Forming groups

Students using grouping strategies…

understand that numbers are capable of being regrouped in flexible ways. These students no longer need to rely on counting by ones but can apply a range of grouping strategies to solve addition and subtraction problems.

When dealing with multiplication and division problems, students at this level are able to form equal groups. Rather than emphasising individual items or numbers, the students develop an increasingly sophisticated idea of "composites". The students are able to focus on a group of items and learn to treat the group as a single item.

Array structures assist students to "see" composite groups and using this structure aids students to quickly find the total using the process of multiplication.

When students are able to group units and treat them as composites, they then need to move from needing the items present, to being able to visualise and use multiple counting strategies to determine the total of the groups.

As students begin to use composite units, it is common to see them use a combination of counting strategies to determine the total.

Just as students counting strategies are limited by their knowledge of the sequence of number words, so too their early multiplication and division strategies are often limited by their knowledge of the sequence of multiples. When a student's knowledge of a sequence of multiples is exhausted, they will switch back to counting by ones.

As students work with larger numbers, the use of grouping begins to draw upon the special role played by "tens" in our number system.

Place value concepts, as with multiplication and division processes, make use of the composite nature of groups, but in this case the groups are units of, tens, hundreds, thousands and so on. Addition, subtraction, multiplication and division all make use of place value for both mental and written computation.

The notion of a composite unit is also important in the development of measurement concepts. Knowledge of the size of units and being able to repeatedly use a unit are critical to the process of measuring. Students need to see how individual units can be combined into composite units, for example a row of tiles or a layer of bricks. These composite units can then be used in calculations such as in determining area multiplication.

In spatial understanding, individual units can also be combined into composite units. For example, combining two squares to make a rectangle, repeating and rotating patterns and tiling activities.

Index of activities

Memory domino

Where are they now?

Students are able to recognise a small number of items instantly, without the need to count each item.

Where to next?

Students are able to recognise larger groups of items when arranged in spatial patterns.

Encourage students to share their strategies.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

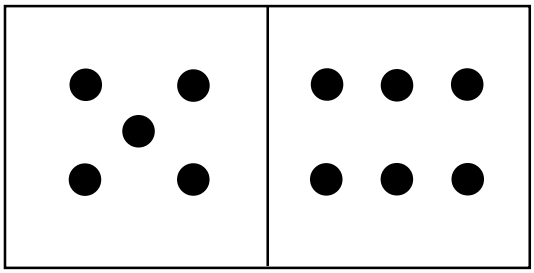
WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

CMIT reference

Subitising: conceptual

How?

Organise a set of dominoes representing number combinations to 12 and a set of numeral cards in the range 0–12. Place the dominoes and the numeral cards face down on the floor. Have the students take turns to firstly flip over one domino. Encourage the student to immediately state the number of dots on each side of the domino and then the total. Then have the students flip over a numeral card to try and match the total on the domino. If the student successfully matches a pair they keep the domino and the numeral card.



Why?

Activities that promote the development of instant recognition of spatial patterns can assist students to recall number facts.

Domino challenge

Where are they now?

Students are able to recognise a small number of items instantly, without the need to count each item.

Where to next?

Students are able to recognise larger groups of items when arranged in spatial patterns.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

CMIT reference

Subitising: conceptual

How?

Organise a set of dominoes so that all the tiles greater than five are placed face down on the floor. Quickly turn over a domino and then place it back on the floor face down. The first student to call out the total number of dots on the domino keeps the tile. If a student calls out an answer, another student may call "challenge" if he or she believes it is incorrect. The "challenger" may keep the tile if he or she states the correct number. If the challenger is incorrect then the tile is returned to the central pile or to the first caller if he or she was correct. Continue until all tiles have been claimed.

Variation

Use a set of "double 9" dominoes. Leave the tile turned over after the student has called out the total. If both the "caller" and the "challenger" are incorrect the tile is then turned over.

Why?

Activities that promote the development of instant recognition of spatial patterns can assist students to recall number facts.

Ten-frame flash

Where are they now?

Students are able to recognise a small number of items instantly, without the need to count each item.

Where to next?

Students are able to recognise larger groups of items when arranged in spatial patterns.

Make a variety of ten-frames including some displaying dots that are not next to each other, for example, one or two spaces between dots.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

CMIT reference

Subitising: conceptual

BLM

Ten-frame, page 148

How?

Prepare flash cards showing ten-frames containing various dot patterns. Hold a ten-frame horizontally and briefly flash the ten-frame. Have the students record how many dots are in the top row, how many dots are in the bottom row and how many dots altogether. This leads to a natural progression of recording a written algorithm, when the numbers are recorded vertically. Have the students share their responses.

Variation

Prepare two sets of ten-frame cards. Each card displaying a filled, or partially filled, ten-frame. Do not use scattered patterns. Flash a ten-frame from each set. Have the students determine how many dots are on each card and then determine how many dots there are altogether.

Why?

Activities that promote the development of instant recognition of spatial patterns can assist students to recall number facts.

Largest number wins

Where are they now?

Students can count forwards and backwards by 10s on and off the decade.

Where to next?

Students can count forwards and backwards by 100s off the 100 and on or off the decade to 1000.

Discuss with the students how they know which is the largest number. Lead discussions and follow-up activities to renaming units within numbers. For example, seeing that "302" is the same as 30 tens and 2 ones, assists consolidation of place value concepts.

Syllabus outcomes

NS2.1: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Counting by 10s and 100s: level 3

Numeral identification: level 4

How?

Organise the students into small groups and provide them with an "operation die" (A cube marked with "+1", "-1", "+10", "-10", "+100", "-100".) Each player starts with a score of 500. The die is rolled and each player adds or subtracts the number rolled to his or her score. In turns, players then have four rolls of the "operation die". After each roll the player calculates and records his or her tally. The winner is the player with the largest number.

Why?

Being able to count on and off the decade and off the hundred is a useful strategy for students to apply when solving some two- and three-digit computations mentally.

Start with four

Where are they now?

Students can count forwards and backwards by tens on and off the decade.

Where to next?

Students can count forwards and backwards by 100 on or off the decade to 1000.

The "operation die" from *Largest number wins* could be used instead of the "operation cards".

Syllabus outcomes

NS2.1: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Counting by 10s and 100s: level 3

Numeral identification: level 4

BLM

Start with four, page 236

How?

Organise the students into pairs or groups of three and provide them with a set of numeral cards 1–9 (make at least three of each number), a set of instruction cards "+1" "+10" "+100" "-1" "-10" "-100" (at least three of each) and a recording sheet each. Alternatively, use the operation die from *Largest number wins*. Ask the students to shuffle the numeral cards and deal out four cards to form a four-digit number. This will be the starting number for the first round. Each student records the starting number on his or her worksheet. The students then take turns to draw an instruction card and add it to, or subtract it from, the starting number and record the new tally on the worksheet. Play continues until all players have had four turns at drawing an "instruction card". The player with the largest number after four draws is the winner.

