeat the activity for the multiples of three.



Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

Relating sequences of multiples

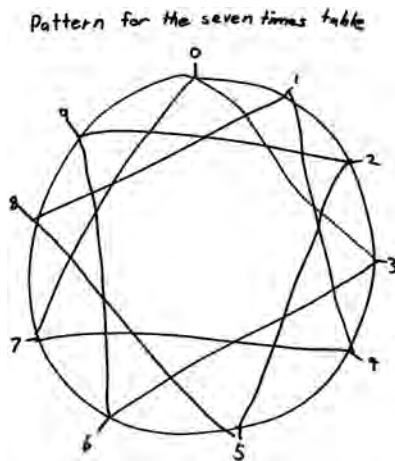
Key idea

Build number relationships by relating multiplication and division facts to at least 10×10

Materials

Copies of BLM, *0-9 wheel* (p. 81)

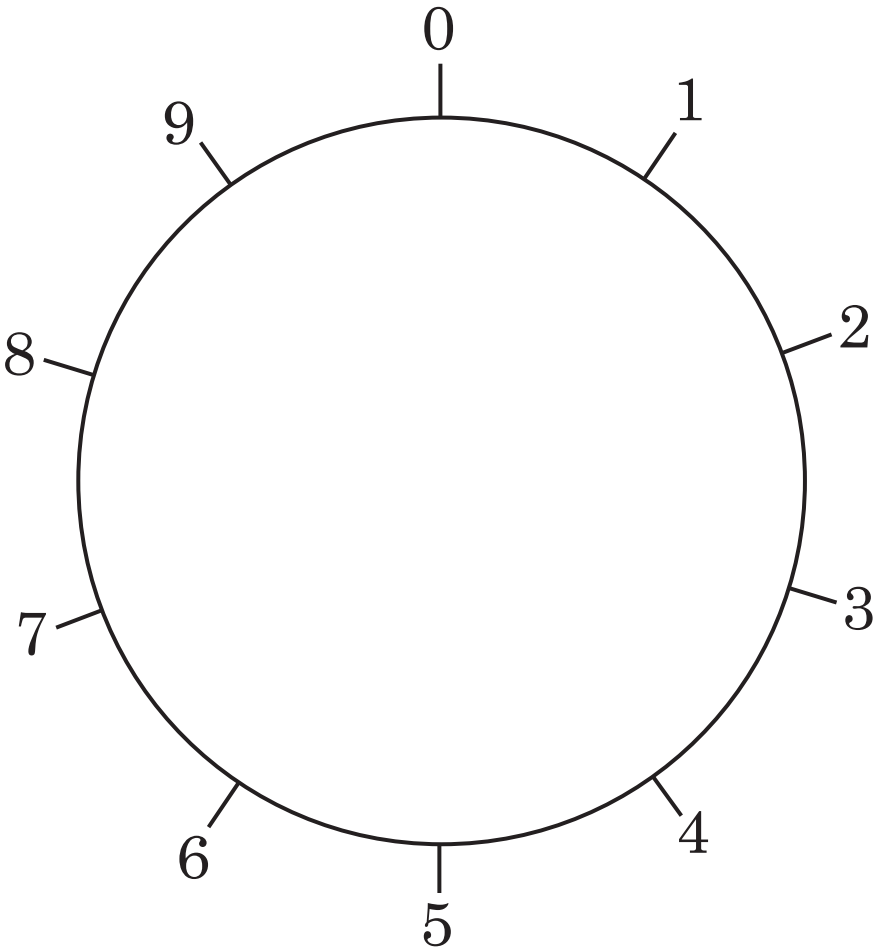
Have the students investigate the relationships among the patterns created on the *0-9 wheel* by the final digits in different sequences of multiples.



Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

0-9 wheel



BLM - Stage 2

Investigating repeated subtraction

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

None

Ask:

What are the numbers that could be repeatedly subtracted from 30 to end at zero?

How will you know that you have found all possible solutions?

Have the students investigate the problem starting from different numbers.

Other outcomes addressed

NS2.2 Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers.

Investigating additive sequences

Repeatedly adding five

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

None

Have the students investigate the sequences created by repeatedly adding *five* to a number of their choice. Have the students record the first ten terms of the sequence in a table.

For example:

1st number	2nd number	3rd number	4th number	5th number	6th number	7th number	8th number	9th number	10th number
6	11	16	21	26	31	36	41	46	51

After the students have completed the task, have them cover the first three numbers in the sequence on their table, then swap with a partner and ask, *What is the starting number of this sequence?*

Have the students cover one number in the sequence and then ask a partner to identify this number.

Have the students repeat the activity repeatedly adding ten to a number of their choice.

Other outcomes addressed

NS2.2 Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers.

Calendar patterns

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

Numeral cards with numerals from 1 to 31
Days of the week cards

Have the students seated in a semi-circle and distribute the numeral cards and the *Days of the week* cards in random order.

Have the students holding the *Days of the week* cards determine the correct placement of their cards to construct on the floor a calendar for one month.

Tell the students that the calendar month has 31 days and that the 15th of the month falls on a Saturday.

Have the student holding the numeral card for 15 place it on the floor to begin the construction of the calendar. Then randomly select students to take turns to place a card and justify the placement.

When the calendar is complete, have the students investigate patterns by asking them questions such as the following:

What diagonal patterns can you find?

Choose three adjacent numbers, horizontally, vertically or diagonally. What is the relationship between the sum of the three numbers and the centre number?

What is the relationship between five consecutive numbers and the centre number?

If the total of the three adjacent numbers is 51, what are the numbers? How did you work that out?

Draw a frame around four numbers that form a square, such as 2, 3, 9, 10. If you add the numbers in pairs diagonally, what do you find? Does a similar relationship apply for larger boxes?

SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

Other outcomes addressed

NS2.2 Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

Generalising about numbers

Yes or no game

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

None

Divide the board into two sections. Write *Yes* at the top of one section and *No* at the top of the other. Tell the students that you are thinking of a sequence of multiples and ask them to suggest numbers that belong to the sequence.

Record correct suggestions in the *Yes* section of the board and incorrect suggestion in the *No* section. When several numbers are recorded in the *Yes* section, ask the students to determine the correct sequence of multiples. Then ask, *Could it be a different sequence of multiples?* Continue until the students have correctly determined the sequence.

Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

Repeated multiplication

Key idea

Generate, describe and record number patterns using a variety of strategies

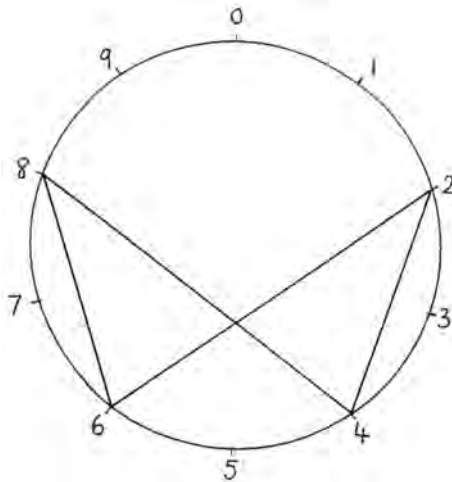
Materials

Calculator

Copies of BLM *0–9 wheel* (p. 81)

Have the students enter the number *two* on a calculator and then repeatedly multiply by two. Record the first ten terms of the sequence on the board. This will generate the sequence 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024.

Focus the students' attention the last digit in each number in the sequence. On a *0–9 wheel*, start from two and join in order with a straight line the last digit of each number in the sequence.



Ask, *Why are some numbers on the wheel not part of the pattern?*

Have the students repeat the activity starting with the number *three* and repeatedly multiplying by three.

Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

Sequences involving fractions and decimals

A decimal pattern

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

None

Write the following sequence on the board: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.

Have the students use a calculator to divide each of the numbers by *four* and record the resulting numbers.

Ask, *Is there a relationship among these numbers? What is the relationship?*

Other outcomes addressed

NS2.4 Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages.

Sequence of mixed numerals

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

None

Write the following sequence of numbers on the board and ask the students to describe it:

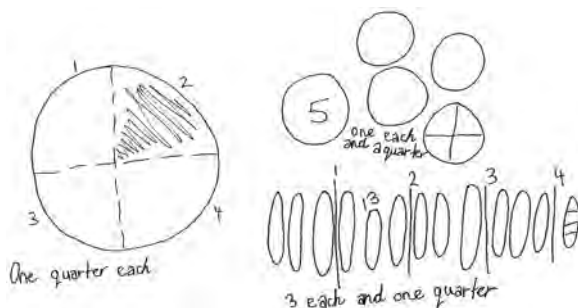
1, 5, 9, 13, 17, 21.

Ask, *Is there a relationship among these numbers? What is the relationship?*

Present the following problem to the students:

Four people are to share 1 pizza, 5 cupcakes, 9 sausages and 13 jelly snakes. What would each person have if they shared everything fairly and there was nothing left over?

Have the students draw and present their solutions.



Have the students write as a sequence the amounts that each person will have.

$\frac{1}{4}$, $1\frac{1}{4}$, $2\frac{1}{4}$, $3\frac{1}{4}$

Ask:

Is there a relationship among these numbers? What is the relationship?

How are the numbers in this sequence related to the numbers in the initial sequence?

Other outcomes addressed

NS2.4 Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages.

Generating decimal number sequences on a calculator

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

Calculators

Have the students use the constant function on a calculator to repeatedly add 0.2. Have the students record the sequence generated.

Ask, *How is this sequence related to the sequence of multiples of two?*

Have the students predict and verify what the sequence generated by repeatedly adding 0.3 will be?

Other outcomes addressed

NS2.4 Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages.

Properties of addition and multiplication

Addition strategies

Key idea

Use a range of mental strategies for addition

Materials

Cards displaying three or four numerals

Show the following card to the students:

70	
	80
20	

Ask the students what is the total of all of the numbers on the card and ask them to explain their strategies for solving the addition. Ask, *Why can we add the numbers in different ways and still get the same answer?*

Repeat the task using numbers such as the following:

	54
	46
36	

	61
17	
	9

82	
	18
22	
	43

	26
13	
	38
	5

Other outcomes addressed

NS2.2 Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers.

Multiplication strategies

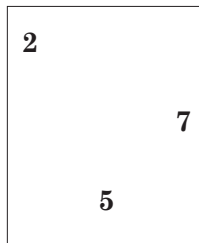
Key idea

Build number relationships by relating multiplication and division facts to at least 10×10

Materials

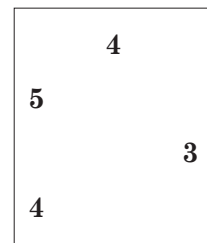
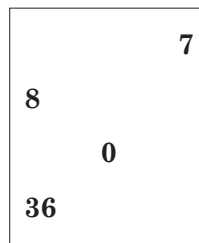
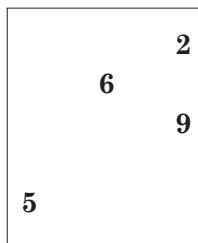
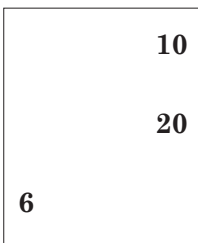
Cards displaying three or four numerals

Show the following card to the students:



Ask the students what the result is when multiplying all of the numbers on the card and ask them to explain their strategies for solving the problem. Ask, *Why can we multiply the numbers in different ways and still get the same answer?*

Repeat the task using numbers such as the following:



Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

Investigating number relationships

Which of these are true?

Key idea

Build number relationships by relating multiplication and division facts to at least 10×10

Materials

None

Write the following number sentences on the board and ask the students to say whether each statement is true and to justify their answers.

$$4 \times 3 = 6 \times 2$$

$$4 \times 0 = 8 \times 0$$

$$6 \times 7 = 7 \times 6$$

$$6 \times 2 \times 5 = 2 \times 5 \times 6$$

$$13 + 11 = 12 + 12$$

$$18 + 27 = 17 + 28$$

$$16 + 9 = 9 + 16$$

$$14 + 18 + 20 = 20 + 14 + 18$$

Other outcomes addressed

NS2.2 Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers.

NS2.3 Uses mental and informal written strategies for multiplication and division.

Array slides

Key idea

Build number relationships by relating multiplication and division facts to at least 10×10

Materials

An enlarged copy of BLM *10 × 10 array* (p. 98)

Sheets of paper

Copies of BLM *10 × 10 array* (p. 98)

Cover the enlarged copy of BLM *10 × 10 array* with two sheets of paper. Slide the sheets of paper to reveal a 5×4 array.

Ask the students to identify the number of rows, the number of dots in each row and the total number of dots in the array.

Draw a retrieval chart as shown below on the board. Have the students record on the chart information and number sentences to describe the array.

