Australian Government

Quality Teacher Programme

Teaching about angles

Stage 2

PROFESSIONAL SUPPORT AND CURRICULUM DIRECTORATE

NEW SOUTH WALES DEPARTMENT OF EDUCATION AND TRAINING

Teaching about angles

Stage 2

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Foreword

Watching a group of young people playing basketball reminds you of why youth is sometimes described as angular. Elbows and knees present a constantly changing landscape of angles. The basketball sometimes moves up and down as it is dribbled on the spot only to move forward in a remarkable way as the player moves rapidly forward. Angles form and quickly reform as players jockey for position to pick up the rebound.

The angular nature of a basketball game is not restricted to the players. The referee puts his hands on his hips to signal a blocking foul and we see two-armed angles aplenty. The second hand on the shot-clock sweeps out its own one-armed angles before the shot misses and rebounds off the backboard.

When we talk about angles it is not always clear what we are describing. For many students the angle is the "pointy bit". For yet other students, the angle is the "arms" and longer arms can mean bigger angles!

Teaching about angles: Stage 2 is a collection of practical lessons designed to assist students to understand what we mean by angles. The materials provide opportunities for students to abstract the angle concept from a range of situations where they can initially see both arms of the angle, then only one arm and finally where they need to imagine both arms of the angle as in the rebound of a ball. All of the lessons have been trialled and I commend *Teaching about angles: Stage 2* to you as a practical resource to assist with teaching two-dimensional space in the new *Mathematics K–6 syllabus*.

Janet Davy

R/Director

Professonal Support and Curriculum Directorate

Contents

Learning about angles

Together with length, angle is possibly the most important tool used to describe shapes in construction, design and navigation. Angle also plays an important role in developing students' understanding of geometry. It is therefore vital that students obtain a good understanding of the concepts of angle at an early stage.

Angles arise in many different contexts. They are used to describe the shape of the corner of a geometrical figure, to specify direction, an amount of turning or opening, and an inclination or slope. This can make angles an interesting topic to teach, because the teacher can draw on many examples in the students' environment. However, angles arise in different ways in different situations, and this can make the topic challenging for students to learn.

The easiest angles for students to learn about are *two-line angles,* in which both arms of the angle are visible. These angles are found in the corners of geometrical figures and corners and intersections in our environment, such as the corners of a room.

Another important group of angles may be described as *one-line angles.* In these angles, only one arm of the angle is clearly visible. The other line of the angle must be imagined or remembered. Objects which can form one-line angles include an opening door, when there is no line on the floor to indicate the closed position of the door. Other examples are the turning of each hand of a clock and the slope on a roof or an incline where the horizontal line has to be visualised.

Many angles arise from situations in which neither arm of the angle is visible. A spinning ball and a turning wheel are examples of *no-line angles.*

A major challenge in learning about angles is to recognise that the same idea is present in many different situations. Some two-line angles are horizontal (the corner of a table top); some are vertical (the corner of a window frame), some are solid (furniture corners) and some are empty (lattice work). Some angles are fixed (picture frames) and some are movable (a pair of scissors). In some cases the two lines are easy to detect (sharp corners) and in others they are more difficult to discern (rounded corners). Students need to recognise that angles in all these objects are similar in that they consist of two linear parts that cross or meet at a point. They are also similar in that the relative inclination of these two parts has some significance; it defines the sharpness of the corner or the *openness* of the arms.

Further challenges arise when students encounter one-line angles. To form an angle, a second line must be constructed and this second line varies according to the context. To describe the opening of a door using angles, the closed position of the door must be remembered or imagined. To describe the turning of a clock hand using angles, the starting position must be used even if there is no hand there. To describe the slope of a roof using angles, a horizontal line must be found in the vicinity or simply imagined. Because the second line is different in each case, one-line angles are more difficult than two-line angles, and no-line angles are even more difficult.

The concept of angle is abstracted from the similarities between many different angle situations. The three critical features of this similarity are:

- two line, the *arms* of the angle
- a point where the lines meet, the *vertex*
- a degree of openness between the line, the *size* of the angle.

The angle concept is the *embodiment* of this similarity.

Using this book

Overview

This book provides support for teaching about angles in Stage 2 or the middle primary years. Students learn about:

- identifying two-line angles
- identifying the arms and vertex of one-line angles
- comparing angles using informal means such as angle testers
- describing angles using everyday language and mathematical terms *right*, *acute* and *obtuse*.

Students learn to work mathematically with these ideas using the processes of questioning, reflecting, communicating, applying strategies and reasoning.

The lessons

Each lesson commences with a general, teacher-led discussion leading into a new idea. The discussion is followed by one or more small-group activities in which students explore the new idea and its properties and consequences. The lesson concludes with another whole-class, teacher-led discussion in which the results of the student activities are highlighted and summarised.

The lessons focus on the teaching of a few key ideas that are clearly identified in the accompanying notes. Teachers are encouraged to adapt or adjust the suggested lesson ideas, to ensure that the program is appropriate to their students' current knowledge and understanding. Extension activities have been included in many of the lessons.

Programming the lessons

The fifteen lessons are presented in a suggested teaching sequence and in three major groups under the headings of:

Introduction to two-line angles

Measuring two-line angles

Measuring one-line angles.

The fifteen lessons, together with suggested extension activities, are intended to be sufficient for a Stage 2 program.

Lessons from the first two groups, *Introduction to two-line angles* and *Measuring two-line angles*, would be suitable for students who are commencing a study of angles.

A program that commences at lesson 8 or lesson 9 and continues to lesson 15 would be suitable for students who have been introduced to identifying and measuring two-arm angles in previous work.

Sequence of angles lessons

Introduction to two-line angles

Lesson 1 Pattern blocks
Students create, describe and draw patterns, using pattern blocks.

Lesson 2 Windmill patterns
Students make *windmill* patterns by fitting pattern blocks of the same colour around a point. They use the patterns to compare the sizes of the pattern block corners.

Lesson 3 Square corners
Students look for right angles in their classroom. They make drawings of the angles and use different methods to measure and compare the angle of the object and the drawn angle.

Lesson 4 Acute and obtuse angles
Students look for acute and obtuse angles in the classroom. They make drawings of the angles, compare the angles with the corners of pattern blocks, and classify the angles according to size.

Lesson 5 Angles in geometrical patterns
Students find and label acute, obtuse and right angles in a pentagram or octagon pattern. Students draw and measure the angles.

Measuring two-line angles

Lesson 6 Equal angles
Students use different methods to check that all of the angles on a windmill pattern are the same size, even though the lines are of different lengths.

Lesson 7 Measuring pattern block angles
Students use the windmill pattern as an angle tester to measure and record the angles on pattern blocks.

Lesson 8 Measuring angles in the classroom
Students use the windmill pattern as an angle tester to measure and record at least three different angles found in the classroom. Students record an acute, an obtuse and a right angle.

Lesson 9 Measuring body angles
Students investigate and record angles made by parts of their body, using the windmill angle tester to measure the angles.

Lesson 10 Drawing two-line angles
Students draw diagrams that can represent angles in any situation. They also investigate the similarity between two-line angles in different locations.

Measuring one-line angles

Lesson 11 Measuring the angle of opening of doors
Students are introduced to the concept of a one-line angle by measuring the angle of opening of a door. Students measure the angle of opening of the door on the house activity sheet and a *floating door*, using pattern block corners.

Lesson 12 Doors that open in different directions
Students investigate the angle of opening in doors that open in different directions. Students use pattern block corners to measure and draw the angle of opening of all doors on the house activity sheet.

Lesson 13 Measuring angles of slope
Students use sloping rulers to identify and investigate angles of slope. Students measure angles of slope with pattern blocks and record by drawing and labelling the angles.

Lesson 14 Clocks
Students identify and describe the angles made by a turning clock hand.

Lesson 15 Drawing two-line and one-line angles
Students match two-line and one-line angles in different situations and explain the main features of an angle.

Teaching materials

The materials for each lesson are listed in the lesson notes. Worksheet masters are included, where necessary.

Pattern blocks: Pattern blocks are used extensively in the angles lessons. Approximately five sets are required for a class completing the pattern-making activities in lessons 1 and 2. In other lessons, individual students or pairs of students need six pattern blocks (one of each shape).

Bent straws: Drinking straws are used as angle testers. The bent straws are made by pushing pipe cleaners into drinking straws. It is preferable to bend the straw so that the arms are not the same length. This will encourage the students to focus on the angle, rather than the length of the arms, when comparing the sizes of angles. The straws retain their shape when transferred from one place to another. Students will generally require one straw each, although the straws can be collected and re-used for further lessons.

Windmill angle tester: Students fold a paper copy of the windmill sheet (page 44) to measure angles.

Windmill copies made on overhead transparencies provide a simple form of a protractor and assist students to identify and measure angles. The use of this material is optional.

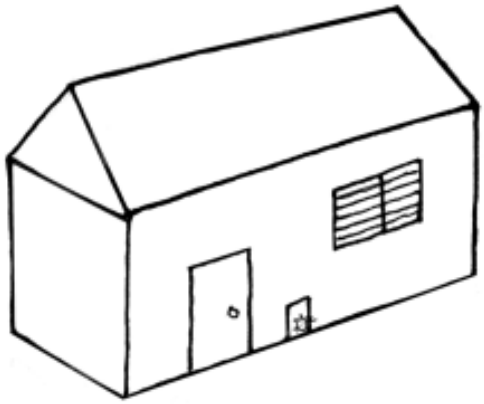
Model house: The lesson notes for lessons 11 and 12 include an optional model house, used to demonstrate opening doors and windows. A simple model may be made from a shoebox.

Instructions:

Draw the door, cat door and shutter openings on one face of a shoebox (similar to the house worksheet in Lesson 11). Do not cut out the openings.

Cut out a door, cat door and shutters from cardboard and tape each of these onto the house along one edge. Note that the cat door opens upwards.

Make and attach a roof.



Student assessment

This book includes sample tasks that can be used to assess students' learning (see pages 54 and 56). Two groups of tasks are provided: one for two-line angles and one for two-line and one-line angles.

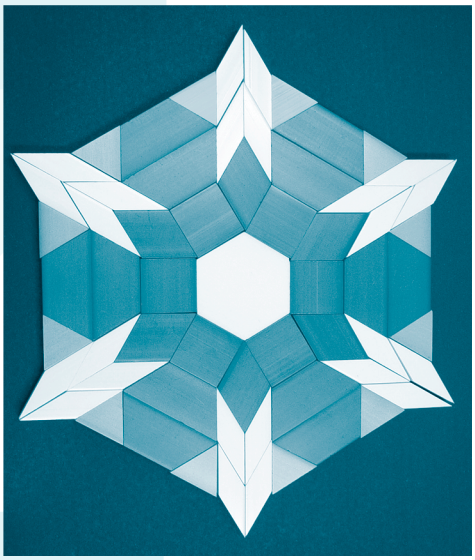
The sample assessment tasks may be used with selected students to monitor the development of the angle concept. The tasks can also act as a source of ideas for constructing class assessment activities.

Glossary

Acute angle	An angle between 0º and 90º
Angle	Two lines meeting at a point, with some meaning given to the relative inclination between the two lines
Arm	One of the two lines which form an angle
Corner	The part of an object where two edges or faces meet
No-line angle	An angle in which no arm is visible
Obtuse angle	An angle greater than a right angle in size
One-line angle	An angle in which only one arm is visible
Reflex angle	An angle between 180º and 360º
Right angle	An angle of 90º
Slope	The acute angle between a line and the horizontal
Straight angle	An angle of 180º
Two-line angle	An angle in which both arms are visible
Vertex	The point where the arms of an angle meet
Zero angle	An angle of 0º

Lesson 1 Pattern blocks

Students create, describe and draw patterns, using pattern blocks.