Variation

Instead of having a "winner", have the students record the final tally of each player as a group recording, by sequencing the numbers from lowest to highest.

Why?

Being able to count on and off the decade and off the hundred is a useful strategy for students to apply when mentally solving some addition and subtraction problems.

Bucket count on: 10s and 100s

Where are they now?

Students can count forwards and backwards by tens on and off the decade.

Where to next?

Students can count forwards and backwards by 100 on or off the decade to 1000.

Syllabus outcomes

NS2.1: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Counting by 10s and 100s: level 3

How?

Drop a small collection of large disks or blocks (all of one colour) into a bucket or container one at a time. Tell the students that the colour of the discs, say red, represents a unit of ten. Ask the students to count aloud by tens as each disc is added. Choose a different coloured disc and tell the students that this colour, say blue, represents units of 100. Drop the discs into the bucket one at a time. Ask the students to continue counting by adding on 100 to the total as each disc is dropped. After adding in this fashion, return to adding discs representing "tens" to the total. Discs of another colour could be used to represent units of "one" and if appropriate, use discs to represent units of "1000".

Why?

Being able to count on and off the decade and off the hundred is a useful strategy for students to apply when mentally solving some addition and subtraction problems.

I have, I want, I need

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

A traditional number line is usually segmented and marked with numerals at regular intervals. The empty number line (without pre-written numerals or interval marks) provides the student with the control to place chosen numerals at any points along the line. The focus is therefore on how the problem was solved and can reflect the student's mathematical thinking without imposing a set procedure.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

CMIT reference

Building place value through grouping: level 2

Combining and partitioning: To 10

BLM

I have, I want, I need, page 237

How?

Provide each student with a worksheet and a set of numeral cards in the range 0–9. Have the students draw two cards from the pile and construct the lowest two-digit number possible from the combination. Record the numeral under the "I have" box. The student then reverses the numerals and records the new number under the "I want" box. Have the students determine the difference between the two numbers and use the empty number line to record their problem solving strategy. Record the difference in the "I need" box. Have the students share their strategies with the class. If the same number is drawn for both cards, have the students return one to the pile and redraw another card.

Variation

Use playing cards instead of numeral cards.

Why?

Students should have the opportunity to develop mental strategies for solving addition and subtraction problems and to record their thinking before introducing algorithmic procedures.

Model the use of the empty number line as a way of recording. Demonstrate strategies such as "building to the next decade", "jumping by multiples of ten" and "counting down to a number".

The empty number line

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

Model the use of the empty number line as a way of recording. Demonstrate strategies such as "building to the next decade", "jumping by multiples of ten" and "counting down to the decade".

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

CMIT reference

Building place value through grouping: level 2

Combining and partitioning: To 10

How?

Demonstrate the use of the empty number line to students, showing how multiples of tens and single digits can be recorded and used to solve addition and subtraction problems.

Provide the students with two piles of numeral cards in the range 0–9. Have the students draw two cards to create a two-digit number. Ask the students to use an empty number line to record the way they found out how many more are needed to make 100.

Variation

Provide the students with two sets of cards. The students then draw four cards and make two, two-digit numbers. Have the students use the empty number line to record their methods of adding the two numbers. Encourage the students to build to the next decade and then count on by tens or groups of ten.

Why?

Students should have the opportunity to develop mental strategies for solving addition and subtraction problems and to record their thinking before introducing formal algorithms.

Hundred chart challenge

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

TP

Discuss the mental calculations students use to solve problems and which strategy the students think was the best. For example, did they count-on by tens and ones and keep track of the total? Did they bridge to the nearest decade? Did they use the hundred chart to track counting on by tens and then add or subtract the ones?

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

WMS2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

CMIT reference

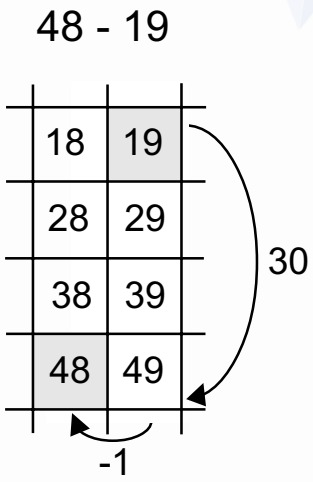
Building place value through grouping: level 2

Combining and partitioning

Recording symbols

How?

Prepare two packs of numerals cards in the range 0–9. Have the students select two cards from each pack to make two, two-digit numbers. Locate and mark the numbers on a hundred chart. Have the students determine the difference between the two numbers in the least number of moves. Encourage the students to explain how they determined the difference.



"Add thirty, go back one."

Why?

Problems involving regrouping of numbers both with and without material should be introduced to the students before teaching procedures for algorithms. Students need significant opportunities to focus on regrouping numbers to develop an understanding of place value.

WR

Record students thinking on the chalkboard as solution strategies are explained.

Addition challenge

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

Model mental computation strategies such as combining numbers to make ten or adding on by tens and then ones.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

WMS2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

CMIT reference

Building place value through grouping: level 2

Combining and partitioning

Recording symbols

How?

Organise the students into small groups and provide each group with a set of playing cards, using the "ace" (to represent number one) through to the "nine", and paper for recording. Divide the cards into two piles. Students take turns to draw two cards to make a two-digit number. Each student then draws another two cards to form a second two-digit number, adds the two numbers together and records the total. The aim is to be the player with a total of 100 or to have the largest total less than 100. A player with a total greater than 100 automatically loses. Instruct the students to keep a tally of their wins and the first to score ten wins and is the "grand champion".

Variation

Use a hundred chart to assist mental calculations.

Why?

Students should have the opportunity to develop mental strategies for solving addition and subtraction problems and to record their thinking before introducing algorithmic procedures.

Use the students recording sheets to discuss and model strategies.

Friends to 100

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

Demonstrate to the students how a subtraction problem may be solved using an addition process. This could be based on the "jump" method of adding tens and ones to the first number.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

PAS2.1: Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

CMIT reference

Building place value through grouping: level 2(a)

BLM

Friends to 100, page 238

How?

Organise the students into pairs and provide each pair with a copy of a hundred chart. The first student calls out a number between 1 and 99. The second student shades over the called number on the hundred chart and then counts on from the number by tens and ones to determine how many more are needed to make 100. After determining the difference, the student locates this number on the hundred chart and shades over the number. Discuss any patterns that the students notice. After practice, have the students determine the difference without the use of the hundred chart.

Variations

Record the number combinations, as they are determined.