Number of rows	Number of columns	Multiplication facts	Division facts
5	4	$5 \times 4 = 20$ $4 \times 5 = 20$	$20 \div 5 = 4$ $20 \div 4 = 5$

Use the covering sheets to keep the number of rows in the array constant while changing the number of dots in each row. Have the students record information for the new arrays on the retrieval chart.

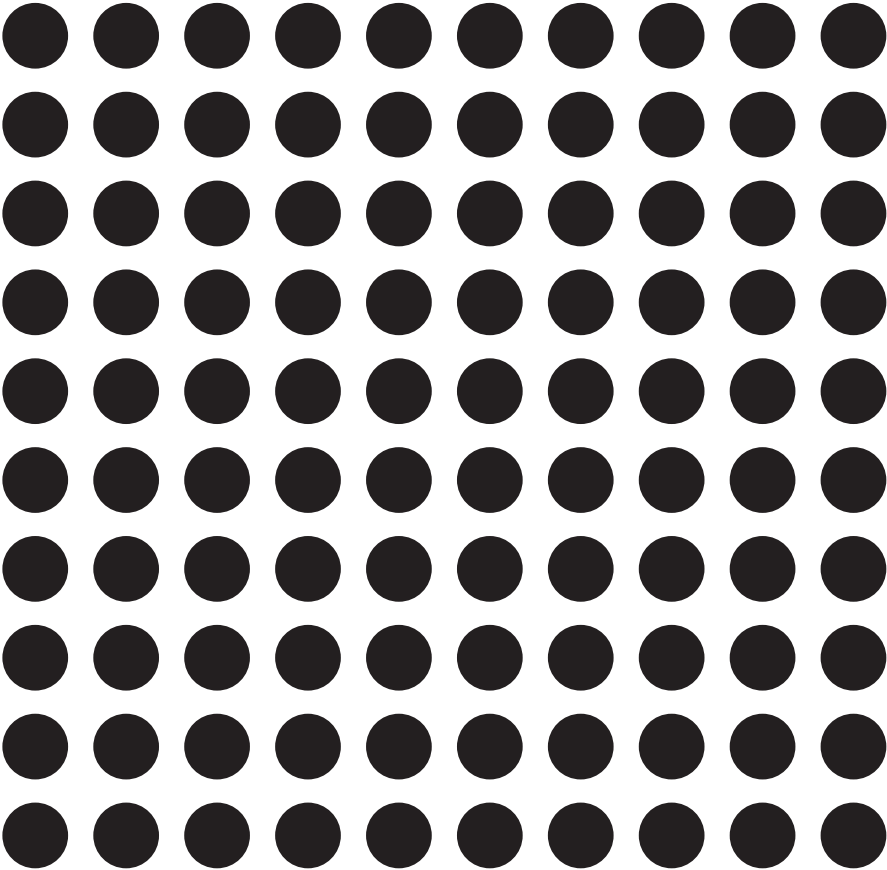
Use the covering sheets to keep the number of dots in each row of the array constant while changing the number of rows. Have the students record information for the new arrays on the retrieval chart.

Provide each pair of students with a copy of BLM 10×10 array and two cover sheets. Have the students investigate other arrays and record information about these on a retrieval chart.

Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

10 x 10 array



BLM - Stage 2

Related multiplication and division facts

Key idea

Build number relationships by relating multiplication and division facts to at least 10×10

Materials

Blank cards

Present to the students a set of related numbers sentences such as the following.

$$3 \times 17 = 51$$

$$17 \times _ = 51$$

$$51 \div 3 = _$$

$$51 \div 17 = _$$

Have the class identify the missing numerals and justify their answers.

Ask, *What can you tell me about these number sentences?*

Have the students record on cards different related number sentences with missing numerals. The students then swap the cards for others to determine the missing numerals.

Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

Investigating addition and subtraction facts

Key idea

Complete simple number sentences by calculating the value of a missing number

Materials

None

Write the following incomplete number sentences on the board.

$$\begin{aligned} 1 &= 7 + \square \\ 21 &= 17 + \square \\ 31 &= 27 + \square \\ 41 &= 37 + \square \\ 51 &= 47 + \square \end{aligned}$$

Have the students identify the missing numerals and discuss their strategies for doing this.

Repeat the activity with other examples such as the following:

$$\begin{aligned} 8 + \square &= 11 \\ 80 + \square &= 110 \\ 800 + \square &= 1100 \\ 8000 + \square &= 11\,000 \end{aligned}$$

$$\begin{aligned} \square - 8 &= 17 \\ \square - 8 &= 27 \\ \square - 8 &= 37 \\ \square - 80 &= 170 \\ \square - 80 &= 270 \\ \square - 80 &= 370 \end{aligned}$$

$$\begin{aligned} 73 - 17 &= \square \\ \square + 17 &= 73 \\ 730 - 170 &= \square \\ \square + 170 &= 730 \end{aligned}$$

Have the students create similar examples for others to solve.

Other outcomes addressed

NS2.2 Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers.

Investigating multiplication and division facts

Key idea

Complete simple number sentences by calculating the value of a missing number

Materials

None

Write the following incomplete number sentences on the board.

$3 \times \square = 21$ $3 \times \square = 210$ $3 \times \square = 2100$
--

Have the students identify the missing numerals and discuss their strategies for doing this.

Repeat the activity with other examples such as the following:

$6 \times \square = 42$ $6 \times \square = 420$ $6 \times \square = 4200$	$7 \times \square = 56$ $8 \times \square = 56$ $56 \div \square = 7$ $56 \div \square = 8$	$8 \div \square = 4$ $80 \div \square = 40$ $800 \div \square = 400$
--	--	--

Have the students create similar examples for others to solve.

Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

The shape of number sequences

Square numbers

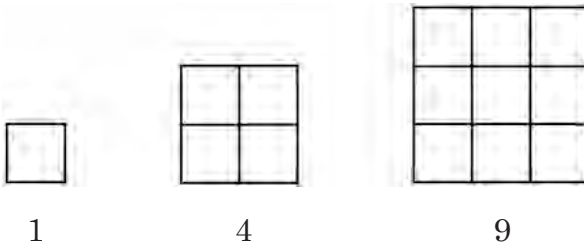
Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

Square tiles
Square grid paper
Calculators

Model the first three square numbers with square tiles and record the number of tiles used for each model.



Ask:

Why do you think these numbers are called square numbers?

How could you describe these arrays?

How could we record these arrays as number sentences?