Main ideas

Construct two-dimensional patterns from pattern blocks.

Recognise that two-dimensional shapes may fit together along their sides or at their corners.

Copy two-dimensional shapes in various orientations.

Syllabus outcomes

SGS2.2a Manipulates, compares, sketches and names two-dimensional shapes and describes their features.

WMS2.3 Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

Notes

1. Each set of pattern blocks consists of orange squares, green equilateral triangles, yellow regular hexagons, red trapeziums with angles of 60º and 120º, and two types of rhombus (a blue rhombus with angles of 60º and 120º, and a white rhombus with angles of 30º and 150º).
2. The most interesting patterns (mathematically speaking) are those that are formed using only one or two different pattern blocks. Students who have not used pattern blocks previously should be encouraged to investigate and create patterns, without any restrictions. If students are already familiar with pattern blocks, they may be asked to use only one or two different blocks.

Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.

Materials

pattern blocks
pencils and paper

Step 1	Questioning
Distribute the pattern blocks so that each group has a large number. Ask the students what they notice about the blocks, in terms of colours and shapes. Discuss the terms *hexagon, rhombus* and *trapezium.* Explain how students will be using the blocks to make patterns.	*What is the same about these shapes?* *What is the same about the red blocks?* *What are these different shapes called?*

Step 2	Check that students:
Have your students work in groups to: • make their own patterns with the pattern blocks • describe their favourite pattern to the group • make a coloured drawing of their favourite pattern.	• select appropriate blocks and make a pattern • use mathematical language when describing their pattern • record the pattern by drawing.

Step 3	Discussion
Discuss the patterns the students have made. Guide students to see that patterns always involve some regular repetition of colours and/or shapes. Discuss the way the pattern blocks fit together. Guide students to see that the blocks all have the same edge-length or multiples of that edge-length, and that the corners fit together in a special way.	*How are these patterns different?* *How are the patterns similar?* *Why do we call them patterns?* *Tell us why you like this pattern.*

Extension

(Barrier Game) Students work in pairs to make and describe patterns with pattern blocks. Student A makes a pattern without Student B seeing it. Student A describes her pattern and Student B makes the same pattern by following her partner's directions.

Lesson 2 Windmill patterns

Students make *windmill* patterns by fitting pattern blocks of the same colour around a point. They use the patterns to compare the sizes of the pattern block corners.

Main ideas

Identify examples of angles as corners of two-dimensional shapes.

Compare corners according to their angular size.

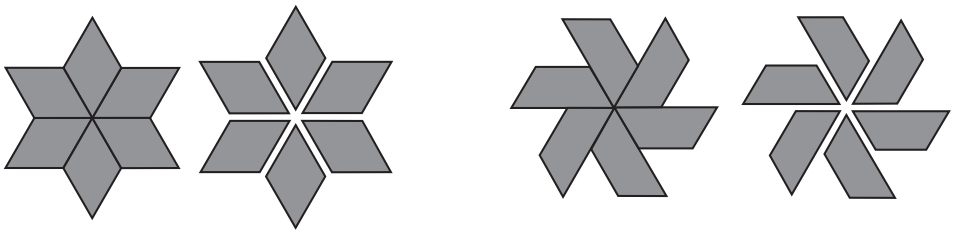
Syllabus outcomes

SGS2.2b Identifies, compares and describes angles in practical situations.

WMS2.2 Selects and uses appropriate mental or written strategies, or technology, to solve problems.

Notes

1. This lesson focuses on the pattern of the lines emanating from a central point that is made when several copies of the same pattern block are fitted around that point.
2. The pattern must initially have all of the edges touching
3. To identify the pattern made by the lines at the centre, slightly separate the blocks.
4. The pattern of lines meeting at the point in the centre has been named a *windmill* in these lessons.



The whole class discussion should result in a table of information:

Hexagon (yellow)	3 corners
Square (orange)	4 corners
Triangle (green)	6 corners
Fat rhombus (blue)	6 small corners or 3 large corners
Thin rhombus (brown or white)	12 small corners
Trapezium (red)	6 small corners or 3 large corners

Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.

Materials

pattern blocks, overhead projector, pencils and paper

Step 1	Questioning
Put a number of blue pattern blocks on the overhead projector. Place them together so that they form a pattern.	*How would you describe my pattern?*
Make sure the students understand that the blocks fit together around a point.	
Separate the blocks and point to the pattern of lines made by the joins between the blocks.	*What do you notice about the lines in the middle of the pattern?*
Decide on a name for such patterns.	*What shall we call this type of pattern?*

Step 2	Check that students:
Have your students work individually to: • make their own windmills using different pattern block corners • draw their windmill patterns • label each drawing to state the number and type of block used, (e.g. *eight of the small red corners*).	• use one colour block to make a pattern • join the blocks together around a point • describe and label their pattern.

Step 3	Discussion
Discuss the different windmill patterns the students have made.	*How are these patterns different or similar?*
Make a table summarising the relationship between the pattern block corner used and the number of pieces needed.	*Which do you think are the best patterns?*
From the table, identify block corners that are the same size. Check by placing one corner on top of the other.	*Why?*
Introduce the mathematical word for corner as *angle*.	*Why do these pattern blocks make such attractive patterns?*
Discuss why some patterns use more blocks than others. Fewer blocks are needed when the angles which are placed around the central point are larger.	*Which blocks can you fit around a central point?*

Extension

Explore which corners combine to make another pattern block corner. For example, two triangle corners make a hexagon corner.

Lesson 3 Square corners

Students look for right angles in their classroom. They make drawings of the angles and use different methods to measure and compare the angle of the object and the drawn angle.

Main ideas

Identify examples of angles as corners of three-dimensional objects.

Recognise right angles in the classroom.

Explain why an angle is a right angle.

Syllabus outcomes

SGS2.2b Identifies, compares and describes angles in practical situations.

WMS2.3 Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

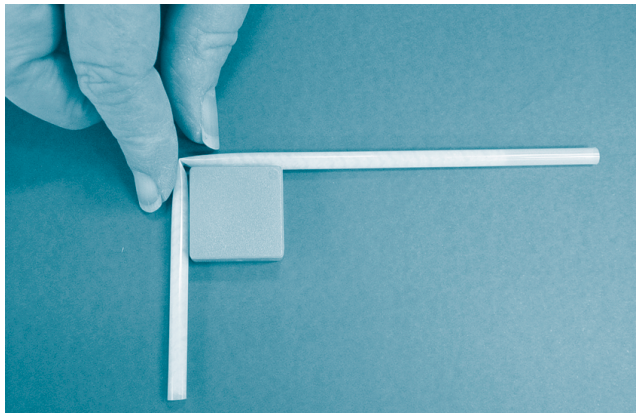
Notes

1 The words *angle* and *corner* can be used interchangeably.

2 Students may find several types of corners. These include the corners of a flat surface, such as a table top, as well as the corners formed by intersections, such as the bars of a window frame. Students may also identify the angle formed in the corner of a room, or the angle between the ceiling and the wall as a corner where two flat surfaces meet.

3 The bent straws are made by pushing pipe cleaners into drinking straws. The straws retain their shape when transferred from one place to another. The bent straws will be used in several lessons.

4 To help students develop the idea of an angle, they need to draw both lines that meet at the corner.



Grouping

Step 1: whole-class explanation and discussion

Step 2: demonstration

Step 3: work in pairs

Step 4: report back to whole class.

Materials

pattern blocks
bent straws
coloured pencils
pencils and paper

Step 1	Questioning
Discuss what an angle is. Use the bent straw to show that an angle has two lines and a point. Explain that the mathematical terms are *arm* and *vertex*.	*What angles can you see in this classroom?*
Introduce the term *right angle* or *square angle*.	*Does anyone know what this kind of angle is called?* (Teacher points to an example of a right angle.)
Students find examples of right angles in the classroom.	*What does it mean to say that a corner is "square"?*
Discuss which pattern block has right angles.	*What does it have to do with squares?*

Step 2	
Demonstrate how to bend a straw into a right angle by folding the straw over one corner of a square pattern block.	*How could we check if this angle really is a right angle?*
Select one of the suggested examples of a right angle and use the straw to demonstrate that the angle is the same size. If possible, check by holding the pattern block against the angle.	
Draw the object and model how to use the bent straw to compare the drawn and the actual angle.	

Step 3	Check that students:
Have your students work in pairs to: • search for objects or locations that have right angles in the classroom • make a sketch of the object and mark the angle(s) in colour • use the bent straw and the square pattern block to check that the drawn angle is the correct size.	• draw an angle with both lines meeting at the corner • use the bent straw or pattern block to compare the size of the angle on the object with the size of the drawn angle.

Step 4	Discussion
Discuss and list the different examples of right angles that students have measured.	*How many right angles do you think there would be in this room?*

Variation

Find right angles in the playground and check the size using the drinking straw angle tester or the square pattern block.

Lesson 4 Acute and obtuse angles

Students look for acute and obtuse angles in the classroom. They make drawings of the angles, compare the angles with the corners of pattern blocks, and classify the angles according to size.

Main Ideas

Identify examples of angles as corners of three-dimensional objects.

Identify acute and obtuse angles in the classroom.

Explain why an angle is obtuse or acute.

Syllabus outcomes

SGS2.2b	Identifies, compares and describes angles in practical situations.
WMS2.3	Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

Notes

1. To help students develop the idea of an angle, they need to draw both lines that meet at the corner.
2. Some angles will not exactly match any pattern block corner. Answers such as *A bit bigger than a small red corner* are acceptable.

Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.

Materials

pattern blocks
bent straws
pencils and paper

Step 1	Questioning
Revise previous work with right angles, and discuss the terminology used to describe angles.	*Find some angles in this classroom that are not right angles.*
Introduce the terms *acute* and *obtuse* and discuss their relationship to the right-angle.	*Can you explain what acute and obtuse angles are?*
Ask students to identify acute and obtuse angles in the classroom, and list several of these.	*Where can you see acute and obtuse angles in the room.*
Select one example of an acute angle. Demonstrate how to use a bent straw to measure and draw the angle on the chalkboard. Use the straw to compare the size of the drawn angle with the angle on the object.	*How do you know the angles are the same size?*
Find a pattern block that has an angle about the same size, and label the angle drawing with the pattern block colour or shape.	*Can you suggest a pattern block angle which is about the same size as our drawn angle?*

Step 2	Check that students:
Have your students work in pairs to: • look for acute and obtuse angles in the classroom • use the bent straw to measure the angle • draw and label the angle and use the bent straw to measure the correct size • find a pattern block that has an angle about the same size as the angle drawn • order the angles from smallest to largest by numbering them.	• draw the angle with both lines which meet at the corner • use the bent straw or pattern block to compare the size of the angle on the object with the size of the drawn angle.

Step 3	Discussion
Discuss different examples of angles that students have measured and list them under the headings *acute* and *obtuse*.	*What is the difference between obtuse and acute angles?*
Write the size of the angle in terms of the pattern blocks, e.g. the same as a small red corner	*How would you describe an acute angle to your friend?* *Which pattern blocks have obtuse angles?*

Extension

Look for reflex angles (angles greater than two right angles) in the classroom. Find examples in the classroom and ask students to describe these. Examples may include the angle outside the corner of a desk or book.

Lesson 5 Angles in geometrical patterns

Students find and label acute, obtuse and right angles in a pentagram or octagon pattern. Students draw and measure the angles.

Main ideas

Identify and copy examples of angles as corners of two-dimensional shapes.

Identify the arms and vertex of an angle.