Pose the problem as a subtraction, e.g. *How many would I have to take away from one hundred to land on the called number?*

Why?

Students should have the opportunity to develop mental strategies for solving addition and subtraction problems and to record their thinking before introducing formal algorithms.

Red or black

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

WMS2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

CMIT reference

Building place value through grouping: level 2

Recording symbols

How?

Each group will need a pack of playing cards and paper to record on. Each player begins with a score of 20 points. Shuffle the cards and place them face down in a pile. Each student in the group takes turns to draw a card from the pile. Before doing so, he or she must say whether the card will be red or black. If the student correctly guesses the colour of the card, the number on the card is added to his or her score. If the student guesses incorrectly, the number on the card is subtracted. If the student guesses incorrectly, but hasn't enough points to take away the number on the card, he or she misses a turn. All cards have a score of their face value. Picture cards score as follows:

Jack = 11, Queen = 12, King = 13, Ace = 1

The winner is the first player to score 100.

Variations

Remove picture cards and play with cards Ace–10 only.

Students invent extra rules for playing. For example, include the "joker" and if turned up, the player reverses the score of an opponent.

Correct guesses are added to the score. Incorrect guesses miss a turn.

Change the winning score. For example, for a shorter game change the winning score to 50.

Highest score after five turns wins.

Why?

Students should have the opportunity to develop mental strategies for solving addition and subtraction problems and to record their thinking before introducing algorithmic procedures.

Have the students record their mental calculations for solving the two-digit additions and subtractions. Different ways of recording can be used as follow-up topics of discussion.

Double dice multi

Where are they now?

Students are able to form equal groups and count perceived items by ones.

Where to next?

Students are able to count forwards or backwards using a pattern of multiples without having to rely on perceiving the items.

Provide many opportunities for students to practise forwards and backwards counting of multiples.

Syllabus reference

WMS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

BLM

Double dice multi, page 239

How?

Provide the students with a baseboard, two dice, one die displaying numerals 1 to 6, the other displaying numerals 1 to 3 and 12 counters. Instruct the students to take turns to roll the dice and multiply the two numbers rolled. Model to the students how skip counting or repeated addition may be used to determine the answer. Once the answer has been determined, the student covers the corresponding numeral on the baseboard. If the number is already covered, the player misses a turn. Continue until all numerals on the baseboard have been covered.

Variations

Modify the dice to have both displaying numerals 1–6 or replace one of the dice with a ten-sided die displaying numerals 1–10. If varying the dice, the base-board will need to be modified.

Use three dice. Have the students roll the dice and choose two of the numbers rolled to multiply.

Why?

Students need to move from being dependent on using groups of physical objects to being able to keep track of the groups when the material is not present.

Teddy target

Where are they now?

Students are able to form equal groups and count perceived items by ones.

Where to next?

Students are able to count forwards or backwards using a pattern of multiples or repeated addition without having to rely on perceiving the items.

Encourage the students to use skip counting or repeated addition to determine the totals.

Point out to the students that the constant function on the calculator is using repeated addition.

Syllabus reference

NS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

How?

Draw a large target on the asphalt with chalk. Write the numerals 6, 5, 4 and 3, on the target, so that one numeral is on one segment of the target. Organise the students into teams and provide each team with a set of "teddy bags". "Teddy bags" are clear plastic bags containing 2, 3, 4, 5 or 6 plastic teddies. Have the students take turns to throw the "teddy bags" onto the target. The team calculates the score by multiplying the number of teddies in the "teddy bag" by the number indicated on the target segment. Have one of the team members record the score and another member check the calculation on a calculator. After each member has had a throw, the team adds the total. The team with the highest total wins.

Variations

Change the number of teddies in the "teddy bags."

Change the numerals on the target.

Use cloth beanbags marked with numerals to encourage students to recall multiplication facts.

Why?

Students need to move from being dependent on using groups of physical objects to being able to keep track of the groups when the material is not present.

Have the students show their thinking when recording the totals. Use students' recordings to discuss and model strategies.

Collecting threes

Where are they now?

Students are able to form equal groups and count perceived items by ones.

Where to next?

Students are able to count forwards or backwards using a pattern of multiples or repeated addition without having to rely on perceiving the items.

Syllabus outcomes

NS2.3: Uses mental and informal written strategies for multiplication and division

PAS2.1: Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3

Recording symbols

How?

Organise the students into small groups. Have the students take turns to roll three dice. Whenever a three or a six is rolled, the player records the number of threes and the total. Each player needs to keep a personal total. The first player to reach "30" is the winner.

Variation

Roll the three dice and use any combinations to form groups of three. For example if a three, five and four are thrown, the student may record four groups of three and a total of twelve.

Why?

Students need to move from being dependent on using groups of physical objects to being able to keep track of the groups when the material is not present.

People markers

Where are they now?

Students are able to form equal groups and count perceived items by ones.

Where to next?

Students are able to count forwards or backwards using a pattern of multiples or repeated addition without having to rely on perceiving the items.

Syllabus outcomes

WMS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

Recording symbols

How?

Prepare ten dot-pattern cards for the multiple to be practised. For example, ten cards showing four dots on each. Distribute the dot pattern cards to the students and tell them they are going to represent the group on the card. Nominate a number of students, say six, to bring their dot pattern cards, face down, to the front of the room and form a line. Have the students use skip counting or repeated addition for the nominated multiple to determine the total as each person turns over his or her card in turn. Repeat the activity using various numbers of students in the line and record the number facts.

Variation

Use numeral cards instead of dot pattern cards.

Why?

Before practising multiplication tables by rote or other methods, students should have a good understanding of using composite units, without having to rely on visual representations of the units.

Students could record the multiples on a number line first and then record the multiplication fact.

Triples plus one

Where are they now?

Students are able to form equal groups and count perceived items by ones.

Where to next?

Students are able to count forwards or backwards using a pattern of multiples or repeated addition without having to rely on perceiving the items.

Model strategies of skip counting and repeated addition to the students and encourage them to use these strategies whilst completing the activity.

Syllabus outcomes

NS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

BLM

Triples plus one, page 240

How?

Prepare a baseboard for each pair of students and also provide them with a pile of counters and a numeral die. Have the students take turns to roll the die. The student then multiplies the number rolled by three, determines the answer, and then adds one more to the total. The student then covers a corresponding numeral on the baseboard with a counter. The first player to cover four numerals in a row, vertically, horizontally or diagonally wins.

Why?

Before practising multiplication tables by rote or other methods, students should have a good understanding of using composite units, without having to rely on visual representations of the units.

Graphing with symbols

Where are they now?

Students are able to form equal groups and find the total by counting by ones.

Where to next?

Students use skip counting or repeated addition to determine the total of the groups.