Provide the students with grid paper and ask them to copy and continue the sequence.

Ask:

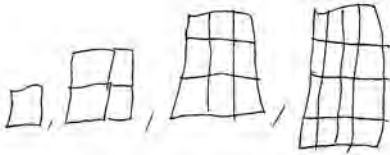
How would you describe the number sequence?

How can you use a calculator to find the first ten square numbers?

Provide the students with calculators and ask them to solve problems such as:

What is the fifteenth square number?

What is the next square number after 289?



I add another row and column.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144
We get a number and times itself.

Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

Cubic numbers

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

Calculators

Interlocking cubes

Use interlocking cubes to make a $3 \times 3 \times 3$ cube.

Ask:

How many interlocking cubes are in this model?

How do you know?

What is this three-dimensional object?

If we call nine a square number, what can we call twenty seven?

What are the previous cubic numbers in this sequence?

Have the students construct models.

Have a student construct a model of the next cubic number in the sequence.

Ask:

How many cubes are there in this model?

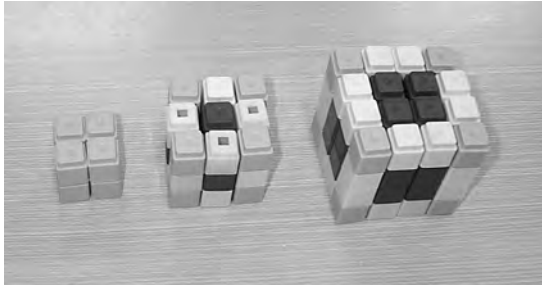
How do you know?

What number sentence could we write to describe each model?

Provide the students with calculators and ask them to solve problems such as:

Determine the twelfth cubic number.

Determine the twentieth cubic number.



Other outcomes addressed

NS2.3 Uses mental and informal written strategies for multiplication and division.

MS2.3 Estimates, measures, compares and records volumes and capacities using litres, millilitres and cubic centimetres

Talking_{about} Patterns
& Algebra

Talking_{about} Patterns & Algebra

Stage 3

3

2

4

8

5

1

Overview of Stage 3 content

PAS3.1a Records analyses and describes geometric and number patterns that involve one operation using tables and words.

PAS3.1b Constructs verifies and completes number sentences involving the four operations with a variety of numbers.

In Stage 3 students create patterns of multiples using whole numbers and decimals. They complete a table of values for patterns and use this to determine a rule. Students describe number sequences in different ways and make generalisations about them.

The syllabus for Stage 3 provides opportunities for students to determine a rule to describe a pattern involving only one operation and using the rule to determine a higher term.

The syllabus for Patterns and Algebra in Stage 3 prepares students for the use of algebraic symbols in Stage 4. Algebraic symbols should not be introduced until the students have had considerable experience describing patterns in their own words.

Patterns and Algebra - Stage 3

Recording number sequences in tables

- | | |
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| 1. Introducing tables | 110 |
| 2. What's this table? | 112 |
| 3. Houses | 114 |

Generalising about number relationships

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Number sequences and relationships

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| 6. What is the same about all of these numbers? | 119 |
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Completing number sentences

- | | |
|---------------------------------|-----|
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Lesson breakers

135

Recording number sequences in tables

Introducing tables

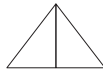
Key idea

Build simple geometric patterns involving multiples
Complete a table of values for geometric and number patterns
Describe a pattern in words in more than one way

Materials

None

Draw the following diagram on the board and ask the students, *How many triangles are there?*



Record the number three on the board above the diagram. Draw a second diagram next to the first one and ask, *How many triangles are there altogether?* Record the number six above the second diagram.

Demonstrate the use of a table to record the first two terms of the number sequence.

Have the students work in pairs to determine and record the first ten terms of the sequence in a table. Ask the students to describe in different ways the sequence of numbers that has been generated.

Ask questions such as the following:

How many triangles would there be if there were 20 diagrams?

If there are 99 triangles, how many diagrams are there?

Have the students explain how they worked out the answers to these questions.

Other outcomes addressed

NS3.3 Selects and applies appropriate strategies for multiplication and division.

What's this table?

Key idea

Build simple geometric patterns involving multiples
 Complete a table of values for geometric and number patterns
 Describe a pattern in words in more than one way

Materials

None

Draw the following table on the board, leaving out the title and row headings. Ask the students to suggest titles for the table and headings for the rows and to justify their suggestions.

	1	2	3	4	5	6	7	8	9	10
	7	14	21	28	35	42	49	56	63	70

Row headings in the above table could include:

Number of weeks and Number of days

Number of heptagons and Number of sides

Number and Number multiplied by 7.

Have the students create their own tables, omitting headings, for others to complete.

Repeat the activity with sequences of decreasing numbers.

For example:

Position	1	2	3	4	5	6	7	8	9	10	11	12
Number	19	18	17	16								

Other outcomes addressed

NS3.3 Selects and applies appropriate strategies for multiplication and division.

Houses

Key idea

Build simple geometric patterns involving multiples
Complete a table of values for geometric and number patterns
Describe a pattern in words in more than one way

Materials

None

Draw the following diagram of a house on the board and ask the students, *How many lines are there?*



Record the number six below the diagram. Draw another house beside the first one and ask, *If I drew two houses, how many lines would I draw altogether?* Record the number 12 below the second diagram.

Have the students continue the sequence to the tenth term and record it in a table.

For example:

Number of houses	1	2	3	4	5	6	7	8	9	10
Number of lines	6	12	18	24	30	36	42	48	54	60

Ask the students to describe the sequence in different ways.

Ask questions such as the following:

How many lines would there be if there were 25 houses? How did you work it out?

If there were 330 lines altogether, how many houses would there be? How did you work it out?

How does the table help determine the relationship between the number of houses and the number of lines?

Have the students investigate number sequences based on the total number of sides of repeating two-dimensional shapes and record the data in a table. For example the following letters could be investigated:



Hexagons



Octagons



Decagons



Dodecagons

Other outcomes addressed

NS3.3 Selects and applies appropriate strategies for multiplication and division.

Generalising about number relationships

Subtraction patterns

Key idea

Describe a pattern in words in more than one way

Materials

Numeral cards 0-9

Paper

Display the ten numeral cards in order along the board. Select the numeral cards displaying *six* and *three*, show them to the students and ask, *What two-digit numbers can be made with these cards?*

Have the students subtract the smaller of the two two-digit numbers from the larger one and record the vertical algorithm and result on the board.