Identify acute, obtuse and right angles.

Syllabus outcomes

SGS2.2b Identifies, compares and describes angles in practical situations.

WMS2.3 Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

Notes

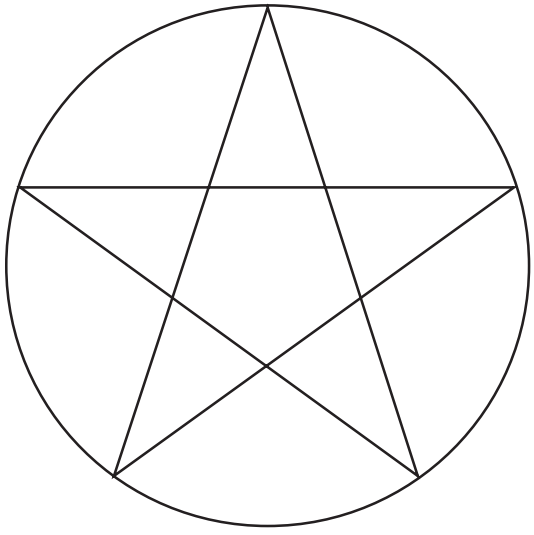
1. This lesson has a pentagram worksheet and an octagon worksheet. Teachers may choose one or both worksheets for the lesson.
2. Some students may need assistance to copy and draw the angles approximately the correct size.
3. The bent straws are made by pushing pipe cleaners into drinking straws. The straws retain their shape when transferred from one place to another.
4. The pentagram (not to be confused with a pentagon) used to be regarded as a symbol of good fortune and was often used to ward off evil spirits.
5. The pentagram was also used by a secret society of mathematicians in ancient Greece (the Pythagoreans) as the way they recognised each other. They chose a regular pentagram that was perfectly symmetrical and inscribed in a circle.

Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.



Materials

bent straws
geometric pattern activity sheets
(pages 42, 43)
coloured pencils

Step 1	Questioning
Revise the terms *right*, *acute* and *obtuse* angles. Ask a student to use a bent straw to demonstrate the mathematical terms *arm, vertex* and *angle*. Introduce the pentagram or octagon worksheet and discuss the instructions.	*What types of angles have we been talking about?* *What are the differences between these angles?* *What kinds of angles can you see in this pentagram? Why do you think it is called a "pentagram"?*

Step 2	Check that students:
Ask your students to mark the angles on their worksheets. Remind students to copy each angle as accurately as possible by using the bent straw to measure and compare the two angles.	• identify obtuse, acute and right angles • identify the arms and vertex of an angle • use the bent straw to measure and draw the angles accurately.

Step 3	Discussion
Lead students in a discussion of the angles they identified. Ask several students to draw their angles on the board, and describe the angle using the terminology *arms* and *vertex*.	*Draw one of your angles on the chalkboard.* *How do you know that it is the right size?* *What can you tell us about your angle?*

Extension

Look for reflex angles (angles larger than two right angles) and straight angles (180º) on the geometrical patterns.

Lesson 6 Equal angles

Students use different methods to check that all of the angles on a windmill pattern are the same size, even though the lines are of different lengths.

Main ideas

Determine whether two angles are the same size.

Understand that the size of an angle depends on the inclination between the two lines and not on the lengths of the lines.

Outcomes

SGS2.2b	Identifies, compares and describes angles in practical situations.
WMS2.3	Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

Notes

1. Many students initially believe that the angles at the top and bottom of the windmill activity sheet (page 44) are bigger than the angles at the sides. Checking that all the angles are equal helps them to distinguish angle from length and area.
2. The windmill pattern used in this lesson has the same angles as the one made in lesson 2 using the acute corner of the thin rhombus. The difference here is that the lines are of different lengths.
3. The teacher should prepare a cardboard segment for each student by cutting cardboard copies of the windmill activity sheet into twelve segments.
4. The windmill sheets will be used again in lessons 7 and 8.

Directions for folding the windmill activity sheet

(a) Fold the paper in two along the horizontal line, and use a fingernail to make a sharp crease.

(b) Fold the paper in two along the vertical line, and check that the first crease folds over onto itself. Sharpen the crease.

(c) Now fold the paper in three along the remaining lines, and sharpen the creases. The windmill angle tester should look like this:

(d) Open out the paper to show the twelve angles again.

This demonstration shows conclusively that the twelve angles of the windmill are of equal size.

Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: demonstration and discussion.

Materials

pattern blocks, windmill sheets, cardboard windmill segments, bent straws

Step 1	Questioning
Show students the windmill pattern and focus on the angles around the centre point. Ask students to estimate whether the angles are the same size. Discuss how to check if the angles are all the same. Possible methods include: 1. cut out one angle and see if it fits the others 2. copy one angle using a bent straw and see if it fits the others 3. see if the same pattern block corner fits all the angles. Introduce any of these methods that students do not suggest. Explain to the students that they will use two of the three methods to measure the angles to see if they are the same size.	*What do you notice about these angles at the centre of the windmill?* *How can we check if the angles are the same size?*

Step 2	Check that students:
Have your students choose two ways to measure the angles on their windmill sheet. Strategies may be: 1. using a windmill segment 2. using a bent straw to measure the angles 3. finding a pattern block corner that fits into each angle of the windmill pattern.	• use two methods to measure and compare the angles • understand that the angles are the same size even though the arms of the angles are different lengths.

Step 3	Discussion
Demonstrate to the students how to fold the windmill sheet and have them fold their windmills. Discuss the results of using the different methods. Discuss that the size of an angle depends on the way the lines meet, not on the length of the lines. Choose a class name for the angle. (Do not mention degrees at this point.)	*What did we find when we measured the angles?* *Why do the angles on the windmill look as if they are different sizes?* *What could we call an angle of this size?*

Lesson 7 Measuring pattern block angles

Students use the windmill pattern as an angle tester to measure and record the angles on pattern blocks.

Main ideas

Measure angles by using an informal unit.

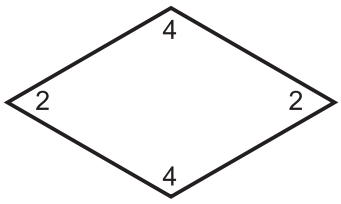
Syllabus outcomes

SGS2.2b Identifies, compares and describes angles in practical situations.

WMS2.3 Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

Notes

1. Some students find it difficult to disembed the angle to be measured from the pattern of lines in the windmill.
2. The intermediate lines must be ignored, while at the same time they indicate how many windmill angles are included. Disembedding is much easier using a transparent windmill.
3. A useful teaching strategy is to copy the windmill onto overhead transparencies (one for each student or pair of students), thus creating informal protractors.
4. The size of an angle is the number of angle units that fit into it, just as the length of a line is the number of length units that fit into it, the area of a region is the number of area units that fit into it, and the volume of a container is the number of volume units that fit into it. Students need to understand that the angle tester is used by counting the number of spaces (angle units), not the number of lines.



5. The completed worksheet should have the results recorded inside each pattern block.

Other answers on the worksheet:

white block: 1, 5, 1, 5

orange block: 3, 3, 3, 3

green block: 2, 2, 2

yellow block: 4, 4, 4, 4, 4, 4

red block: 2, 4, 4, 2

Grouping:

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.

Materials

pattern blocks
windmill sheets (page 44)
optional windmill protractors (see Note 3)
pattern blocks worksheets (page 45)
pencils

Step 1	Questioning
Using a square pattern block, discuss how the windmill sheet can be used as an angle tester. Place any right-angled corner at the centre of the windmill sheet and show that it covers 3 windmill angles. Find some more right angles on the sheet. Use the windmill sheet to measure the size of the corners of the blue pattern block. Discuss how to complete the pattern blocks worksheet.	*How could I use the windmill sheet to measure the corners of this square pattern block?* *How many right angles are there on the windmill sheet?* *Can you estimate how many windmill angles will measure the corners on the blue rhombus block?*

Step 2	Check that students:
Have your students work in pairs to: • measure the other pattern block corners and record their results on the recording sheet • discuss their results with a partner or small group, focusing on a comparison of the sizes of the angles.	• measure and record all of the pattern block angles • check the angles by putting the corners of pattern blocks on top of each other • discuss results with their partner.

Step 3	Discussion
Discuss the students' results which may include: – all the yellow angles are the same – a small blue angle is the same as a green angle – two small blue angles make a large blue angle – a small blue angle and a small white angle make an orange angle. Discuss the unit that has been used to measure and compare the angles.	*Which corners are the same size?* *What does this tell you about those corners? How could you check?* *What do our results tell us about the size of the two blue corners? How can you check this?* *Can you explain any other relationships between the sizes of the block corners? How can you check these?*

Extension

Many students will know that a right angle is also referred to as 90º. Since three windmill angles make a right angle, it seems reasonable to call this angle 30º. Students can then work out the sizes of the other pattern block corners (30º, 60º, 90º, 120º and 150º) and may be able to measure angles in the classroom to the nearest 10º.

The Babylonians invented the degree as a unit for measuring angle long ago. There are 360º around a point, and the Babylonians believed there were 360 days in a year.

Lesson 8 Measuring angles in the classroom

Students use the windmill pattern as an angle tester to measure and record at least three different angles found in the classroom. Students record an acute, an obtuse and a right angle.

Main ideas

Measure angles by using an informal unit.

Copy angles by drawing, and labelling the size of the angle.

Outcomes

SGS2.2b	Identifies, compares and describes angles in practical situations.
WMS2.3	Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

Notes

1. When students use the windmill angle tester to measure angles, some angles will be close to a whole number of windmill angles, but most will not. Students may need to approximate when recording, e.g. *between one and two windmill angles* or *a bit bigger than five windmill angles*. Students may also use fractions where appropriate, as in *a little more than 2 1/2 windmill angles*.
2. Acute angles can range from zero, to just under three windmill angles. A right angle is exactly three windmill angles. Obtuse angles range from just over three to just under six windmill angles (reflex angles lie between six and twelve windmill angles).

Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.

Materials

bent straws
windmill sheets or transparencies (page 44)
pencils and paper

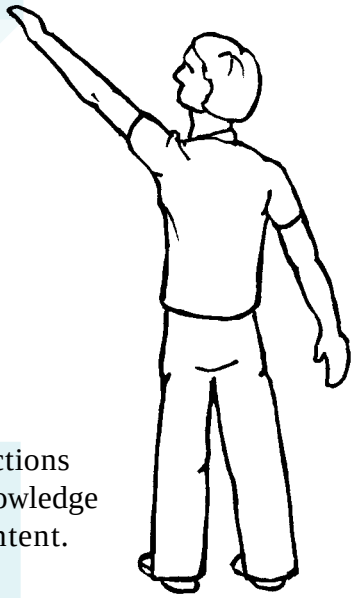
Step 1	Questioning
Discuss how the angles on the windmill sheet can be used as informal units to measure other angles. Discuss strategies that students might use to copy and measure angles in the classroom in terms of *windmill* units. Students may suggest: take the windmill sheet or transparency to the object and lay over or into a corner measure the angle with the folded windmill angle tester measure the angle with a bent straw and then lay the straw on the windmill sheet. Discuss the activity: students measure, draw and label at least three corners (including an acute, an obtuse and a right angle) in the classroom.	*What have we learnt about angles?* *How would I measure an angle?* *How have we used the windmill angle tester to measure other angles?*

Step 2	Check that students:
Have your students work in pairs or small groups to identify, measure, copy and label angles.	• use the angle tester to measure different angles in the classroom • copy angles accurately • measure and record the results • can identify acute, obtuse and right angles.