Students' strategy use in solving multiplication and division problems will be dependent upon their knowledge of multiple sequences.

Syllabus outcomes

DS2.1: Gathers and organises data, displays data using tables and graphs, and interprets the results

NS1.3: Uses a range of mental strategies and concrete materials for multiplication and division

WMS2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

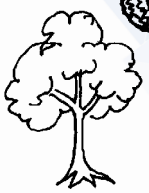
CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

How?

Organise the students to investigate number problems where they have to find the total number and represent the quantity in symbols.

For example:

- The number of fingers in the room.  = 5 fingers
- The number of car doors in the car park.  = 2 doors
- The number of wheels on cars in the carpark  = 4 wheels
- The number of trees in the playground  = 10 trees

Have the students represent the information as a picture graph and have other students in the class interpret the graphs.

Why?

These activities allow the student to be removed one step from direct use of concrete materials and counting by ones. Here the symbols act as markers for the students to keep track of the groups as they count the multiples.

High rollers 2

Where are they now?

Students are able to form equal groups and find the total by counting by ones.

Where to next?

Students use skip counting or repeated addition to determine the total of the groups.

Allow the students to discuss ways of dealing with any single units left over from each group.

Syllabus outcomes

DS2.1: Gathers and organises data, displays data using tables and graphs, and interprets the results

NS1.3: Uses a range of mental strategies and concrete materials for multiplication and division

NS2.5: Describes and compares chance events in social and experimental contexts

WMS2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

How?

Organise the students into groups and provide each group with a die. Each student in the group takes a turn to roll the die. The group records the number of times each number is rolled using tally marks. Continue until the die has been rolled a nominated number of times. Have the group then transfer the information onto a whole class picture graph, where groups of five are represented by a single symbol. Have the class use skip counting, repeated addition or recall of facts to determine how many times each number was rolled.

Variations

Transfer the tally marks to ten-frames and count in multiples of ten.

Before commencing to roll the die, ask the students to predict the outcome after a nominated number of rolls and then compare the results to the actual number rolled.

Why?

These activities allow the student to be removed one step from direct use of concrete materials and counting by ones. Here the symbols act as markers for the student to keep track of the groups as they count the multiples.

Jelly bean pans

Where are they now?

Students are able to form equal groups and find the total by counting by ones.

Where to next?

Students use skip counting or repeated addition to determine the total of the groups.

This activity may also be used to develop place value concepts.

Syllabus outcomes

DS2.1: Gathers and organises data, displays data using tables and graphs, and interprets the results

NS1.1: Counts, orders, reads and represents two- and three-digit numbers

WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

Building place value through grouping: level 0 and 1

How?

Present students with a large jar of jelly beans, beads, plastic teddies or other small objects and a pile of patty pans. Instruct the students to fill each patty pan with ten jelly beans, or similar object, until all have been removed from the jar. Keep a tally of any left over jelly beans. The objects may now be removed from the patty pans. Use the empty patty pans to form a picture graph. Encourage the students to count by tens and add on remaining units to determine the total. The students could determine a way of representing any remaining units using the patty paper.

Variation

To encourage off-the-decade counting, begin the count from the single units and add "ten" for each patty pan.

Why?

These activities allow the student to be removed one step from direct use of concrete materials and counting by ones. Here the symbols act as markers for the student to keep track of the groups as they count the multiples.

Fractured fairy tales

Where are they now?

Students are able to form equal groups and find the total by counting by ones.

Where to next?

Students use skip counting or repeated addition to determine the total of the groups.

Syllabus outcomes

DS2.1: Gathers and organises data, displays data using tables and graphs, and interprets the results

NS1.1: Counts, orders, reads and represents two- and three-digit numbers

WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

WMS2.1: Asks questions that could be explored using mathematics in relation to Stage 2 content

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

Building place value through grouping: level 0 and 1

How?

Pose open questions to the students such as:

- Little Bo Beep has lost her sheep. How many were there?
- How many rats followed the Pied Piper?
- How many "hundreds and thousands" on a piece of fairy bread?
- How many sultanas in a kilogram pack?

Have the students represent the answer as a picture graph. Other students could then read the graph and determine the total. Compare the results.

Variation

Students pose their own problems.

Why?

These activities allow the student to be removed one step from direct use of concrete materials and counting by ones. Here the symbols act as markers for the student to keep track of the groups as they count the multiples.

How do I know?

Where are they now?

Students use the row structure repeatedly to measure area and count the total number of units by counting in multiples.

Where to next?

Students use a row and column structure to measure area and use a variety of counting strategies that do not rely on visual material to determine the total number of units.

Model various counting strategies for multiplication such as skip counting, repeated addition or recalling number facts.

Syllabus outcomes

MS2.2: Estimates, measures, compares and records the areas of surfaces in square centimetres and square metres

NS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.5: Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 2 content

CMIT reference

Count Me Into Measurement: Area 3.1

Building multiplication and division through equal grouping: level 3, 4 and 5

How?

Provide the students with a variety of cardboard rectangles and a sheet of grid paper. Have the students place the rectangles on top of the grid paper and use the grid structure to determine the total number of units covered by the rectangle. Have the students share their results with others and explain how they determined the total. Discuss with the students why grid paper is useful for measuring and if they could suggest other things to use to help them measure area.

Why?

Students need to be able to visualise and use the spatial grid structure to effectively understand *area* concepts.

Provide the students with paper to record their solutions. Use these recordings as models for recording thinking.

Using nets 2

Where are they now?

Students use the row structure repeatedly to measure area and count the total number of units by counting in multiples.

Where to next?

Students use a row and column structure to measure area and use a variety of counting strategies that do not rely on visual material to determine the total number of units.

Discuss with the students that this practical form of measurement will be approximate and the accuracy will depend on the measuring instrument and the experience of the person who is measuring.

Syllabus outcomes

MS1.2: Estimates, measures, compare and records areas using informal units

NS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Count Me Into Measurement: Area 3.1

Building multiplication and division through equal grouping: level 3, 4 and 5

How?

Provide the students with a selection of packages that they can open to form a net. Each student will also need a small cardboard square. Have the students use the square to determine the number of units that are needed to make a row across the net and the number needed to make a column. Encourage the students to mark a line at the end of each unit. Have the students then use the row and column structure to determine the total number of units needed to cover the net. The students may need to be reminded to add any single units needed to completely cover the area. Have the student use multiple counting, repeated addition or recall known facts to determine the number of units needed to cover the rectangle and count on the additional units.

Why?

Students need to be able to visualise and use the spatial grid structure of grids to effectively understand *area* concepts.

WR

Have the students draw the 3D shapes showing the rows and layers of units.

How many more?

Where are they now?