Repeat the activity using the numeral cards *eight* and *five* and then using the numeral cards *three* and *zero*.

Have the students discuss the results.

Have the students repeat the process with two other pairs of numbers, recording the results on paper.

Have the students display their algorithms and then group them according to their results.

Ask:

What patterns can you see in the results of the subtractions?

What sort of numbers produce nine as a result?

What sort of numbers produce 18 as a result?

Who can write a subtraction that has 36 as its answer?

How could you state in words what the pattern for the answers is?

$$\begin{array}{r} 63 \\ -36 \\ \hline 27 \\ \hline \end{array} \quad \begin{array}{r} 98 \\ -89 \\ \hline 9 \\ \hline \end{array}$$

$$\begin{array}{r} 53 \\ -35 \\ \hline 18 \\ \hline \end{array} \quad \begin{array}{r} 72 \\ -27 \\ \hline 45 \\ \hline \end{array}$$

$$\begin{array}{r} 41 \\ -14 \\ \hline 27 \\ \hline \end{array} \quad \begin{array}{r} 32 \\ -23 \\ \hline 9 \\ \hline \end{array}$$

$$\begin{array}{r} 86 \\ -68 \\ \hline 18 \\ \hline \end{array} \quad \begin{array}{r} 75 \\ -57 \\ \hline 18 \\ \hline \end{array}$$

$$\begin{array}{r} 84 \\ -48 \\ \hline 36 \\ \hline \end{array}$$

All the answers are a multiple of 9.
Also some answers are the same.

What am I doing to the numbers?

Key idea

Describe a pattern in words in more than one way

Materials

None

Ask a student to suggest a number between 1 and 20 and write it on the board. Without telling the rule to the students, multiply the number by three then draw an arrow from the numeral and record the result at the end of the arrow. Repeat the process for other suggested numbers.

For example:

$$7 \longrightarrow 21$$

$$3 \longrightarrow 9$$

$$20 \longrightarrow 60$$

$$15 \longrightarrow 45$$

$$4 \longrightarrow 12$$

Ask the students to identify what operation was performed on the suggested numbers.

Repeat the activity by having students select a new rule.

Other outcomes addressed

NS3.3 Selects and applies appropriate strategies for multiplication and division.

Number sequences and relationships

What is the same about all of these numbers?

Key idea

Describe a pattern in words in more than one way

Materials

None

Record the following numerals on the board in random order.
For example:

	22	99
		33
55		66
		88
	11	
		44
	77	

Have the students write the numbers in ascending order.

Ask:

What is the same about all of these numbers?

What is the rule for this sequence of numbers?

What would be the twentieth number in the sequence? How did you work it out?

Present the students with different sequences of numbers including decimals and ask similar questions.

For example:

	0.009	
		9000
90		0.09
	9	
900		0.9

	1	$3\frac{1}{4}$
		$5\frac{1}{2}$
4		$1\frac{3}{4}$
	$4\frac{3}{4}$	$2\frac{1}{2}$

Other outcomes addressed

NS3.4 Compares, orders and calculates with decimals, simple fractions and simple percentages

What number is missing?

Key idea

Describe a pattern in words in more than one way

Materials

None

Record the following numerals on the board in random order.
For example:

	24		74
		54	
	94		
84		34	
	44		
		104	
	14		

Tell the students that these numbers are part of a sequence. Have the students order the numbers to identify the missing term. Then continue the sequence to identify the first 15 terms.

Have the students share their results and strategies for determining the terms of the sequence.

Repeat the activity with other sequences.

For example:

		81	
9	63		
			18
	54		
		45	
	27		
90	36		

	3.5		8.5
		1.5	
			9.5
	6.5		
		4.5	
	7.5		0.5
		2.5	

		2		
3.5			1.5	
				5
		4		
			0.5	
3				
	1		2.5	

Other outcomes addressed

NS3.4 Compares, orders and calculates with decimals, simple fractions and simple percentages.

Which number doesn't belong?

Key idea

Describe a pattern in words in more than one way

Materials

None

Record the following numerals on the board in random order.
For example:

	15		
		32	
	25		
		30	
70			5
		50	

Tell the students that all but one of these numbers are part of a sequence.

Ask:

How could you describe this sequence of numbers?

Which number doesn't belong to the sequence? Why?

Repeat the activity with other sequences.

For example:

		24	
96			
		32	
16			
	18		
40		28	

	14		108
26		38	
		15	
12			52
	86		

	1		36
		25	
			2
16			
		4	
100			

Continuing number sequences

Key idea

Describe a pattern in words in more than one way

Materials

None

Record the following numerals on the board: 10, 20.

Have the students suggest different sequences of which these numbers could be the first two terms. For example, the sequence could be 10, 20, 30, 40 ... or it could be 10, 20, 40, 80 ...

Repeat the activity using other numbers. For example, 1, 2, 4 ... could be the first three terms of the sequence 1, 2, 4, 8, 16 ... or of the sequence 1, 2, 4, 7, 11 ...

Have the students write numbers which could be the first terms of different sequences and have other students determine the sequences.

Area sequences

Key idea

Build simple geometric patterns involving multiples
Describe a pattern in words in more than one way

Materials

1 cm grid paper
Rulers

Provide the students with 1 cm grid paper. Have the students draw rectangles with dimensions of $1\text{ cm} \times 3.5\text{ cm}$, $2\text{ cm} \times 3.5\text{ cm}$, $3\text{ cm} \times 3.5\text{ cm}$ and $4\text{ cm} \times 3.5\text{ cm}$. Have the students determine and record in ascending order the area of each rectangle.

Ask:


What similarities and differences do you notice about these rectangles?


What do you notice about the area of these rectangles in relation to the length of their sides?


Focus the students' attention to the sequence of the areas of the rectangles.


How are the numbers in this sequence increasing?


Have the students create sequences of multiples for other decimal numbers.

1.  $1\text{cm} = 3.5\text{cm}^2$

2.  $2\text{cm} = 7\text{cm}^2$

3.  $3\text{cm} = 10.5\text{cm}^2$

4.  $4\text{cm} = 14\text{cm}^2$

5.  $5\text{cm} = 17.5\text{cm}^2$

They are all multiples of three point five. One side is always three point five

Other outcomes addressed

NS3.4 Compares, orders and calculates with decimals, simple fractions and simple percentages.

MS3.2 Selects and uses the appropriate unit to calculate area, including the areas of squares, rectangles and triangles.