Step 3	Discussion
Discuss students' responses and the range of angle sizes found.	*Who had the smallest angle?* *Who had the largest angle?* *How big are these angles?* *How many windmill angles fit into an acute angle?* *How many windmill angles fit into an obtuse angle?*

Lesson 9 Measuring body angles

Students investigate and record angles made by parts of their body, using the windmill angle tester to measure the angles.



Main ideas

Make angles with parts of the body.

Measure angles by using an informal unit.

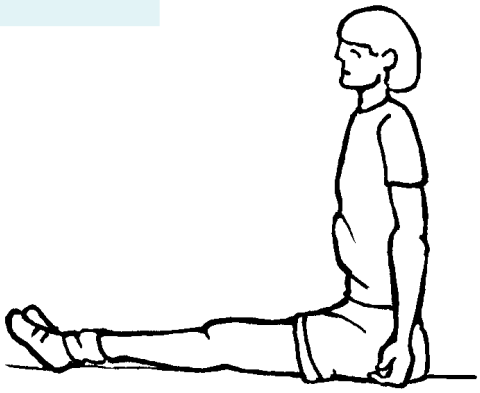
Find the range of variation in the size of a movable angle.

Outcomes

SGS2.2b	Identifies, compares and describes angles in practical situations.
WMS2.5	Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 2 content.

Note

Most students will use their windmill angle testers to measure the angles, but students who have learnt about degrees may give answers in terms of degrees.



Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.

Materials

windmill sheets, body angles sheets (page 46), pencils and paper

Step 1

Discuss different angles that can be made with the human body.

Stand with one arm straight out to the side, then bend your arm at the elbow. Have students do the same, and discuss the angles they can make.

Revise the use of terminology *arms* and *vertex*, before discussing the arms and vertex in the body angles.

Ask a student to hold her arm out straight and ask what the angle is at the elbow. Introduce the term *straight angle*.

Questioning

Can you show us how to make an angle with a part of your body?

What angles can you see when I bend my arm like this?

How would you describe these angles?

What angles can you make with your elbow?

When I make an angle with my elbow, where are the arms and the vertex of the angle?

How big is the angle at the elbow when an arm is held straight out from the body?

What could we call this angle?

Step 2

Have your students work in pairs to:

- make different body angles and discuss these with their partner
- complete the body angles sheet.

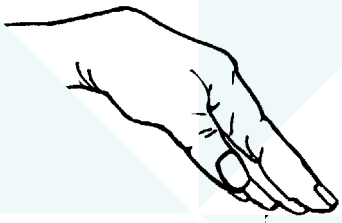
Check that students:

- identify the arms and vertex in the body angles
- use the windmill angle tester to measure and draw the angles.

Step 3

Discuss students' answers to the body angles questions.

Focus on the largest and smallest angles which students can make by bending their wrists.



Discussion

What were the easiest angles to find or make?

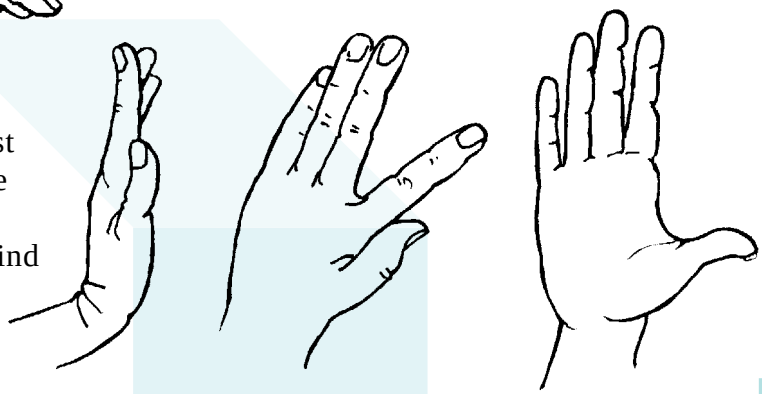
Can anybody tell us about body angles which we haven't already discussed?

What are the largest and smallest angles you can make with your wrist?

Can you estimate the size of these in windmill angles?

Extension

Point out that some angles go beyond a straight angle, e.g. most people can bend their wrist more than six windmill angles. Such angles are called reflex angles. Find some more examples.



Lesson 10 Drawing two-line angles

Students draw diagrams that can represent angles in any situation. They investigate the similarity between two-line angles in different locations.

Main ideas

Identify and describe angles in two-dimensional shapes and three-dimensional objects.

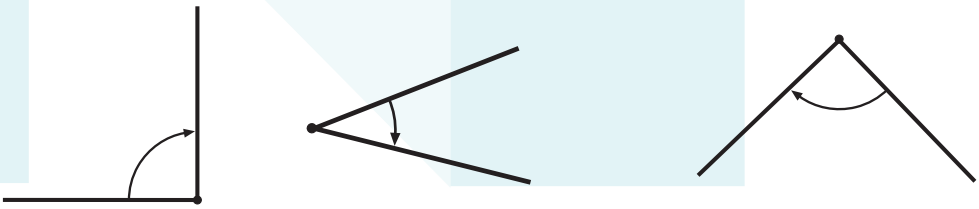
Draw abstract diagrams to represent angles in various contexts.

Outcomes

SGS2.2b	Identifies, compares and describes angles in practical situations.
WMS2.2	Selects and uses appropriate mental or written strategies, or technology, to solve problems.

Notes

1. Drawings of angles may include the following diagrams:



2. Each diagram shows the three crucial features of an angle: the lines show the arms of the angle, the dot shows the vertex, and the arc shows the opening. The opening must be the correct size, but the length of the arms is unimportant.
3. *Angle* is an **abstract concept** because it represents the same idea occurring in different situations; it is *abstracted from* all those contexts. Similarly, the angle diagrams above are called **abstract** diagrams because they do not represent any particular angle but what is common to all angles of that size, in different situations.
4. It is difficult for students to say exactly what an angle is. An appropriate definition at this stage is to say that it consists of two lines, a point, and an amount of opening.
5. Teachers may wish to highlight the language involved in angles. For example, the arms of an angle may be called edges if the angle refers to a corner, but they would probably be called lines in the case of a sloping or turning object. Similarly, the vertex of an angle may be called a point, a pivot, a joint or a hinge, depending on the context.

Grouping

Step 1: whole-class explanation and discussion

Step 2: work individually or in pairs

Step 3: report back to whole class or larger group.

Teacher's materials

objects with movable arms

Students' materials

drawing two-line angles sheets (pages 47 and 48), pencils and paper, access to angle testers and pattern blocks

Step 1	Questioning
Revise and discuss situations in which the size of an angle may change. These may include body angles, the hands of a clock, or scissors.	*We have discussed how the angles on some objects are fixed or don't change, and angles on other objects can change by opening or turning.*
Discuss how angles on objects or in different situations can be fixed or changeable.	*Tell us about some angles in this room that are fixed.* *Tell us about objects in this room that have changeable angles*
Discuss how to draw an angle diagram that could represent any of these situations (see Note 1) and ask students to demonstrate on the chalkboard.	*How can you draw an angle so that it can look like either a fixed angle or one that can be changed?*
Ask students to suggest the angles on objects or shapes that could be represented by the angle diagrams on the chalkboard.	*You can't tell from these diagrams what objects or shapes they come from. What real things could have made these angles?*
Introduce and discuss the drawing two-line angles sheet.	

Step 2	Check that students:
Have your students work individually or in pairs to complete the drawing two-line angles sheet.	• understand a moveable angle and a fixed angle • measure the angles to compare the size • draw all three parts of an angle • can identify objects that make different angles.

Step 3	Discussion
Discuss students' answers to the worksheet questions.	*What is the same about all the angles you have found?*
Review the different types of angles students have identified.	*What can you tell us about the parts of these angles?*
Review the different parts of angles on a variety of objects.	*What have you learnt about angles?*

Optional extension

Discuss what it means to say that angle is an **abstract** concept (see Note 3).

Lesson 11 Measuring the angle of opening of doors

Students are introduced to the concept of a one-line angle by measuring the angle of opening of a door. Students measure the angle of opening of a door using the house activity sheet and a *floating door*, using pattern block corners.

Main ideas

Identify the arms and vertex of the angle of opening for a door.

Describe the imaginary closed position in an opening.

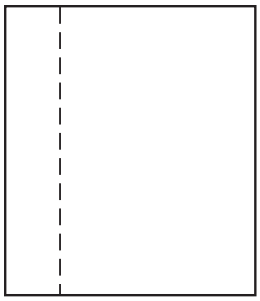
Outcomes

SGS2.2b Identifies, compares and describes angles in practical situations.

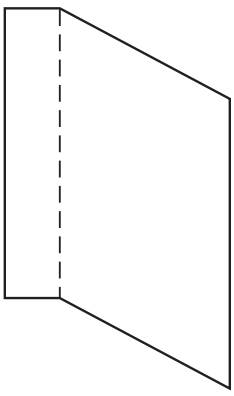
WMS2.3 Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

Notes

1. The angle of opening is the angle between the top edge of the door and the top of the doorframe. If there is a line on the floor marking the closed position of the door, the angle of opening is also the angle between this line and the bottom edge of the door.



2. On the house activity sheet, students should cut along the vertical line and remove the strip of paper. Students then cut along the broken lines on the main door so that it can be opened. Do not cut the other doors for this lesson. The house worksheet will be used again in lesson 12.
3. Teachers may choose to make a simple model house to demonstrate door angles in lessons 11 and 12. Instructions for making a house are in *Teaching Materials*, page 10.
4. The floating door is made by folding the piece of card and holding it upright on the desk to represent a door. To form the angle of opening, the closed position has to be imagined or remembered.



Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.

Materials

For each pair of students: house activity sheet (page 49), A5 card, set of six pattern blocks, scissors and bent straw; optional model house for teacher's demonstration

Step 1

Open and close the classroom door slowly. Discuss how the door turns or pivots on the hinges.

Discuss the angle of opening of the door by looking at the top edge and then the bottom edge. Discuss how to visualise the arm formed by the doorway at the bottom edge.

Demonstrate opening the classroom door to about 45º and the door on the house worksheet or the model house to about 45º and use a bent straw to check that the angles are equal. Discuss how the angle could be measured with pattern block corners.

Questioning

Describe what is happening when this door opens and closes?

What allows the door to swing this way?

How could we describe this in mathematical terms?

How could I measure the angle of opening?

How could I make the same angle of opening with the model door or house worksheet door? How could I measure this angle?

Step 2

Activity A

Have pairs of students prepare their house worksheets (see notes) and lay the sheets on their desks. Explain how Student A will select a pattern block angle and open the house door to match the angle without their partner seeing. Student B will estimate which pattern block angle was chosen. The players measure the angle and then reverse the roles.

Activity B

Demonstrate to the students how to fold the A5 card to make a floating door. Hold the floating door upright on a desk. Discuss how one arm of the angle must be imagined when the door is opened. Ask your students to make a floating door and repeat the activity of measuring the opening with a pattern block.

Check that students:

- identify the arms and vertex of the angle
- place the block in the opening correctly to open the door
- can describe the imaginary closed position of a door opening.

Step 3

Discuss the different angles that can be made when the door is opened.

Ensure students understand that part of the angle when a door is opened needs to be imagined or remembered, as it cannot be seen.

What are the largest and smallest angles you can make when you open the door?

In an angle of opening, where is the vertex? Where are the arms of the angle?

Lesson 12 Doors that open in different directions

Students investigate the angle of opening in doors that open in different directions. Students use pattern block corners to measure and draw the angle of opening of all doors on the house activity sheet.

Main ideas

Match angles in doors opening in different directions.

Identify the imaginary arm in an angular opening.