Students use the row structure repeatedly to measure area and count the total number of units by counting in multiples.

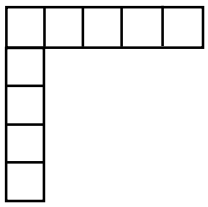
Where to next?

Students use a row and column structure to measure area and use a variety of counting strategies that do not rely on visual material to determine the total number of units.

TP

Students may need guidance in forming the row and column structure. Discuss how one square will be used as the first square in the row as well as the first square in the column, and why.

e.g.



Syllabus outcomes

MS1.2: Estimates, measures, compare and records areas using informal units

NS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Count Me Into Measurement: Area 3.1

Building multiplication and division through equal grouping: level 3, 4 and 5

How?

Provide the students with square tiles and two dice. Have the students take turns to roll the dice. The students then collect a corresponding number of tiles and form a row equal to one of the rolled numbers and a column equal to the other rolled number. The students then need to determine the number of additional tiles needed to make a "mat" from the tiles. Encourage the students to use the row and column structure to determine how many additional tiles are needed. Have the students share their solutions.

Variation

Replace the tiles with cubes.

Why?

Students need to be able to visualise and use the spatial structure of grids to effectively understand *area* concepts.

What's in the box?

Where are they now?

The student is able to generate static visual images of shapes and objects in a variety of orientations.

Where to next?

The student is able to mentally modify images of shapes and objects.

Syllabus outcomes

SGS2.1: Makes, compares, describes and names three-dimensional objects, including pyramids, and represents them in drawing

WMS2.4: Checks the accuracy of a statement and explains the reasoning used

CMIT reference

Count Me Into Space: Part-whole relationships, pattern and dynamic imagery strategies

How?

Conceal a variety of objects such as assorted tissue boxes, balls and containers inside a box. Select an object in the box and lift it up within the box to reveal part of the object to the students. Ask the students to name what the three-dimensional object could be and explain why. Continue to reveal part of the object and repeat questioning. Repeat with various 3D objects such as prisms, pyramids, cylinders, cones and spheres.

Variation

Replace the 3D object with a 2D shape and place on an overhead projector, partially covered. Gradually reveal part of the shape and question the students as to what shape it might be and why.

Why?

Students need to explore shapes to help them develop strong concept images that focus on properties that make up the shape, such as angles, sides and faces.

Moving 3D objects

Where are they now?

The student is able to generate static visual images of shapes and objects in a variety of orientations.

Where to next?

The student is able to mentally modify images of shapes and objects.

Syllabus outcomes

SGS2.1: Makes, compares, describes and names three-dimensional objects, including pyramids, and represents them in drawing

WMS2.4: Checks the accuracy of a statement and explains the reasoning used

CMIT reference

Count Me Into Space: Orientation and motion, pattern and dynamic imagery strategies

How?

Select a square pyramid from a set of geometric solids. Show the solid so that it is on its base to the students. Ask the students to draw the shape they will see if you were to push the pyramid over so that the point lands away from them. Have the students share their drawings and explain their reasoning before checking by moving the solid. Repeat the activity using other solids and appropriate instructions.

Variation

Organise the students into pairs and have them complete the activity.

Why?

Students need to explore shapes to help them develop strong concept images that focus on properties that make up the shape, such as faces and angles.

Time to shape up!

Where are they now?

The student is able to generate static visual images of shapes and objects in a variety of orientations.

Where to next?

The student is able to predict changes to shapes and objects by mentally visualising and modifying the image of the shape.

Syllabus outcomes

SGS2.2a: Manipulates, compares, sketches and names two-dimensional shapes and describes their features

SGS2.2b: Identifies, compares and describes angles in practical situations

WMS2.5: Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 2 content

CMIT reference

Count Me Into Space: Orientation and motion, pattern and dynamic imagery strategies

How?

Show a round clock face to the students and ask them to imagine what shape would be drawn if you drew a line starting at the 12 and going to 3, 6, 9 and back to 12. Have the students suggest other numbers that could be used to form squares or rectangles on the clock face.

Variations

Have the students draw the shapes that would be formed in the same orientation as represented on the clockface.

Find other shapes that can be drawn on the clock. For example start at a number and count by twos or fours.

Trace the shape on the clock using fingers if the students are having difficulty visualising shapes.

Why?

Students need to explore shapes to help them develop strong concept images that focus on properties that make up the shape, such as angles and sides.

Rolling boxes

Where are they now?

The student is able to generate static visual images of shapes and objects in a variety of orientations.

Where to next?

The student is able to mentally modify images of shapes and objects.

Syllabus outcomes

SGS2.2a: Manipulates, compares, sketches and names two-dimensional shapes and describes their features

WMS2.5: Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 2 content

CMIT reference

Count Me Into Space: Orientation and motion, pattern and dynamic imagery strategies

BLM

Rolling boxes, page 241

How?

Collect a variety of cardboard boxes. Place a symbol in a corner or along an edge on one face of each box. Hold up one of the boxes so that the students can see the face displaying the symbol. Ask the students to draw the shape of the face if it was rotated a quarter turn to your left and to draw where the symbols would now be on the face. Share and discuss the students' drawings. Repeat the process, turning the box another quarter turn. Continue for one or two more rotations then use a different shaped box or container.

Variation

After having experience with the above activity, provide the students with a copy of the *Rolling boxes* worksheet and ask them to draw where the symbols would be on the boxes if they were rotated a quarter turn each time.

Why?

Development of visual images helps students focus on the properties of 2D shapes and 3D objects.

Pattern block angles

Where are they now?

The student recognises shapes in different orientations and proportions and checks by physical manipulation of materials.

Where to next?

The student is able to use the relationship between parts of shapes.

Syllabus outcomes

SG2.2b: Identifies, compares and describes angles in practical situations

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Count Me Into Space: Part-whole relationships:

pictorial imagery strategies

pattern and dynamic imagery strategies

How?

Provide the students with pattern blocks. Remove the hexagonal blocks. Instruct the students to draw a central dot on a piece of paper. The students then choose to investigate one of the pattern blocks. Using as many of the one type of block as needed, have the students place the same type of "corner" of each pattern block on the dot, ensuring the pattern blocks fit next to each other without any gaps or overlaps. The students should place enough blocks on the paper until they are able to form a straight line going through the dot. Have the students then draw radiating lines from the dot where additional blocks would be placed to complete one revolution around the dot. Remove the blocks and draw in all of the radiating lines.

Have the students share their discoveries and try to describe the angles. This description does not have to be in terms of degrees.

Variation

Tell the students that some angles on the pattern blocks cannot be joined together to make a straight line. (Note: If the angle is greater than 90º.) Ask them to discover which ones. They will still be able to draw the radiating lines to go around the dot.

Why?