Decimal sequences

Key idea

Complete a table of values for geometric and number patterns
Describe a pattern in words in more than one way

Materials

Calculators

In an untitled table, record the following sequence of numbers:

Term in the sequence	1	2	3	4	5
	0.22	0.44	0.66	0.88	

Ask the students to determine the fifth number in the sequence, then check the answer using a calculator. Have the students share their strategies for determining the answer and discuss any errors.

Ask the students to label the second row in the table and give the table a title. Ask them to determine a rule for the relationship between the top and bottom rows and then to use the rule to find the tenth term in the sequence.

Have the students generate other sequences of decimals, using a calculator to check their calculations.

Position Number	1	2	3	4	5
Sequence of decimals	0.22	0.44	0.66	0.88	1.1

The tenth number would be 2.2 because I worked it out by xing 0.22 by 10

Other outcomes addressed

NS3.4 Compares, orders and calculates with decimals, simple fractions and simple percentages.

Polygons and triangles

Key idea

Complete a table of values for geometric and number patterns

Describe a pattern in words in more than one way

Materials

None

Have the students investigate and discuss the least number of triangles needed to form a quadrilateral. Have the students continue their investigations with other polygons.

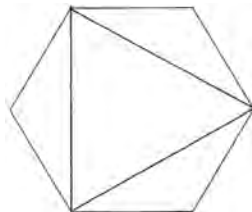
Ask the students to record their findings in a table, beginning with a triangle.

For example:

Forming polygons from triangles

Number of sides	3	4	5	6	7	8	9
Number of triangles	1	2	3	4	5	6	7

If the students are having difficulty with the task, suggest that they use the diagonals of each shape.



Using the tables that the students have created, ask them to describe the relationship between the number of sides of a polygon and the least number of triangles needed to form it.

Polygons and triangles

Number of sides	4	5	6	7	8	9
Number of triangles	2	3	4	5	6	7

square pentagon hexagon heptagon

Other outcomes addressed

SGS3.2a Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties.

Completing number sentences

Incomplete number sentences

Key idea

Construct, verify and complete number sentences involving the four operations with a variety of numbers

Materials

None

Write the following series of incomplete number sentences on the board and have the students complete them.

$5 + _ = 15 - 5$ $6 + _ = 15 - 4$ $7 + _ = 15 - 3$ $8 + _ = 15 - 2$ $9 + _ = 15 - 1$

Discuss the patterns and relationships present in these number sentences.

Repeat the activity for the following incomplete number sentences:

$2 \times 3 = 4 + _$ $2 \times 4 = 4 + _$ $2 \times 5 = 4 + _$ $2 \times 6 = 4 + _$ $2 \times 7 = 4 + _$

$8 \times _ = 8.8$ $8.8 \div 8 = _$ $_ \times 2.2 = 17.6$ $17.6 \div _ = 2.2$ $17.6 \div 2.2 = _$
--

Discuss strategies for verifying the answers.

Present a series of number sentences where the operation signs, rather than the numbers, are missing, such as the following:

$6 _ 2 = 4 _ 4$
$6 _ 2 = 4 _ 3$
$6 _ 2 = 4 _ 2$
$6 _ 2 = 4 _ 1$

Have the students create similar problems for others to solve.

Other outcomes addressed

NS3.3 Selects and applies appropriate strategies for multiplication and division.

NS3.2 Selects and applies appropriate strategies for addition and subtraction with counting numbers of any size.

NS3.4 Compares, orders and calculates with decimals, simple fractions and simple percentages.

Find the hidden numbers


Key idea



Construct, verify and complete number sentences involving the four operations with a variety of numbers


Materials



None

Present pairs of students with the first problem below. Have the students solve the problem, then share and discuss their strategies for determining the answer. Repeat the process with the other problems.

Imagine there is a number hidden under this circle: 
 If I double the hidden number and add 3, the total is 15.
 Can you work out what the hidden number is?

Imagine there is a number hidden under each circle:  
 If I multiply the two numbers, the answer is 42.
 Can you work out what the hidden numbers could be?

Imagine there is a number hidden under this circle: 
 If I subtract 18 from the hidden number and divide by 6, the answer is 4.
 What is the hidden number?

Imagine there is a number hidden under each circle:  
 If I add the two numbers and then multiply the result by 2, the answer is 22.
 Can you work out what the hidden numbers could be?

Imagine there is a number hidden under each circle: ○ ●
If I multiply the two numbers, the answer is 7.
Can you work out what the hidden numbers could be?

Have the students create their own questions for others to solve.

Other outcomes addressed

NS3.3 Selects and applies appropriate strategies for multiplication and division.

Lesson breakers

How many numbers are in this sequence?

How many terms are in each of these sequences if they start and stop with the numbers as shown?

$$3, 6, 9, 12 \dots 60$$

$$10, 20, 30, 40, 50, \dots 140$$

$$4, 8, 12, 16, 20, \dots 424$$

$$45, 40, 35, 30, 25, \dots 5$$

Incomplete number sentences

Have the students provide incomplete number sentences for the whole class to solve.

For example:

$$_ + 6 = _ \times 3$$

$$24 \div _ = \frac{1}{2} \times _$$

$$_ + 11 = _ \times 6$$

$$100 - _ = 12 + _$$

Problem sentences

Ask questions such as the following:

What number is 14 more than the product of 15 and 7?