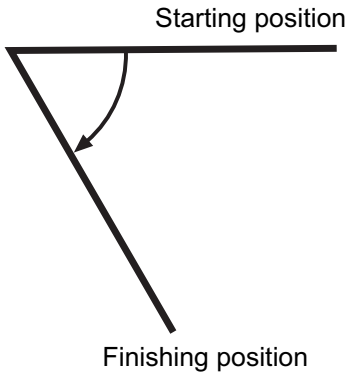
Differentiate between *angular opening* and *gap opening* for doors.

Outcomes

SGS2.2b	Identifies, compares and describes angles in practical situations.
WMS2.3	Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

Notes

1. Guide students to prepare the house activity sheet by cutting the broken lines on the cat door and shutters. To cut the shutters, fold the paper gently across the middle of one of the broken lines and make a small snip. Then smooth out the fold and cut the remaining lines.
2. The shutters are smaller than the door, and open in both directions.
3. The cat door opens in a vertical direction. The difference in direction often makes it difficult to compare angles of opening between such objects and *normal* doors.
4. There are two ways of saying how far open a door or shutter is: (a) by the angle it has turned through, and (b) by the *gap* it leaves. Both meanings are legitimate, but students need to realise that difference. If two doors are open the same amount, the gap will be smaller on the smaller door.
5. Emphasise that an angle of opening is measured from the starting position to the finishing position. A directional arc, joining the starting position to the finishing position, can represent the angle of opening.



Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.

Materials

For each pair of students: house activity sheet (page 49), set of six pattern blocks, scissors, pencils and paper; optional model house for teacher's demonstration

Step 1

Demonstrate how to cut the cat door and shutters.

Discuss the way the shutters and cat doors open. The angle of opening is still measured between the open position and the closed position, with the vertex of the angle at the hinge.

Demonstrate how to draw an angle of opening. Discuss how to measure and compare the angles.

Questioning

What do you notice about the different ways in which the doors and shutters open?

What kinds of angles could we show with each of the opening doors?

In an angle of opening, one line is the object itself. Where is the other line? How can you check that the angles are the same?

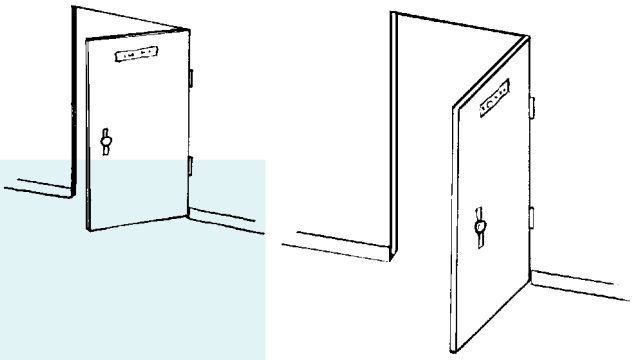
Step 2

Have your students working in pairs with the house activity sheet, pattern blocks, pencils and paper. Explain how to complete the activity:

- player A selects a pattern block and nominates one of its corners
- player B uses estimation to open the shutter, door and cat door to that amount
- player A tests the angles with the pattern block
- students then reverse the roles
- students draw one of their door, shutter or cat door angles, by drawing the arms and a directional arc.

Check that students:

- open the shutter or cat door to the exact size of the selected pattern block corner
- estimate before checking the angles
- draw the angle of the open door.



Step 3

Demonstrate on the model house or house worksheet. Open the doors and shutters to about 45º (half a right angle). Observe that a man could get through the door but not through the cat door.

Open the doors so that the *gaps* are the same.

Discuss the sizes of the angles and note that they are different. Discuss the two meanings of *opening* for door-like objects.

Discussion

What's the difference between these two angles?

How could you check whether the angles are different?

What is happening to the angles as I open the cat door wider?

How can we describe the size of an angle formed by an opening door?

Extension

Identify other objects that open like the cat door, e.g. tilting windows, window awnings or in other directions, e.g. box lids. If possible, use the green triangle pattern block to set them all to a 60º opening. Discuss the different openings made by the various objects.

Lesson 13 Measuring angles of slope

Students use sloping rulers to identify and investigate angles of slope. Students measure angles of slope with a bent straw and record by drawing the angles.

Main ideas

Identify the arms and vertex of an angle of slope.

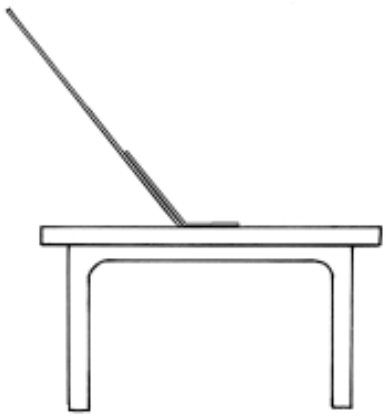
Describe the imaginary horizontal arm in slopes of various orientations.

Outcomes

SGS2.2b	Identifies, compares and describes angles in practical situations.
WMS2.2	Selects and uses appropriate mental or written strategies, or technology, to solve problems.

Notes

1. The intuitive idea of *horizontal* is crucial to the understanding of slope.
2. Many students will say that the ruler is *getting higher* (meaning that the end of ruler is getting higher.) This is correct, but it has a problem: Two rulers of different length can have the same slope but one will be higher than the other. It is better to measure slope by the angle between the ruler and the horizontal.
3. Emphasise that the angle of slope is the angular amount of movement from the horizontal.
4. By convention, an angle of slope cannot be obtuse. For a ruler sloping up to the left, the angle of slope is taken as the acute angle rather than the obtuse angle. The reason is that as the slope increases (gets steeper) the acute angle increases but the obtuse angle decreases.



Grouping

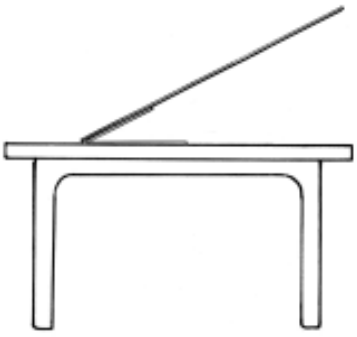
Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.

Teacher's materials

metre ruler, 30 cm ruler



Students' materials

For each pair of students: 30 cm ruler, slopes sheet (page 50)
set of six pattern blocks
pencils

Step 1	Questioning
Discuss the meaning of *horizontal* and identify familiar surfaces such as the horizon, a tabletop and still water. Lay the metre ruler flat on the table and discuss that the ruler is horizontal wherever it has been placed. Raise one end of the ruler from the horizontal and talk about its increasing steepness. Discuss how the angle it makes with the horizontal, or the slope, is also increasing. Hold the metre ruler at a slope of about 30°, and have a student hold a 30 cm ruler on the table at the same slope. Use a bent straw to check that the angles of slope are the same. Emphasise that one line of this angle is the horizontal on the table. Discuss the range of possible slopes. An angle of slope can vary between zero (when the ruler is horizontal) to a right angle (when the ruler is vertical).	*This tabletop is horizontal. What does that mean?* *Is this ruler horizontal? Does it stay horizontal when I move it about?* *The ruler is getting steeper. What does that mean?* *How does the sloping ruler make an angle? What can you tell us about the parts of this angle?* *How can we measure slope?* *What do you notice about the slope of the two rulers?* *How can we compare the slope of the two rulers?* *What is the smallest possible slope?* *What is the largest possible slope?*

Step 2	Check that students:
Have your students work in pairs with the slopes worksheet, 30 cm ruler, pattern blocks and pencils. Explain how to complete the worksheet: • Student A holds the slopes sheet vertically. Student B holds a 30 cm ruler with one end on the table to match the slope in task 1. • Student A visually checks that the slope is correct. • Student B draws a horizontal line on the worksheet to represent the table in task 1. • Student B copies the angle of slope by drawing the two arms on the right-hand side of the worksheet. Student A checks the angle of slope with a bent straw.	• make the slope of the ruler the same as the worksheet • look for the horizontal when completing the worksheet. • alternate roles between Student A and Student B.

Step 3	Discussion
Discuss students' responses to the worksheet exercises. Hold the metre ruler in the air. Ensure students understand how to find the angle of slope.	*What is the same about all of the angles on your worksheet?* *How can I find the angle of slope?* *What slopes in our playground could be measured?*

Lesson 14 Clocks

Students identify and describe the angles made by a turning clock hand.

Main ideas

Understand that time is shown by the angle a clock hand turns from the commencing position.

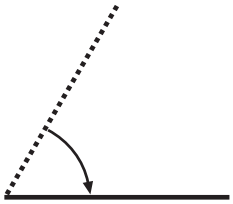
Identify the arms and vertex of the angle in a turn where one arm is visible.

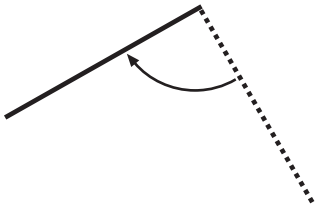
Outcomes

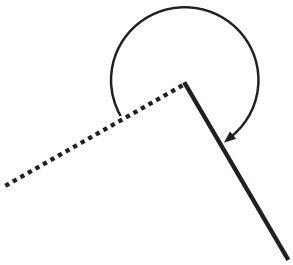
SGS2.2b	Identifies, compares and describes angles in practical situations.
WMS2.3	Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

Notes

1. It may be helpful to draw a line through the middle of each hand of the demonstration clock in order to make the angle of turn clearer. If so, do this before the lesson.
2. When demonstrating the clock angles activity, show students how to draw the angle on a diagram with a directional arc. (This may have been done in Lesson 10.) Emphasise the way the arrow points *from* the starting position *to* the finishing position. If students have already learnt about degrees, they could give their answers in degrees instead of windmill angles.







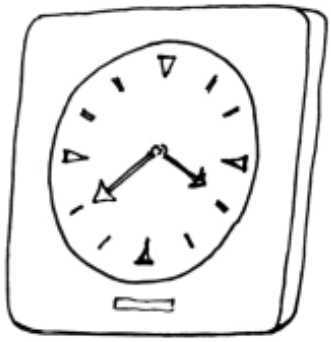
3. Students may also be asked to describe different types of angle in terms of time. For example, a right angle corresponds to three hours, an acute angle to any time between zero and three hours, and an obtuse angle to any time between three hours and six hours. A straight angle corresponds to six hours and a zero angle to zero hours. This discussion may extend to reflex angles and angles equal to or greater than a full turn.

Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.



Teacher's materials

teacher's demonstration clock, windmill sheet, clock face sheet

Students' materials

Windmill sheets, clock face sheet for each pair of students (page 51), clock angles sheet (page 52), pencils

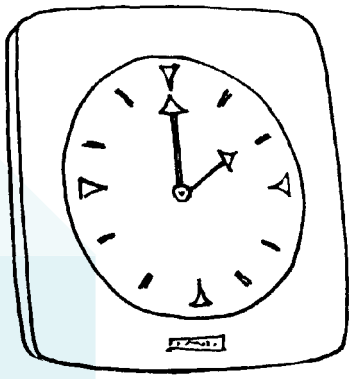
Step 1	Questioning
Hold both hands of the demonstration clock so they move as one. Turn the hands from 12 to 2 and ask how much the hands have turned.	*How does that turn make an angle?*
Return both hands to 12 and hold one hand while turning the other hand to 2.	*Where are the parts of the angle?*
Show that the turn from 12 to 2 makes an angle about the central pivot point.	
Repeat these movements with a different finishing point, (e.g. from 12 to 5) and then from a different starting point, (e.g. from 6 to 10).	*Where have you seen a pattern like this before?*
	How can you check that the angles are all the same size?
Discuss how the clock face is similar to the windmill sheet.	*When the hand moves from 12 to 3, how many hours is that? How many windmill angles is that? How could you draw a diagram of that angle?*
Demonstrate the clock angles sheet by working through the first two examples.	