Students need to develop the language to describe angles. They also need to be able to analyse the parts, such as corners, that make up a shape.

Rectangle count-up

Where are they now?

The student recognises shapes in different orientations and proportions and checks by physical manipulation of materials.

Where to next?

The student is able to use the relationship between parts of shapes.

Remind the students that a square is a type of rectangle and that rectangles can be placed together to form other rectangles.

Syllabus outcomes

SGS2.2a: Manipulates, compares, sketches and names two-dimensional shapes and describes their features

WMS2.5: Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 2 content

PAS2.1: Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

CMIT reference

Count Me Into Space: Part-whole relationships:
pattern and dynamic imagery strategies

BLM

Rectangle count-up, page 242

How?

Provide the students with a copy of the *Rectangle count-up* worksheet and ask them to determine the number of rectangles on the top diagram. Have the students explain or write how they know they have found all of the rectangles. Show the students the second diagram and repeat the questioning. Ask the students to extend the pattern on the diagram and determine the number of rectangles. Discuss number patterns and ask the students to predict how many rectangles there would be if the pattern was extended again. Share results.

Why?

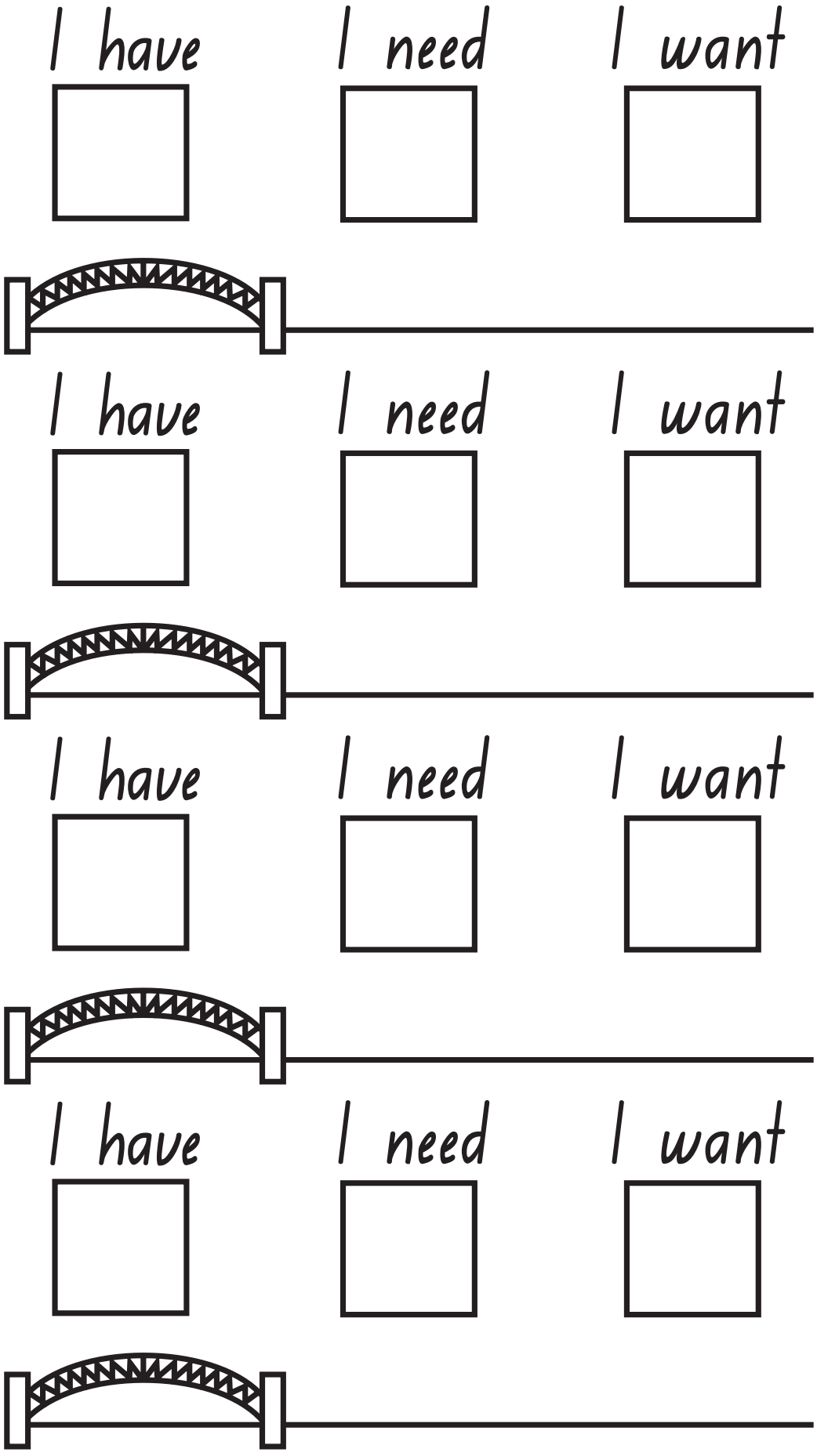
Development of visual images and part-whole knowledge helps students focus on the properties of 2D shapes.