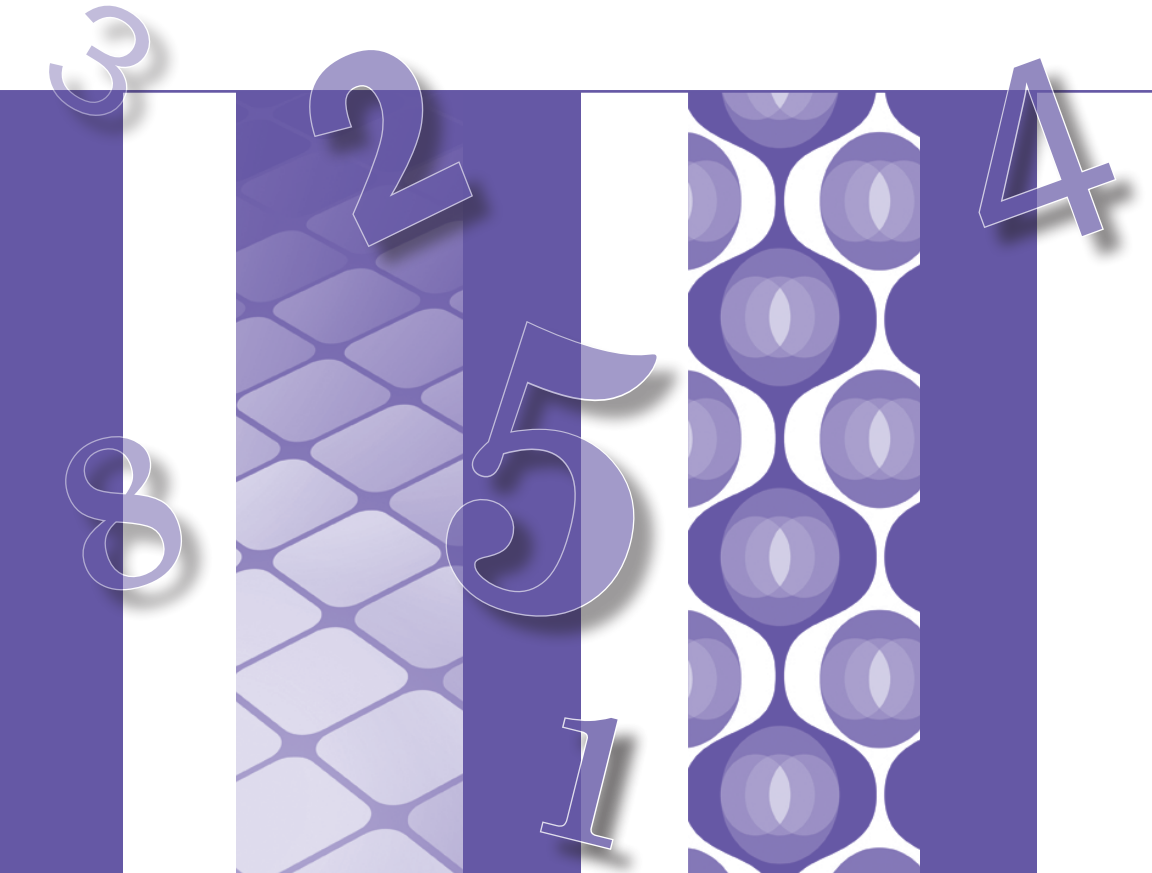
If you add 37 to a number, you get 95. What is the number?

What number is half the sum of 35 and 57?

What number, when divided by six, gives the answer 14?

Talking about Patterns & Algebra

Enrichment



Patterns and Algebra - Enrichment

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Blobs

Key idea

Construct, verify and complete number sentences involving the four operations with a variety of numbers

Materials

None

Have the students identify the number relationship in the following example:

$$7 \longrightarrow 14$$

$$1 \longrightarrow 2$$

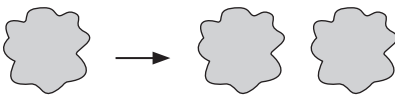
$$19 \longrightarrow 38$$

$$4 \longrightarrow 8$$

$$9 \longrightarrow 18$$

Draw a symbol on the board to represent the starting number and ask, *How could the same number relationship be represented using symbols?*

For example:



Present a different number relationship to the students.

$$5 \longrightarrow 11$$

$$8 \longrightarrow 17$$

$$20 \longrightarrow 41$$

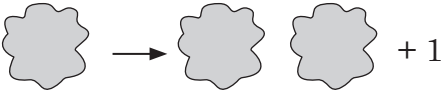
$$11 \longrightarrow 23$$

$$13 \longrightarrow 27$$

$$6 \longrightarrow 13$$

Have the students identify the number relationship and use symbols to represent it.

For example:



How does this work?

Key idea

Construct, verify and complete number sentences involving the four operations with a variety of numbers

Materials

None

Write the following problem on the board:

Think of a number from 1 to 10.
Double it.
Add 10.
Divide by 2.
Subtract the number you started with.
What number do you have?

Have the students suggest a symbol that could be drawn to represent the starting number and then discuss and represent each step of the problem using symbols.

For example:

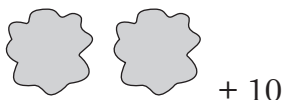
Think of a number from 1 to 10.



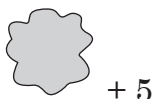
Double it.



Add 10.



Divide by 2.



Subtract the number you started with.

5

Ask the students to discuss with a partner why the answer is always *five*.

Ask, *Will the result change if the number is larger than ten? Why?*

Have the students repeat the process for the following problems.

Think of a number from 1 to 10.

Halve it.

Add 5.

Multiply by 2.

Subtract 10.

What number is left?

Why?

Think of a number from 1 to 100.

Add 20.

Multiply by 4.

Divide by 2.

Subtract 40.

Divide by 2.

What number is left?

Why?

Flying geese

Key idea

Describe a pattern in words in more than one way
Complete a table of values for geometric and number patterns

Materials

Counters

Display the following pattern.



Explain that geese fly in formation to reduce wind resistance when flying over long distances.

Ask:

How would you describe this pattern?

How many dots would there be in the next term of the sequence?

How do you know?

Draw a table of values and complete it for the first three terms of the sequence.

Term in the sequence	1	2	3	4	5
Number of dots	3	5	7		

Provide the students with counters and ask them to complete the table for the fourth and fifth terms.

Have the students write a rule that could be used to determine the number of dots of any term in the sequence.

Have the students share their rules and verify them for specific terms in the sequence.

Patterns in nature

Key idea

Complete a table of values for geometric and number patterns

Materials

None

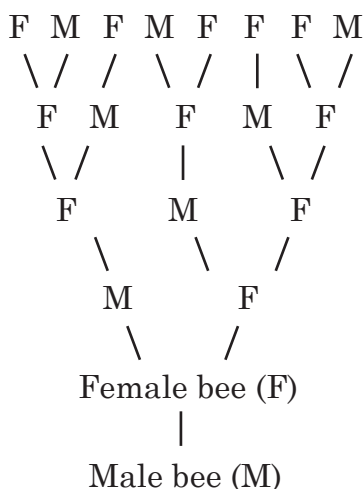
Explain how a tree diagram is constructed.

Inform the students that a male bee has only a mother, as it develops from an unfertilised egg, whereas a female bee develops from a fertilised egg and thus has a mother and a father.

As a class, jointly construct the family tree of a male bee showing the previous three generations.

Have the students work individually or in pairs to continue the diagram for the preceding two generations.