Step 2	Check that students:
Have your students work in pairs to complete the clock angles sheet (page 52).	• understand how to make the angle with the hands of the clock • measure the angle in *windmill* angles • draw and label the angles correctly.

Step 3	Discussion
Discuss students' responses to the clock angles exercises and focus on the two lesson objectives (see Note 3).	*What angle on the clock represents one hour?*
On the demonstration clock, turn the hour hand from 12 to 3. Then turn the minute hand from 12 to 3. Emphasise that the angle is the same although the hands are different lengths.	*How can the angle be the same when the hands are different lengths?*

Extension

Discuss *reflex* angles (angles greater than a straight angle) in terms of how the hands on a clock turn. A clock-hand can also make *a full turn* (12 hours) or even larger angles.



Lesson 15 Drawing two-line and one-line angles

Students match two-line and one-line angles in different situations and explain the main features of an angle.

Main ideas

Match angles across situations where either one or both arms of the angle are visible.

Draw and describe angles in a wide variety of angle situations.

Outcomes

SGS2.2b	Identifies, compares and describes angles in practical situations.
WMS2.5	Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 2 content.

Note

Teachers may wish to highlight the language involved in angles. For example, the arms of an angle may be called edges if the angle refers to a corner, but they would probably be called lines in the case of a sloping or turning object. Similarly, the vertex of an angle may be called a point, a pivot, a joint or a hinge, depending on the context.

Grouping

Step 1: whole-class explanation and discussion

Step 2: work in pairs or small groups

Step 3: report back to whole class.

Teacher's materials

demonstration clock, pattern blocks, bent straw, windmill angle tester

Students' materials

task cards
pencils and paper

Step 1	Questioning
Demonstration: Face the front with one arm stretched out in front of you. Turn to the right so that your arm turns through 90º. Ask a student to draw the angle on the board. Ask a volunteer student to make the same angle on the demonstration clock. Repeat for a 180º turn and different starting times on the clock. On the demonstration clock, turn both hands together from 12 to 2. Ask a student to open the classroom door to the same angle. Ask how to check that the angles are equal. Identify the matching parts of the angles in the clock and door situations. Discuss how to make one drawing that could represent both situations.	*Where is the angle in the turn I mad[illegible]?* *What sort of angle is that?* *If a clock hand started at 12 and turned through the same angle, what time would it be? How can you check?* *Can you open the classroom door to the same angle as the clock has turned?* *How will we know that the two angles are the same?* *When you match those two angles, which parts match?* *How can I make one diagram to show the angle in both these situations?* *How does this diagram represent both the clock angle and the door angle?*

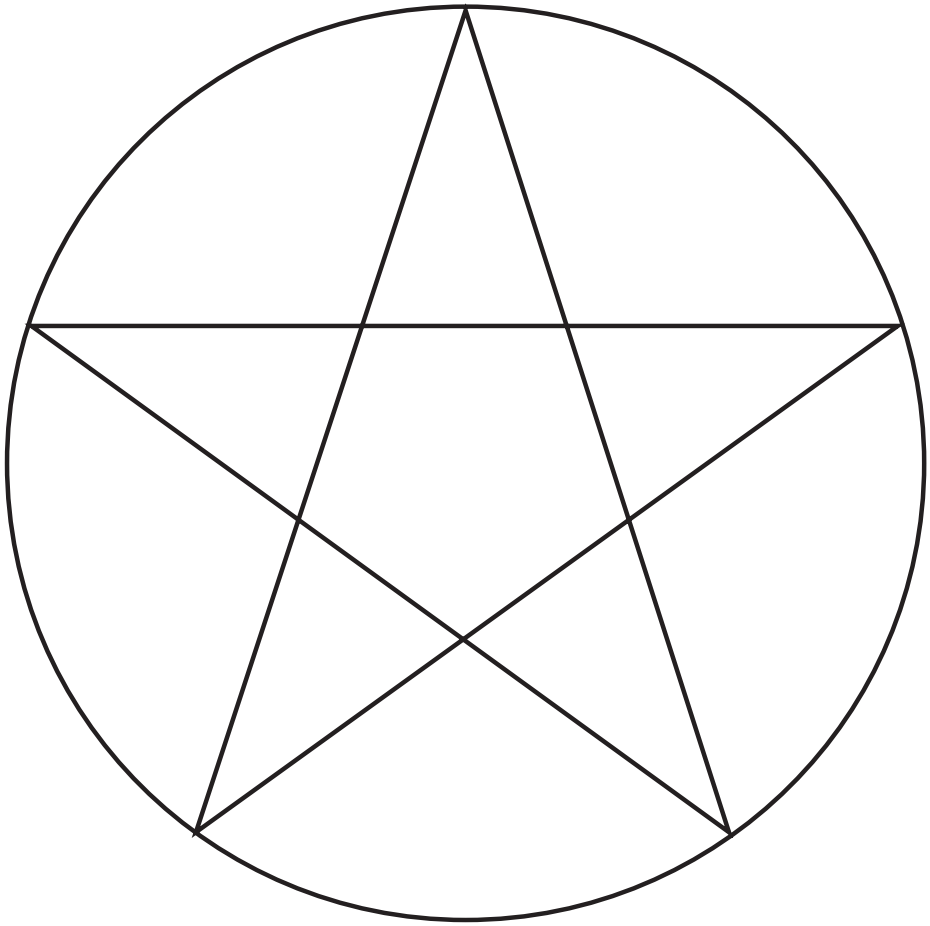
Step 2	Check that students:
Have your students work in pairs. Each pair of students selects a task card (page 53). Ask students to find three angles that could be represented by the angle on their task card, identifying two and one-line angles. Students draw and label the three situations or objects and highlight the angles.	• recognise angles on different objects and in different situations • draw all parts of the angle • measure to see if the angles drawn are the same size as the one on the task card

Step 3	Discussion
Discuss student responses to the activity and look at the situations where they found each angle. Review the three critical features of an angle: arms, vertex and opening. Discuss the meaning of the word *angle*.	*What types of angles have you found? How can so few angle drawings show so many different things?* *How do we measure and compare angles?* *What is an angle?*

Extension

Find some 0-line angles. By performing the body turn at the beginning of this lesson without stretching arms out in front, a 0-line angle will be demonstrated. Look for other examples such as car wheels, a trundle wheel and a ballet dancer. Discuss how to describe such turns using angles, and identify the arms and vertex in each case.

Pentagram for lesson 5



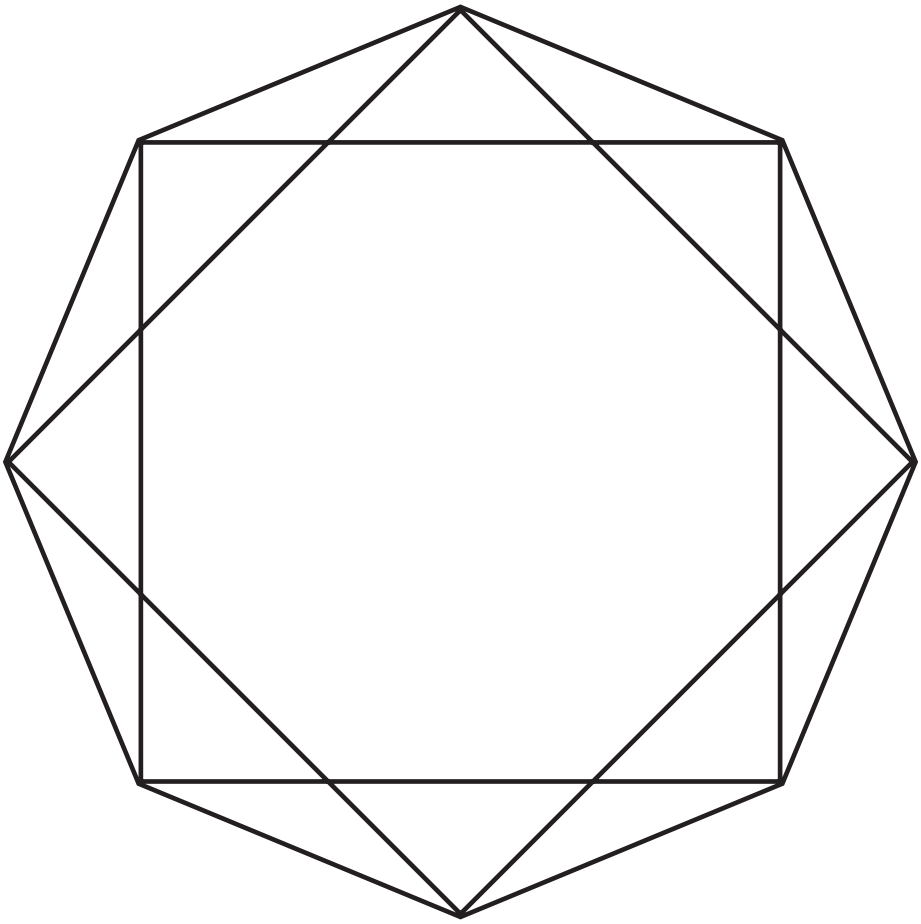
Finding angles in the pentagram

Find and label an acute angle and an obtuse angle.

Copy these angles in the space below, and label each one. Check that your angles are the correct size.

Use coloured pencils to mark the angles that are the same size. Count and record your total number of angles.

Octagon for lesson 5



Finding angles in the octagon

Find and label an acute angle, an obtuse angle and a right angle.

Copy these angles in the space below, and label each one. Check that your angles are the correct size.

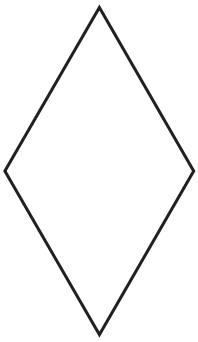
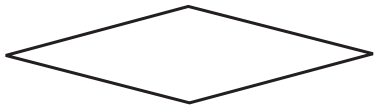
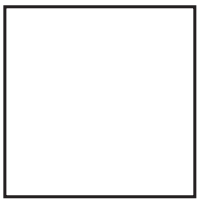
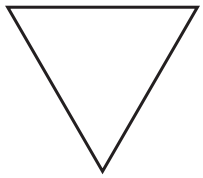
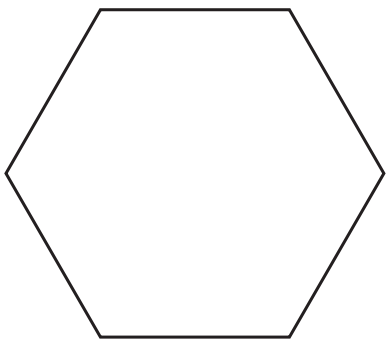
Use coloured pencils to mark the angles that are the same size. Count and record your total number of angles.

Windmill for lessons 6, 7 and 8

1. Make one copy on paper for each student.
2. Make two or three copies on light cardboard. Cut out the twelve segments to make one segment for each student.

Pattern blocks for lesson 7

How many windmill angles are there in each corner of these pattern blocks?

1. Blue block 	2. White block 
3. Orange block 	4. Green block 
5. Yellow block 	6. Red block 

Body angles for lesson 9

Raise one arm at your side, like this:

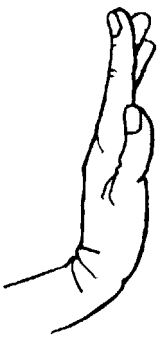
What angle sizes can you make?

Draw the smallest angle and the largest angle.