Forming groups blackline masters

Start with four

+1	+10
+100	−1
−10	−100

I have, I want, I need



Friends to 100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

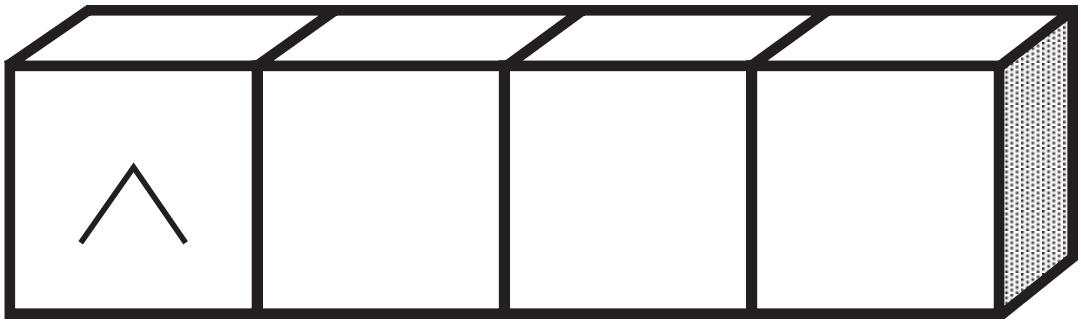
Double dice multi

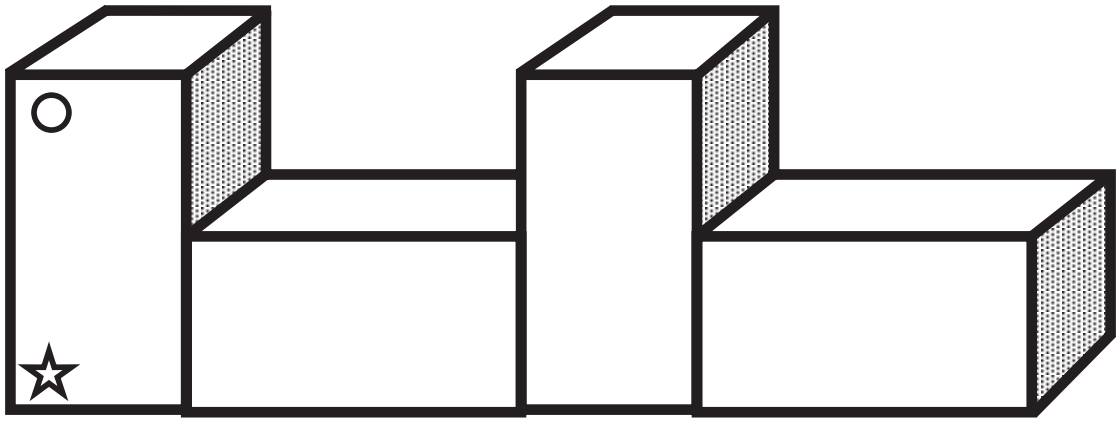
1	2	3
4	5	6
8	9	10
12	15	18

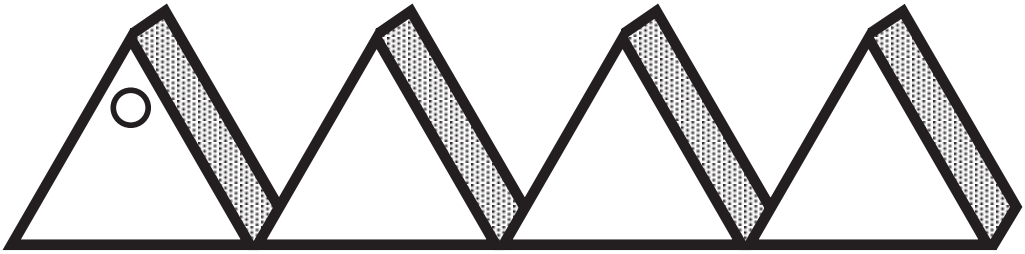
Triples plus 1

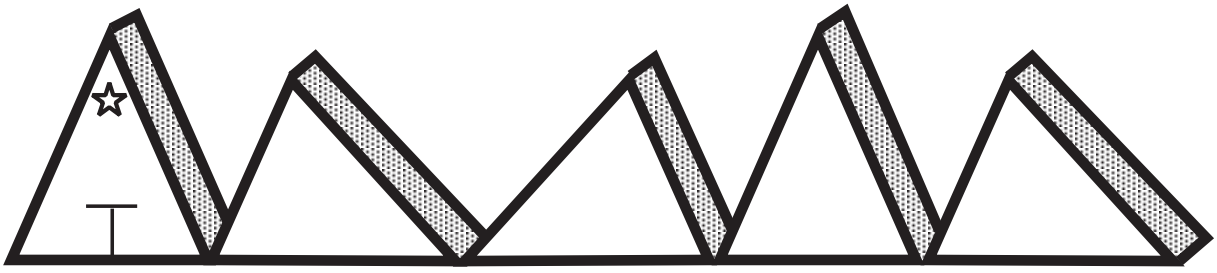
4	16	19	10
19	7	10	4
16	4	13	7
13	19	10	16