For example:



Represent the data from the diagram in a table as follows:

Number of ancestors

Level	Male	Female	Total
6th	3	5	8
5th	2	3	5
4th	1	2	3
3rd	1	1	2
2nd	0	1	1
1st	1	0	1

Ask the students to describe the sequence represented in each column.

Explain that these sequences of numbers are *Fibonacci sequences*, where each term is the sum of the previous two terms.

Follow-up activities could include researching Fibonacci.

Adding consecutive numbers

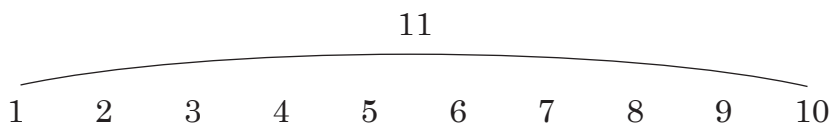
Key idea

Construct, verify and complete number sentences involving the four operations with a variety of numbers

Materials

None

Write the numerals 1-10 on the board. Discuss possible strategies to add quickly all of the numbers from one to ten. If the students are having difficulty providing suggestions beyond adding consecutive numbers, then draw an arc joining the first and last numerals and write the number 11 above the arc.



Ask, *What other relationships can you see between pairs of numbers?*

When the students identify that there are five pairs of numbers that add to 11, draw additional arcs to show the combinations.

Ask, *How can you use these number relationships to quickly find the total of all of the numbers?*

Verify the answer by mentally adding all of the numbers.

Have the students investigate other sequences of consecutive numbers that can be added in the same way.

Exploring the hundred chart

Key idea

Construct, verify and complete number sentences involving the four operations with a variety of numbers

Materials

Copies of BLM *1–100 chart* (p. 150)

Select a 2×2 square on the hundred chart.

For example:

24	25
34	35

Ask:

What happens when you add the diagonals? Why?

Does the same thing happen with other 2×2 squares? Why?

Have the students investigate the relationships among the numbers in the columns, rows and diagonals of a 3×3 square and share their findings.

1-100 chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Row of houses

Key idea

Complete a table of values for geometric and number patterns

Describe a pattern in words in more than one way

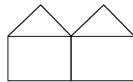
Materials

None

Draw the following diagram on the board.



Ask the students to identify how many lines you have drawn. Record the numeral 6 below the diagram. Draw five more lines to create a second house joined to the first one.



Ask the students to identify how many lines have been drawn altogether, then record the numeral 11 below the diagram.

Repeat for a third house.

Have the students continue the sequence up to ten houses and record it in a table.

For example:

Row of houses

Number of houses	1	2	3	4	5	6	7	8	9	10
Number of lines	6	11	16	21	26	31	36	41	46	51

Have the students describe the sequence and write a rule to determine the number of lines for any number of houses.

Terrace houses

Key idea

Complete a table of values for geometric and number patterns

Describe a pattern in words in more than one way

Materials

None

Draw the first house on the board and tell the students that it is the first house in a row of terrace houses. Ask, *How many windows can be seen?*

First house



Record the number of windows in a table, as below.

Add an adjoining house, and ask, *How many windows can be seen?* Record the total number of windows in the table.



Continue adding up to five houses and record the cumulative number of windows.

Terrace house windows

Number of terrace houses	Total number of windows
1	5
2	8
3	11
4	14
5	17

Have the students describe the sequence of numbers in each column and write a rule to determine the total number of windows for any number of houses.

Triangular numbers

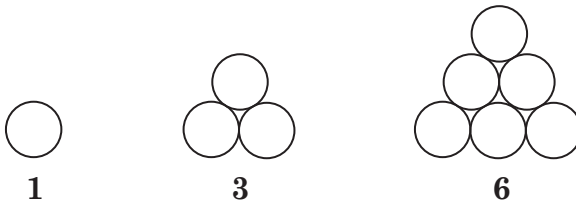
Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

Counters

Draw and discuss the following diagram on the board.



Ask:

Why do you think that these numbers are called triangular numbers?

What will be the next arrangement of circles in the sequence?

Why?

Have the students continue drawing the next two elements of the pattern on the board and record the number of circles in each element.

Provide the students with counters and ask them to make and continue the pattern.

Ask, *How could you describe this sequence of numbers?*

Stacking square numbers

Key idea

Generate, describe and record number patterns using a variety of strategies

Materials

Cubic blocks

Have the students construct models of the first four square numbers. Stack the models in consecutive order.

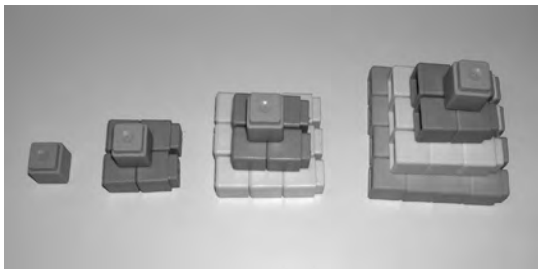
Have the students determine the number of blocks in the resulting model.

Select students to repeat the process to construct models using the first three and then the first two square numbers. Have the students determine and record the number of cubes used to construct each model.

Have the students determine and record the differences between consecutive numbers in this sequence.

How could you describe the numbers in this sequence?

Ask the students to determine how many cubes would be needed to construct the next two models.



Glossary

Algebra	The mathematics of working with generalised <i>number relationships</i> .
Array	A set of objects arranged in equal rows and columns.
Associative property	Property of an operation when the result remains the same regardless of how the numbers are grouped if the operation is carried out upon three or more numbers. For example: $(4 + 5) + 8 = 4 + (5 + 8)$
Commutative property	Property of an operation when the result remains the same if the order of the numbers is reversed, regardless of the numbers involved. For example: $3 \times 4 = 4 \times 3$ or $3 + 5 = 5 + 3$
Inverse of an operation	An operation that reverses the result of another operation. For example, subtraction and addition are <i>inverse operations</i> , as are multiplication and division.
Number relationship	The comparison of one quantity relative to another, described in mathematical terms and symbols.
Pattern	Numbers that are arranged in a special way. A repeating arrangement of shapes that can be described in terms of number. Repeating <i>patterns</i> can be described by indicating the number of elements that repeat.
Sequence	An ordered set of numbers.

Series	The sum of the <i>terms</i> of a <i>sequence</i> .
Skip counting	Counting forwards or backwards in multiples of a number.
Term	One of the numbers in a <i>sequence</i> or <i>series</i> .
Variable	A symbol whose numerical value may change.