Make your hand flat and then make an angle at your wrist, like this:

What angle sizes can you make?

Draw the smallest angle and the largest angle.



Complete the drawing of a school student to make the following angles:

angle right arm raised = 3 windmill angles

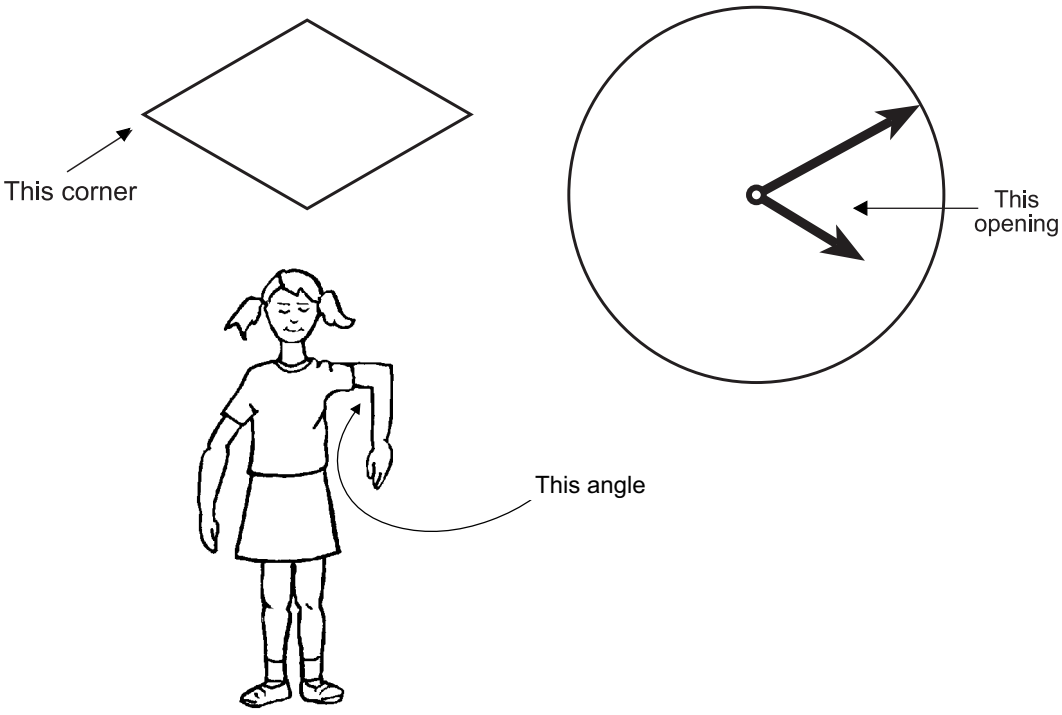
angle at right elbow = 2 windmill angles

angle left arm raised = 5 windmill angles

angle at left elbow = 4 windmill angles

Drawing two-line angles for lesson 10

Part 1. Each of these objects makes an angle. Draw the angles on each object.

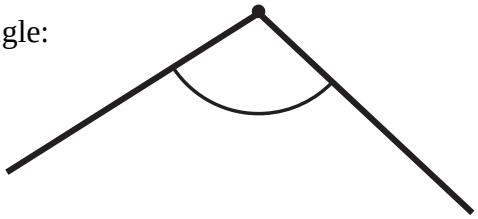


Part 2. Draw the three angles separately here:

Part 3. Find a way to check that the angles you drew in Part 2 are the same size as the angles you found in Part 1. Write how you measured the angles.

Drawing two-line angles for lesson 10

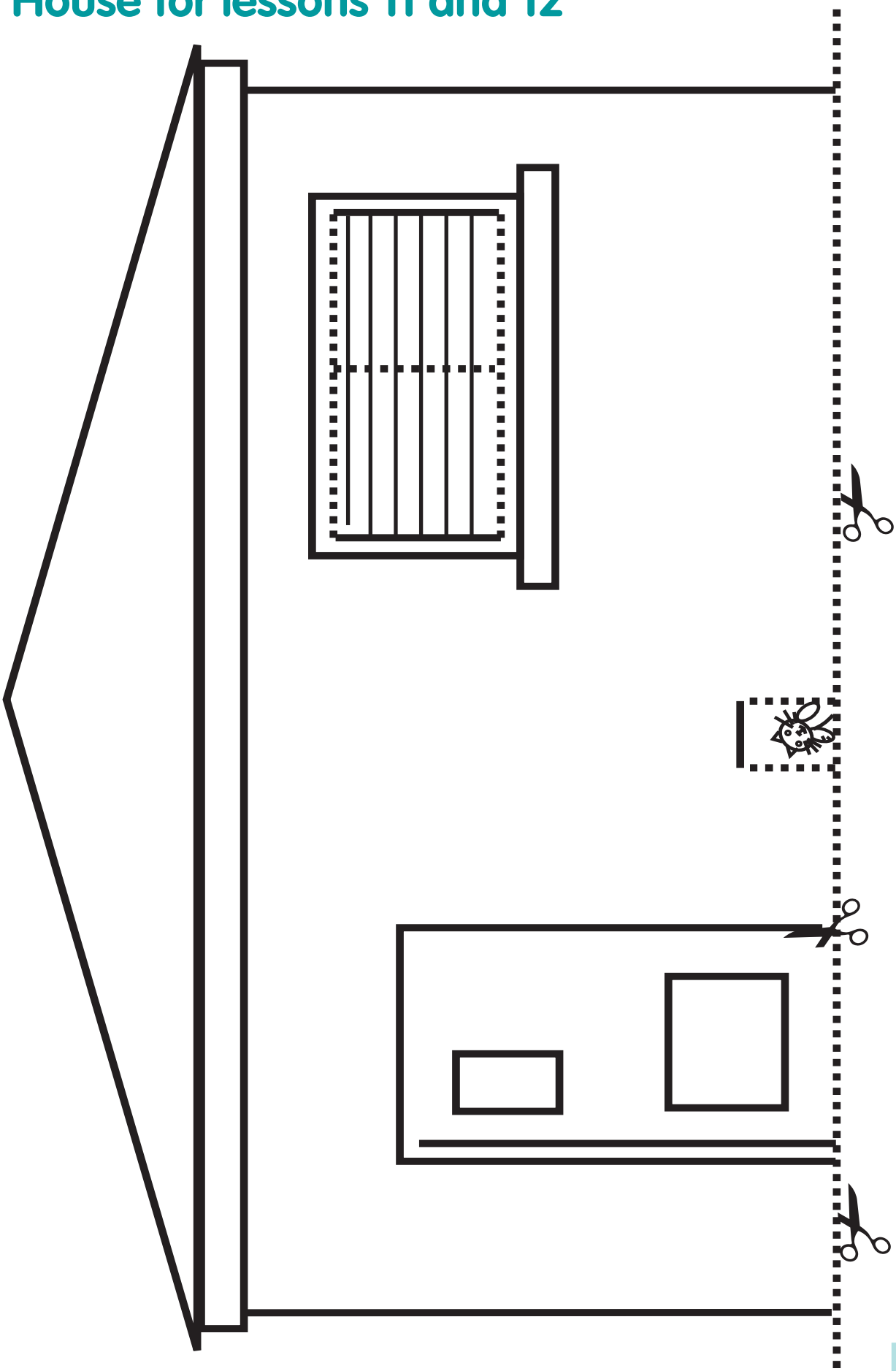
Part 4. Here is another angle:



Draw three *different* objects that make an angle this size:

Part 5. What is an angle?

House for lessons 11 and 12

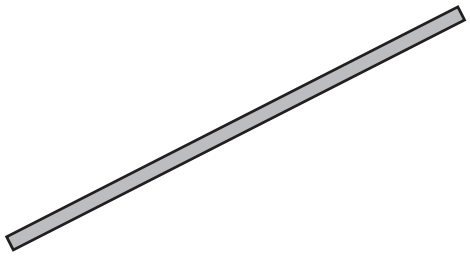
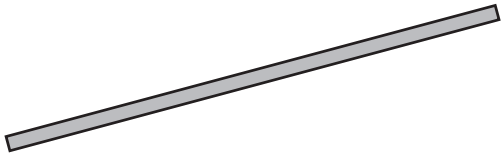
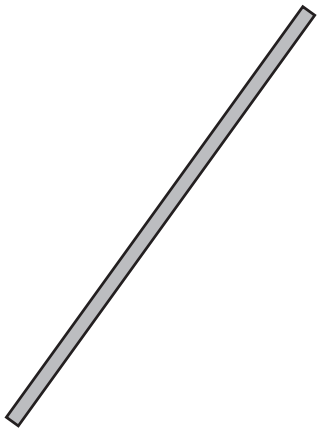
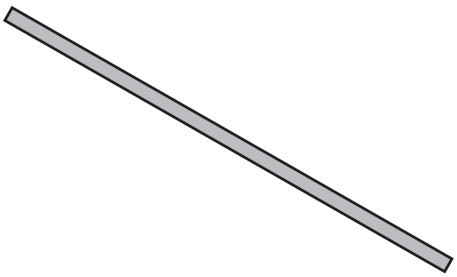
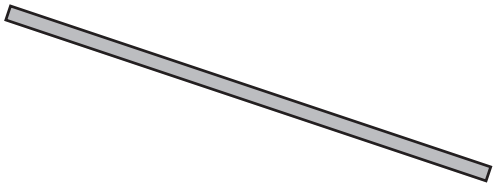


Slopes for lesson 13

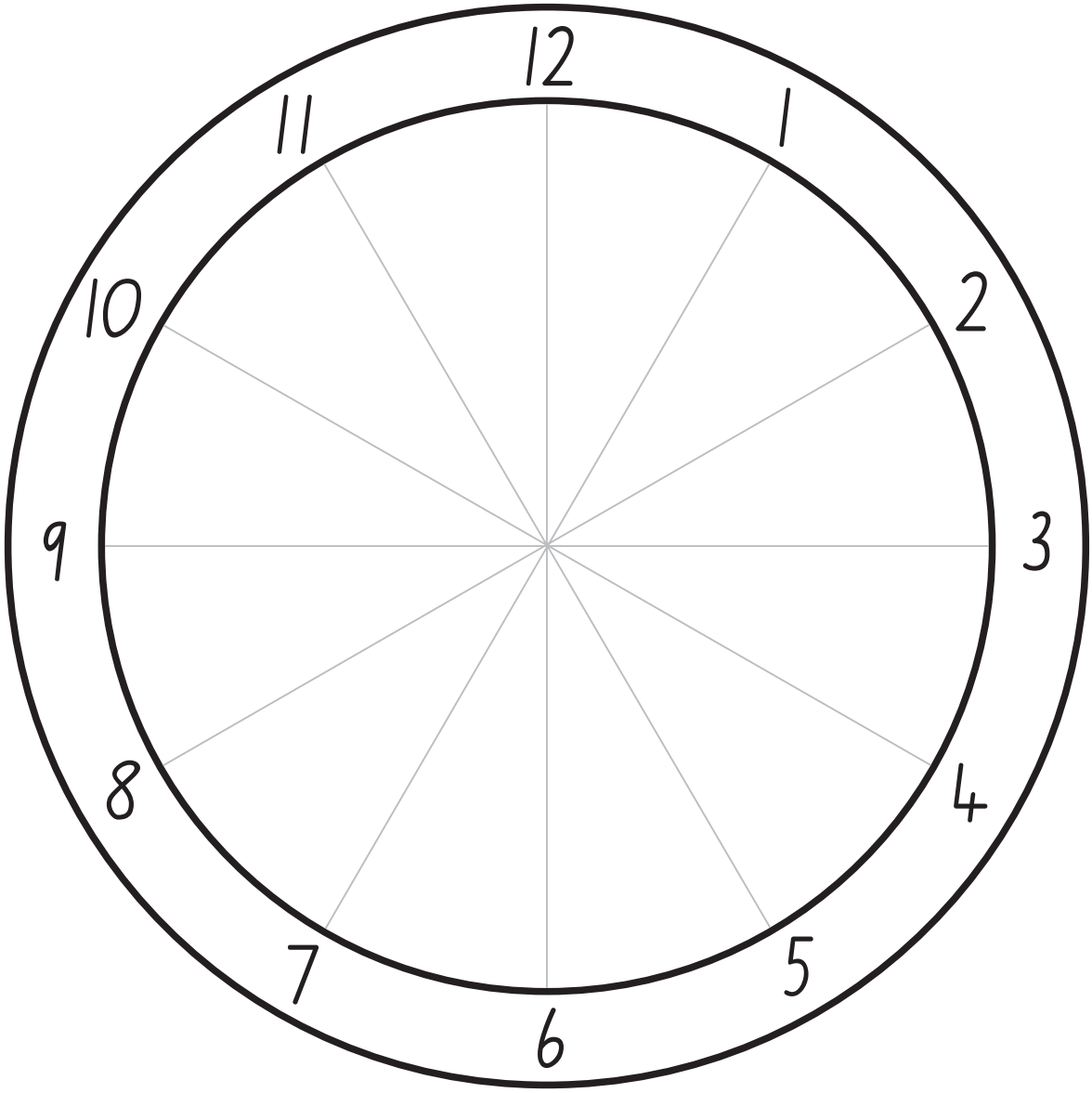
The drawing in the left-hand column shows a sloping ruler standing on a table.