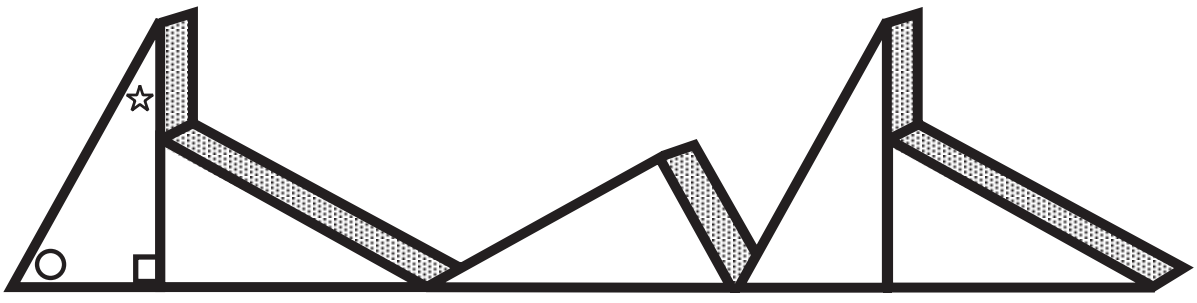
Rolling boxes



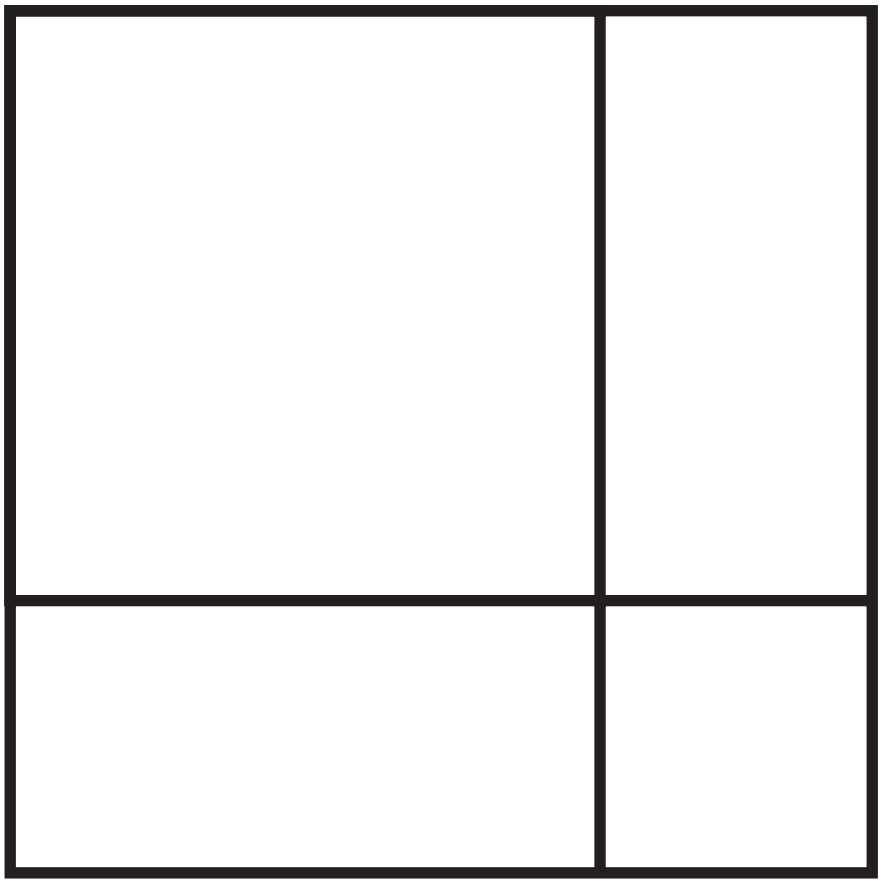


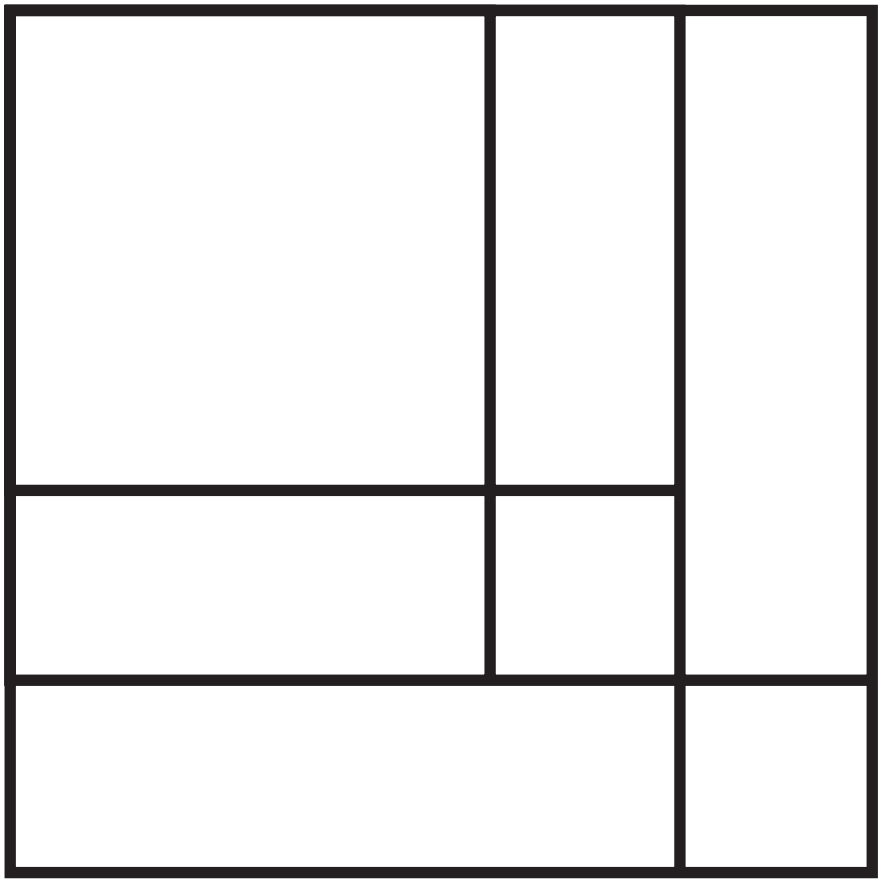




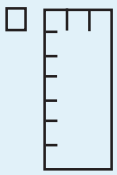


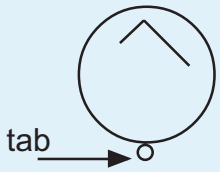
Rectangle count-up





Assessment tasks

Task	Student response	Assessment
Display the following cards **43 + 21** **37 + 19** **50 - 27** T: (For each card) *What is the answer to this? How did you work it out?*	Uses mental strategies to solve the addition and subtraction tasks.	Does the student use a jump or split method to solve the tasks? Does the student use other non-count-by-one strategies?
T: *Start from 24 and count by 100, i.e. adding 100 each time.*	Counts by 100s from 24 to 924	Can the student count forwards by 100s, off the 100?
T: *There are 12 biscuits and the children are given 2 biscuits each. How many children are there?*	Correctly solves the division task (how many groups) without the use of materials.	Does the student use a double count to keep track of the groups and the total? Other strategies used?
Show the student a unit square and a 7 x 3 rectangle.  T: *How many squares like this would you need to cover the rectangle completely?*	Calculates the total number of units needed to over the rectangle without having to use material.	Does the student: • count by ones • skip count the rows • use the row and column structure to find the total?

Task	Student response	Assessment
Show the student the following design on a cardboard circle.  Place another cardboard circle next to it so that the tabs match. Give the student two sticks. T: *Watch carefully and think about the design as I cover and turn these two circles.* Cover both circles and turn them 90º anticlockwise. T: *Build the design on your circle so that it will look like my design now.* If correct: T: *How did you know what to do?*	Quickly makes the design in the correct orientation without having to remove the cover.	Does the student make the design with the angle and position within 15º? When explaining how to build the design does the student refer to size of angles, relative position of the sticks and what happened to the sticks as the circle turned?

Maths bites

Using a calendar

- Choose a date from a calendar. Write it in a shortened form. For example, 12.6.99. Use the digits to make number sentences using the four operations. Such as 1+2+6+9+9 = 18.

- After completing the above activity, ask the students to find a date which they could use to make a nominated number. For example, find a date where the digits could be used to make the total 20.

- Look for number patterns going down the calendar, diagonally right and diagonally left. Ask the students why these patterns occur.

- Look at any four numbers that form a square on the calendar. Add the two numbers which go diagonally right and then the two going diagonally left. *What do you find? Does this work for all numbers that form a square?*

- State a number, say "12". Ask the student to find the "square" of four numbers that add to the stated number on both diagonals.

- Look at nine numbers on the calendar which form a 3 x 3 grid. *Can you see any patterns using these numbers?* Repeat using a 4 x 4 grid of numbers.

- Ask questions such as: *If there are 31 days, which month could it be?*

- Look at today's date. Use the number representing the day to write a list of "number stories". For example if the date is the 12th, students could record 2 x 6 = 12, 8 + 4 = 12, 3 x 4 = 12, 12 ÷ 6 = 2, double 6 = 12.

The following activity can be used as a whole class introduction to investigating number patterns on a calendar.

- Distribute numeral cards 1–31 and cards displaying the "days of the week" to the students. Pose a problem whereby the students have to place the cards in the correct row and column to complete a calendar. For example: *The only information I have is that the 13th is a Saturday. Now complete the calendar.* Ask the students with the "Saturday" card and the numeral card showing "13" to place their cards down on the floor first. Ask questions which will then allow each student to place his or her card down. For example, *Which number comes before "13"? Which number comes after "13"? Which number will be directly under "13" on the calendar? Where will number "1" go?* If a student makes an error, let the class discover the mistake and state how it should be corrected.