Student A: draws the table in the left-hand column.

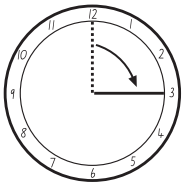
Student B: draws the angle of slope in the right-hand column.

Sloping ruler on table	Angle of slope
	
	
	
	
	

Clock face for lesson 14

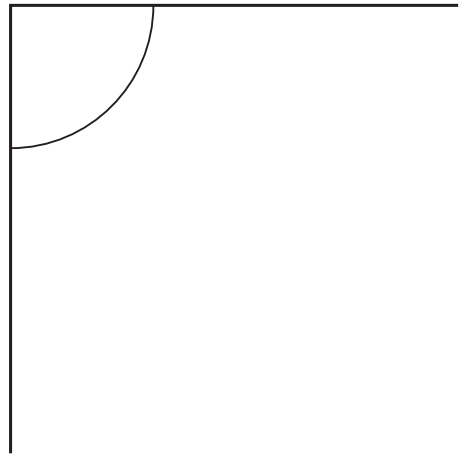


Clock angles for lesson 14

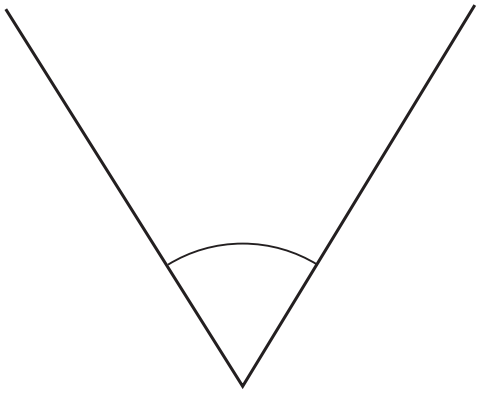
Start time	Finish time	How many hours?	How many windmill angles?	Draw the angle of turning of the hour hand and label as a right, obtuse, acute or straight angle.
12:00	3:00			
9:00	11:00			
3:00	6:00			
12:00	5:00			
5:00	10:00			
2:00	8:00			
1:00	4:00			

Task cards for lesson 15

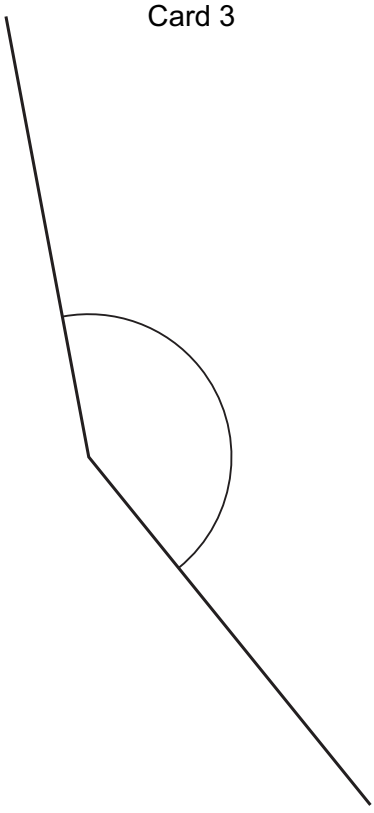
Card 1



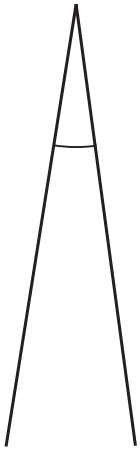
Card 2



Card 3



Card 4



Two-line angle tasks

These tasks are designed to assess students' current knowledge and understanding of angles. Teachers may find it helpful to use these questions with a small number of individual students before or after the implementation of a sequence of angles lessons. An analysis of students' responses may assist in planning an appropriate program of lessons.

Materials

Place on the table six pattern blocks (one of each shape), the windmill activity sheet from Lesson 7, a blank sheet of paper, and a drinking straw strengthened with a pipe cleaner.

Print one copy of the recording sheet for each student.

Procedure

First tell the student that you want to ask some questions about angles.

Indicate that any of the materials on the table may be used to answer the questions.

Show how the straw can be bent, and let the student investigate the other objects if they wish.

Tasks

1. Hold the red block so that an acute corner is pointing towards the student.

   Say to the student: *Show me an angle on the windmill that is the same as this corner.*

2. Put the red block down on the table and ask: *How can you check that it is the same angle?*

3. Hold the red block with the acute corner pointing towards the student and ask: *With your finger, show me the angle in this corner.* If the student indicates the point or points vaguely at the inside of the corner, ask: *Is that the **angle**?*

4. Show the student that you can make angles by bending your arm at the elbow and by opening a book (but do not indicate exactly where the angles are). Open the book to make an angle of about 120º. Ask the student: *Make the same angle as this book opening by bending your arm at the elbow*.

5. Ask the student: *Show me a pattern block where there is a right angle.*

6. Ask the student: *Put your arm down at your side. Now raise it to make an acute angle.*

7. Give the student the blank piece of paper and say: *Draw an obtuse angle.*

8. When the student is finished, sketch the student's response on the recording sheet and ask: *With your finger, show me the **arms** of the angle in your drawing*.

9. *With your finger, show me the **vertex** of the angle in your drawing.*

10. Remind the student that the tasks have involved talking about angles and ask: *Can you tell me exactly what an angle is*? Record the student's response verbatim.

Two-line angle recording sheet

If implementing these tasks before and after a teaching sequence, the recording sheet can be used twice for each student if a different-coloured pen is used on each occasion.

Student's name: ..... Date(s): .....

1. *Show me an angle on the windmill, which is the same as this (red) corner.*

Indicates angle on windmill:
Correct ☐ Incorrect ☐

2. *How can you check that it is the same angle?*

Physically matches both arms and the vertex on the two objects *or* matches both with third object:
Yes ☐ No ☐

3. *With your finger, show me the angle in this (red) corner.*

Correctly traces out the two arms and the vertex of the angle:
Immediately ☐ After prompt ☐
Not at all ☐

4. *Make the same angle as this book opening by bending your arm at the elbow.*

Bends arm to approximately 120°
Correct ☐ Incorrect ☐

5. *Show me a pattern block where there is a right angle.*

Correct ☐ Incorrect ☐

6. *Put your arm down at your side. Now raise it to make an acute angle.*

Correct ☐ Incorrect ☐

7. *Draw an obtuse angle.*
Sketch student's drawings:

First sketch	Second sketch

8. *With your finger, show me the arms of the angle in your drawing.*

Correct ☐ Incorrect ☐

9. *With your finger, show me the vertex of the angle in your drawing.*

Correct ☐ Incorrect ☐

10. *Can you tell me exactly what an angle is?* Student response:

Two-line and one-line angle tasks

These tasks are designed to assess students' current knowledge and understanding of angles. Teachers may find it helpful to use these questions with a small number of individual students before or after the implementation of a sequence of angles lessons. An analysis of students' responses may assist in planning an appropriate program of lessons.

Materials

Place on the table a set of pattern blocks, a copy of the windmill from lesson 7, a 30 cm ruler, a metre ruler, a wooden demonstration clock with the hands fixed together, a blank sheet of paper, and a drinking straw strengthened with a pipe cleaner.

Print one copy of the recording sheet for each student.

Procedure

First tell the student that you want to ask some questions about angles.

Indicate that any of the materials on the table may be used to answer the questions.

Show how the straw can be bent, and let the student investigate the other objects if they wish.

Tasks

1. Hold the red block so that an acute corner is pointing towards the student.

   Say to the student: *Bend the straw to make an angle the same as this corner.*
2. Put the red block down on the table and ask: *How can you check that it is the same angle?*
3. Hold the red block with the acute corner pointing towards the student and say: *With your finger, show me the angle in this corner.*
4. Set the clock with both hands on 2 and ask the student: *Move the hands of the clock to make a right angle. (*Both clockwise and anticlockwise angles are acceptable.)
5. Place one end of the metre ruler on the desk and raise the other end to a slope of about 45º. Say *Can you hold this short ruler so that the angle of slope is the same as the long ruler?*
6. Give the student the blank piece of paper and say: *Draw an obtuse angle. How do you know it is an obtuse angle?*
7. When the student is finished, sketch the student's response on the recording sheet and ask: *With your finger, show me the* ***arms*** *of the angle in your drawing.*
8. *With your finger, show me the* ***vertex*** *of the angle in your drawing.*
9. *Show me an acute angle on one of the pattern blocks.*
10. Remind the student that the tasks have involved talking about angles and ask: *Can you tell me exactly what an angle is*? Record the student's response verbatim.

Two-line and one-line angle recording sheet

If implementing these tasks before and after a teaching sequence, the recording sheet can be used twice for each student if a different-coloured pen is used on each occasion.

Student's name: ..... Date(s): .....

1. *Bend the straw to make the same angle as this (red) corner.*

   Bend straw to approximately 60º:
   Correct ☐ Incorrect ☐

2. *How can you check that it is the same angle?*

   Physically matches both arms and the vertex on the two objects:
   Correctly ☐ Incorrectly ☐
   Not at all ☐

3. *With your finger, show me the angle in this (red) corner.*

   Correctly traces out the two arms and the vertex of the angle:
   Immediately ☐ After prompt ☐
   Not at all ☐

4. *Move the hands of the clock to make a right angle (from 2 o'clock).*

   Correct ☐ Incorrect ☐

5. *Can your hold this short ruler so that the angle of slope is the same as the long ruler?*

   Correct ☐ Incorrect ☐

6. *Draw an obtuse angle. How do you know it is an obtuse angle?*
   Sketch student's drawings and write description

   Correct ☐ Incorrect ☐

First sketch	Second sketch

7. *With your finger, show me the arms of the angle in your drawing.*

   Correct ☐ Incorrect ☐

8. *With your finger, show me the vertex of the angle in your drawing.*

   Correct ☐ Incorrect ☐

9. *Show me an acute angle on one of the pattern blocks.*

   Correct ☐ Incorrect ☐

10. *Can you tell me exactly what an angle is?* Student response:

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